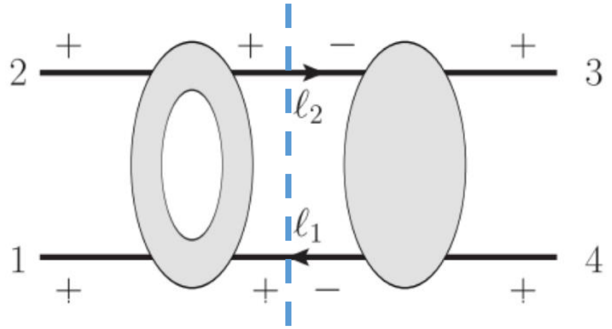
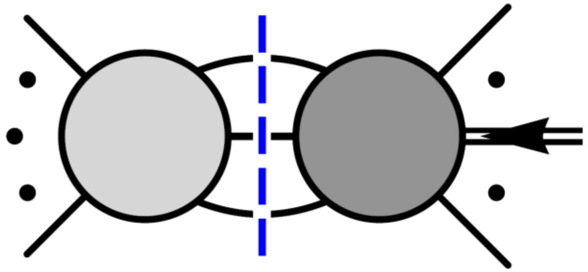
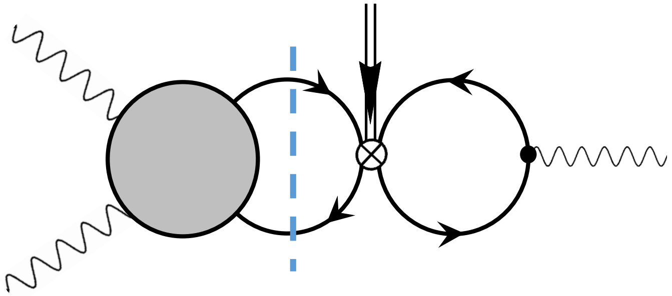






Anomalous Dimensions from On-Shell Methods

Based on 1910.05831 and work in progress

Eric Sawyer

with Julio Parra-Martinez and Zvi Bern

UCLA The Mani L. Bhaumik Institute for Theoretical Physics

Importance of Renormalization

Many major questions involving the UV behavior of field theories

- UV divergence in N=8 SUGRA

  (Bern et al. 1201.5366; 1804.09311)

- Renormalization group flow of higher dimension operators

  (eg. Alonso, Jenkins, Manohar, Trott 1312.2014)

Traditional ways of calculating:

- Calculate entire amplitude, extract UV dependence

  (e.g. Brandhuber, Kostacinska, Penante, Travaglinia 1804.05703)

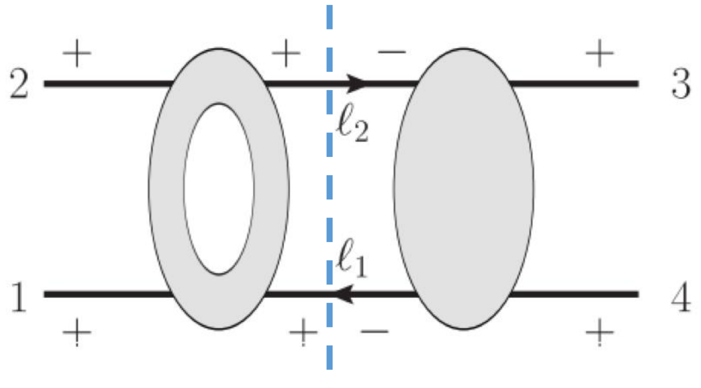
- Off shell methods

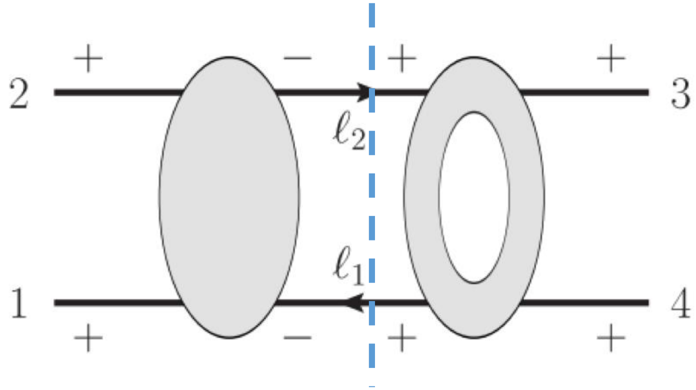
Borrowing Ideas used in Gravity

Two loop UV 1/ε pole is affected by evanescent effects in Einstein gravity

Renormalization scale dependence captured by cuts in kinematic variables

$$\left(\frac{209}{24}\right)\frac{1}{\varepsilon} \qquad \text{vs.} \qquad -\frac{1}{4}\log\mu$$





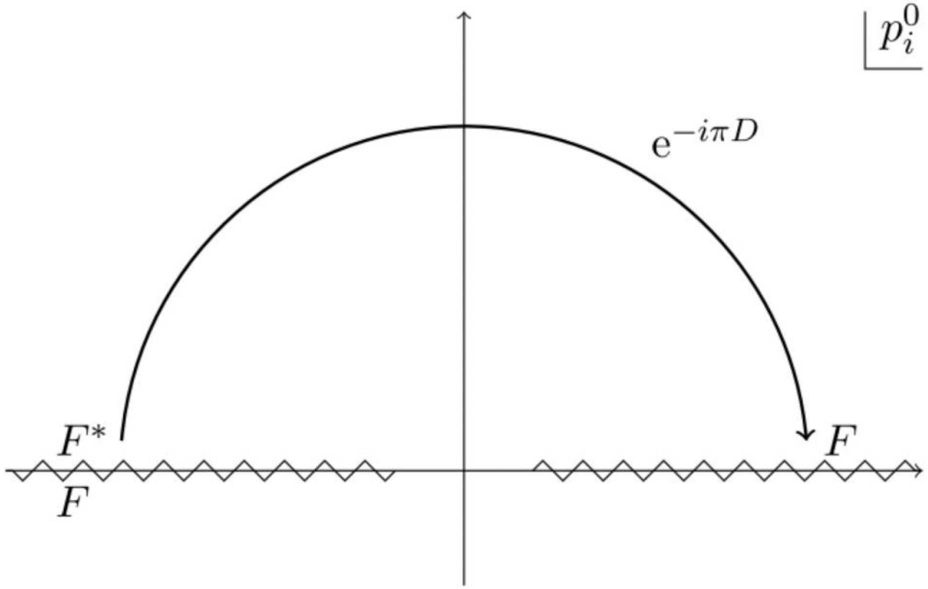
Renormalization scale dependence determined through on shell unitarity cuts

(Bern, Cheung, Chi, Davies, Dixon, Nohle 1507.06118; Bern, Chi, Dixon, Edison 1701.02422)

Form Factors and Renormalization

From Caron-Huot and Wilhelm (1607.06448)

$$F_j[p_1 ... p_n; q; \mu] = \langle p_1 ... p_n | \mathcal{O}_j(q) | 0 \rangle$$



Unitarity: $F = SF^*$

Analyticity: $F = e^{-i\pi D} F^*$

$$\boxed{e^{-i\pi D} F^* = SF^*}$$

$$D \equiv \sum_i p_i^\nu \frac{\partial}{\partial p_i^\nu}$$

"the dilatation operator is minus the phase of the S-matrix, divided by π"

Form Factors and Renormalization

Unitarity: $$F = SF^*$$

Analyticity: $$F = e^{-i\pi D} F^*$$

$$\boxed{e^{-i\pi D} F^* = SF^*}$$

Renormalization scale related to dilatation operator:

$$D \equiv \sum_i p_i^\nu \frac{\partial}{\partial p_i^\nu} \simeq -\mu\partial_\mu$$

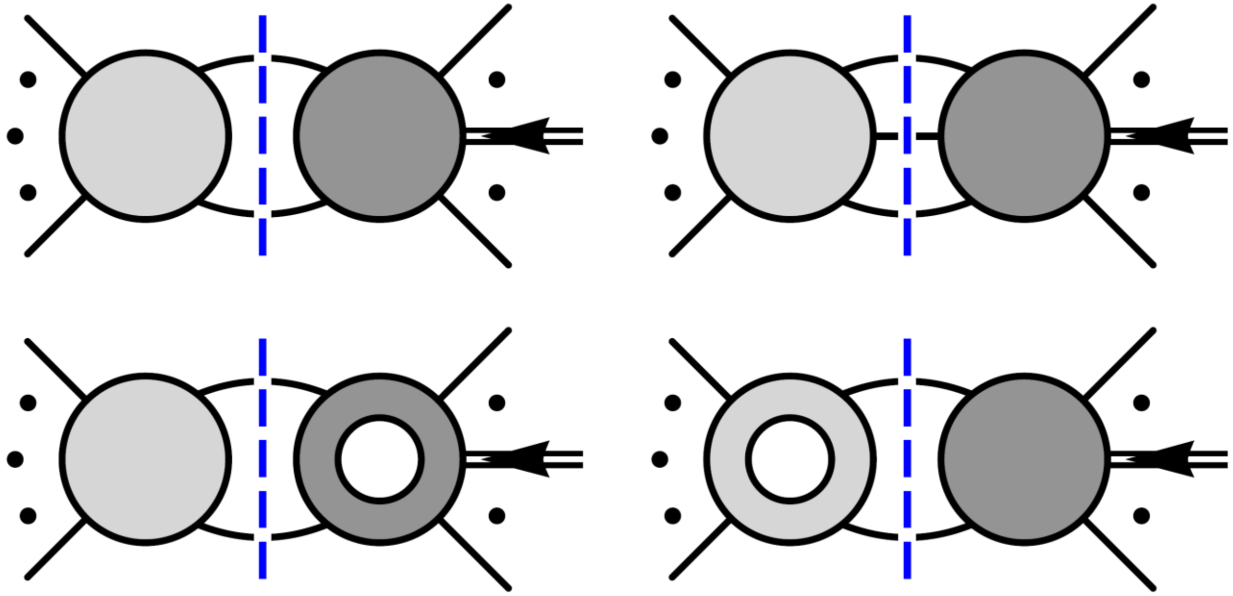
$$DF = \left(\gamma_{\mathcal{O}} - \gamma_{\mathrm{IR}} + \beta(g^2)\frac{\partial}{\partial g^2}\right) F$$

(Caron-Huot and Wilhelm 1607.06448)

Form Factors and Renormalization: Perturbative Expansion

One loop anomalous dimensions:

$$\left(\gamma_{ij}^{\mathrm{UV}} - \gamma_{ij}^{\mathrm{IR}} + \beta(g)\partial_g\right)^{(1)} \langle p_1, \ldots, p_n|\mathcal{O}_i|0\rangle^{(0)}$$
$$= -\frac{1}{\pi}\langle p_1, \ldots, p_n|\mathcal{M} \otimes \mathcal{O}_j|0\rangle$$



(Caron-Huot and Wilhelm 1607.06448)

Form Factors and Renormalization: Perturbative Expansion

$$\left(\gamma_{ij}^{\mathrm{UV}} - \gamma_{ij}^{\mathrm{IR}} + \beta(g)\partial_g\right)^{(1)} \langle p_1, \ldots, p_n | \mathcal{O}_i | 0 \rangle^{(0)}$$
$$= -\frac{1}{\pi} \langle p_1, \ldots, p_n | \mathcal{M} \otimes \mathcal{O}_j | 0 \rangle$$

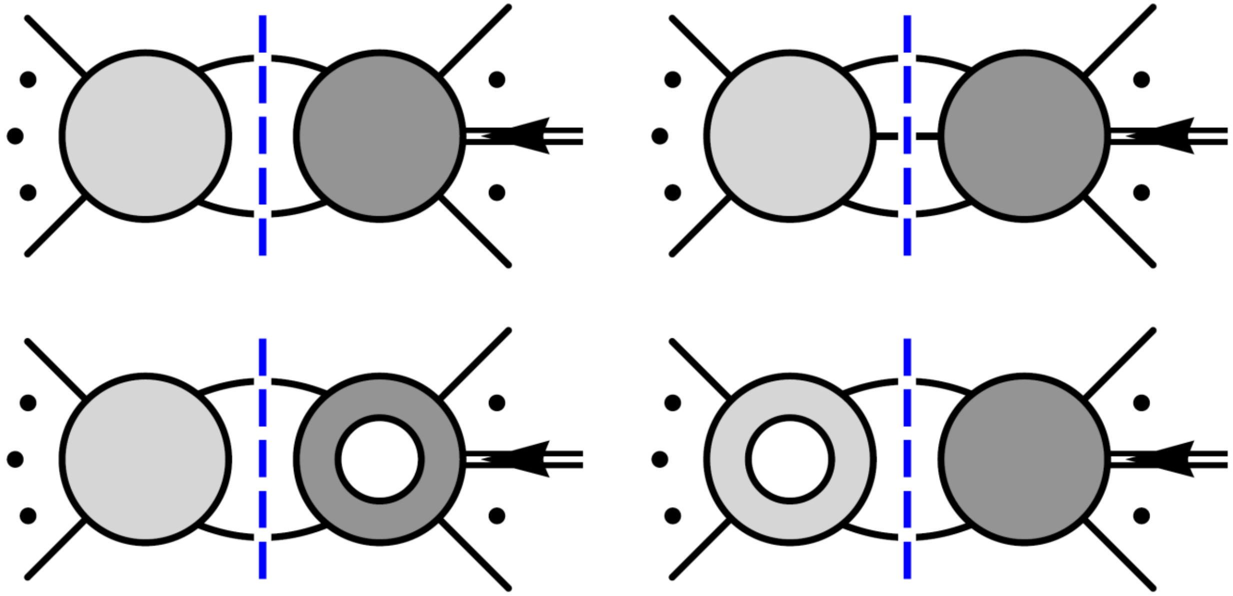
Minimal form factors

$\cancel{\beta(g)\partial_g}$

First chance for operator mixing

$\cancel{\gamma_{ij}^{\mathrm{IR}}}$

$$\left(\gamma_{sl}^{\mathrm{UV}}\right)^{(L)} {}_s\langle p_1, \ldots, p_n | \mathcal{O}_s | 0 \rangle^{(0)}$$
$$= -\frac{1}{\pi} {}_s\langle p_1, \ldots, p_n | \mathcal{M} \otimes \mathcal{O}_l | 0 \rangle$$



(Caron-Huot and Wilhelm 1607.06448)

Standard Model Effective Field Theory

SM EFT: more systematic way to explore physics beyond the Standard Model than model building

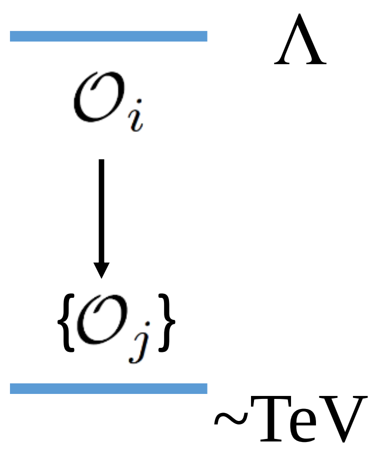
- No need to worry about details of high-energy completion

$$\mathcal{L} = \mathcal{L}_{\text{SM}}^{(4)} + \frac{1}{\Lambda^2} \sum_k C_i^{(6)} \mathcal{O}_i^{(6)}$$

Operators $\mathcal{O}_i^{(6)}$ built out of Standard Model fields

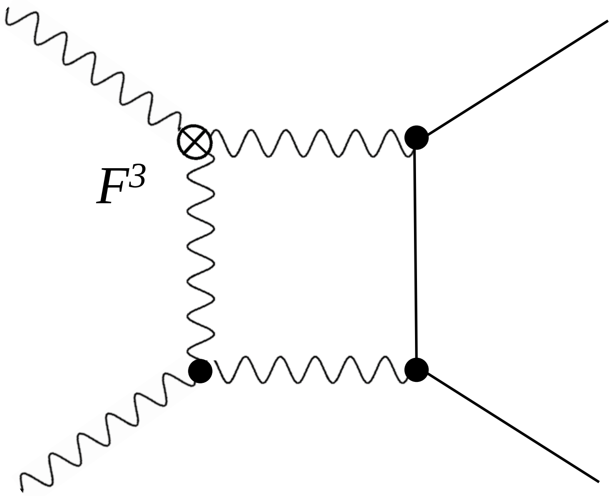
Mixing of operators can produce variety of effects at LHC scale from a single operator at scale Λ

$$16\pi^2 \frac{\partial c_i}{\partial \log \mu} = \gamma_{ij}^{\text{UV}} c_j$$

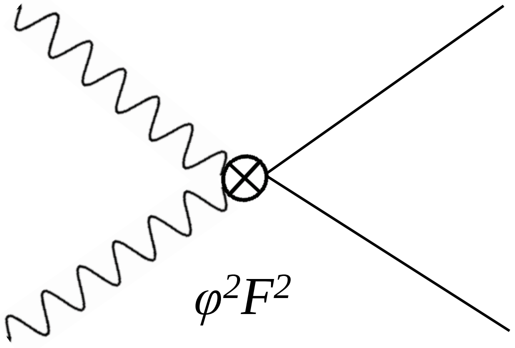


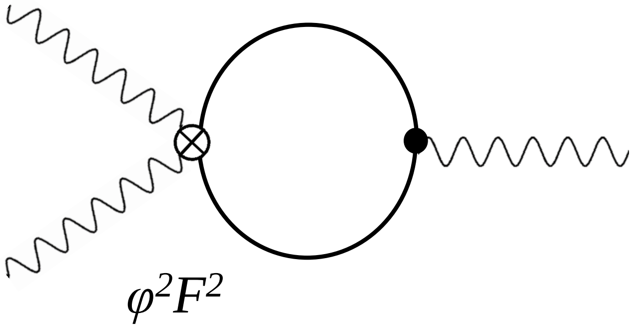
Non-Renormalization

Unexpected zeros in anomalous dimension matrix
(Alonso, Jenkins, Manohar, Trott 1312.2014)



$$\Longrightarrow \quad \frac{\gamma_{ij}}{\varepsilon}$$





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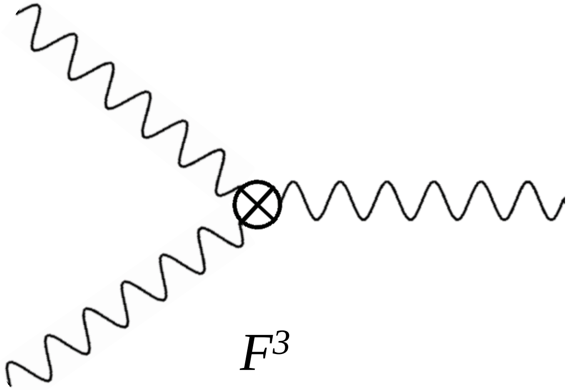


Non-Renormalization

Unexpected zeros in anomalous dimension matrix (Alonso, Jenkins, Manohar, Trott 1312.2014)

Explained using helicity selection rules (Cheung and Shen 1505.01844)

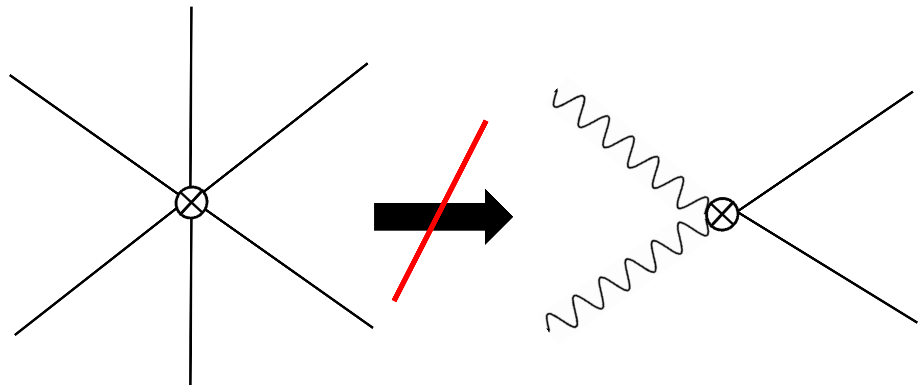
		F^3	$F^2\phi^2$	$F\psi^2\phi$	ψ^4	$\psi^2\phi^3$	$\bar{F}^3$	$\bar{F}^2\phi^2$	$\bar{F}\bar{\psi}^2\phi$	$\bar{\psi}^4$	$\bar{\psi}^2\phi^3$	$\bar{\psi}^2\psi^2$	$\bar{\psi}\psi\phi^2 D$	$\phi^4 D^2$	ϕ^6
	$(w,\bar{w})$	$(0,6)$	$(2,6)$	$(2,6)$	$(2,6)$	$(4,6)$	$(6,0)$	$(6,2)$	$(6,2)$	$(6,2)$	$(6,4)$	$(4,4)$	$(4,4)$	$(4,4)$	$(6,6)$
F^3	$(0,6)$			$\times$	$\times$	$\times$			$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
$F^2\phi^2$	$(2,6)$				$\times$	$\times$				$\times$	$\times$	$\times$			$\times$
$F\psi^2\phi$	$(2,6)$									$\times$				$\times$	$\times$
ψ^4	$(2,6)$	$\times$	$\times$			$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	y^2		$\times$	$\times$
$\psi^2\phi^3$	$(4,6)$	$\times^*$									y^2				$\times$
$\bar{F}^3$	$(6,0)$			$\times$	$\times$	$\times$			$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
$\bar{F}^2\phi^2$	$(6,2)$				$\times$	$\times$				$\times$	$\times$	$\times$			$\times$
$\bar{F}\bar{\psi}^2\phi$	$(6,2)$				$\times$									$\times$	$\times$
$\bar{\psi}^4$	$(6,2)$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$			$\times$	$\bar{y}^2$		$\times$	$\times$
$\bar{\psi}^2\phi^3$	$(6,4)$					$\bar{y}^2$	$\times^*$								$\times$
$\bar{\psi}^2\psi^2$	$(4,4)$		$\times$		$\bar{y}^2$	$\times$		$\times$		y^2	$\times$			$\times$	$\times$
$\bar{\psi}\psi\phi^2 D$	$(4,4)$														$\times$
$\phi^4 D^2$	$(4,4)$				$\times$					$\times$		$\times$			$\times$
ϕ^6	$(6,6)$	$\times^*$		$\times$	$\times$		$\times^*$		$\times$	$\times$		$\times$			

(from Cheung and Shen 1505.01844)

New Non-Renormalization Theorem

1. *Consider operators* $\mathcal{O}_s$ *and* $\mathcal{O}_l$ *such that* $l(\mathcal{O}_l) > l(\mathcal{O}_s)$. $\mathcal{O}_l$ *can renormalize* $\mathcal{O}_s$ *at* L *loops only if* $L > l(\mathcal{O}_l) - l(\mathcal{O}_s)$.

   $l(\mathcal{O}_l)$: number of field insertions in $\mathcal{O}_l$



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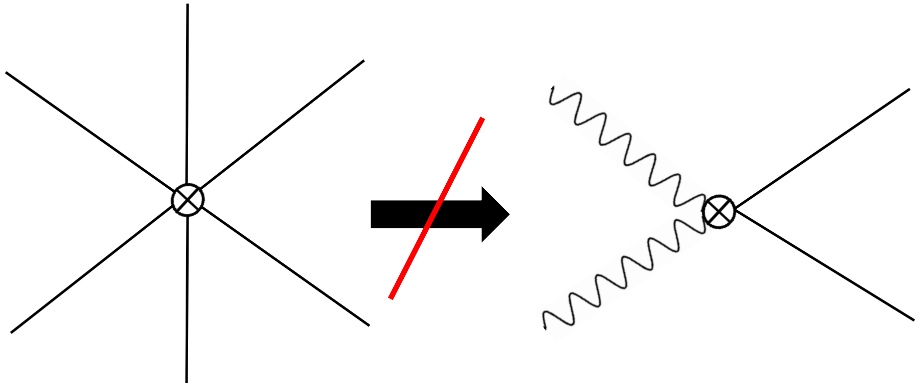
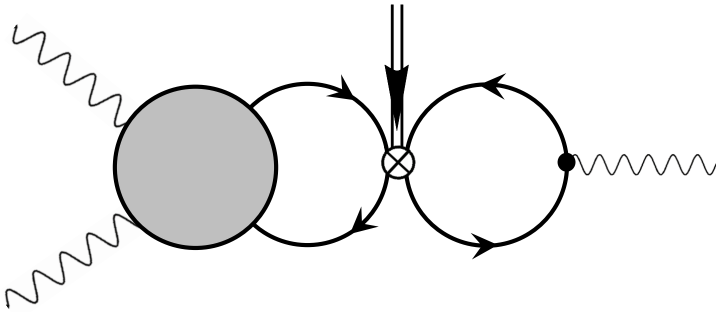
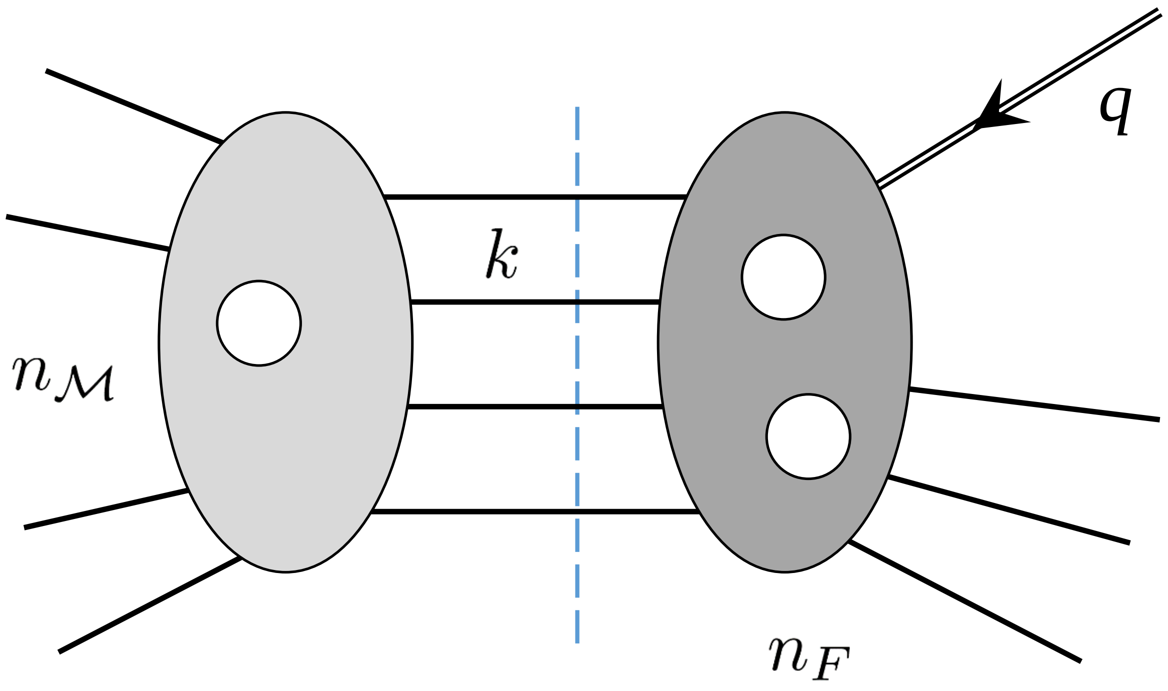


$l(\mathcal{O}_l)$: number of field insertions in $\mathcal{O}_l$

2. *If at any given loop order, the only diagrams for a matrix element with the external particle content of* $\mathcal{O}_s$ *but an insertion of* $\mathcal{O}_l$ *involve scaleless bubble integrals, there is no renormalization of* $\mathcal{O}_s$ *by* $\mathcal{O}_l$.



Longer operators often cannot renormalize shorter operators, even when diagrams are available



L : total loop number

L_F : loops in form factor

$l(\mathcal{O}_l)$: number of field insertions in $\mathcal{O}_l$

Minimal form factor

$$n_{\mathcal{M}} + n_F - 2k = l(\mathcal{O}_s)$$

Required for nonzero result:

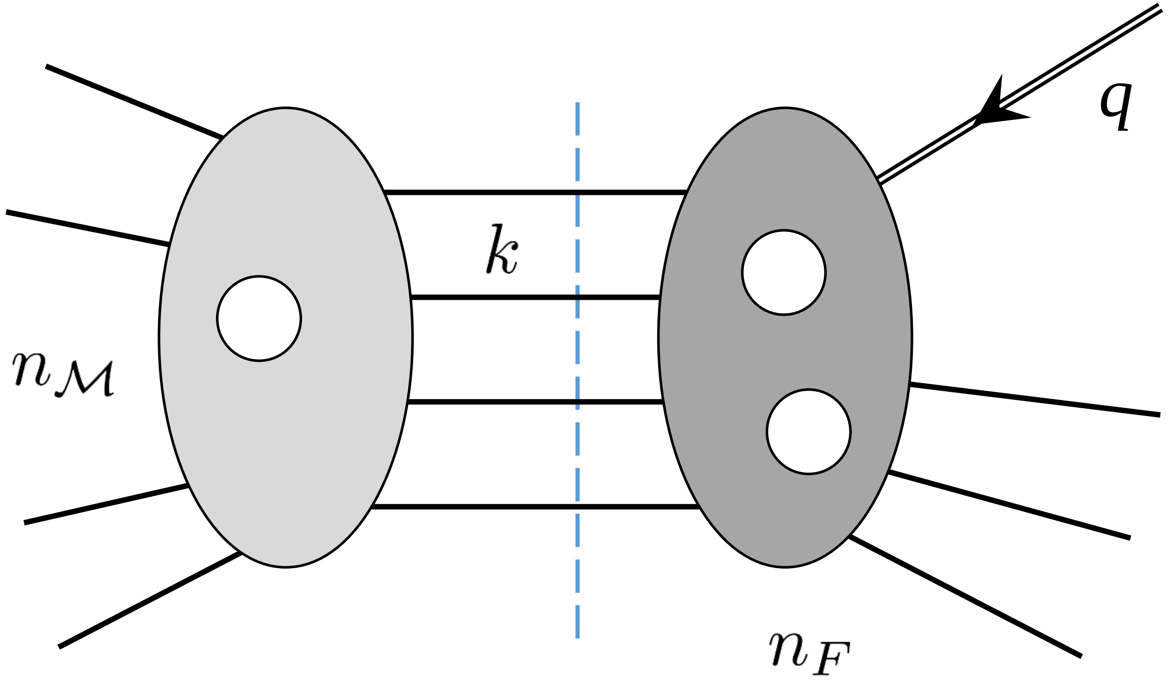
- ≥2 external legs on the left

$$n_{\mathcal{M}} \geq k + 2$$

- No scaleless bubbles on the right

$$n_F \geq l(\mathcal{O}_l) - (L_F - 1) - \delta_{L_F,0}$$

Longer operators often cannot renormalize shorter operators, even when diagrams are available



L : total loop number

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$$n_{\mathcal{M}} + n_F - 2k = l(\mathcal{O}_s)$$

$$n_{\mathcal{M}} \geq k + 2$$

$$n_F \geq l(\mathcal{O}_l) - (L_F - 1) - \delta_{L_F,0}$$

$$L_F \leq L - (k - 1)$$

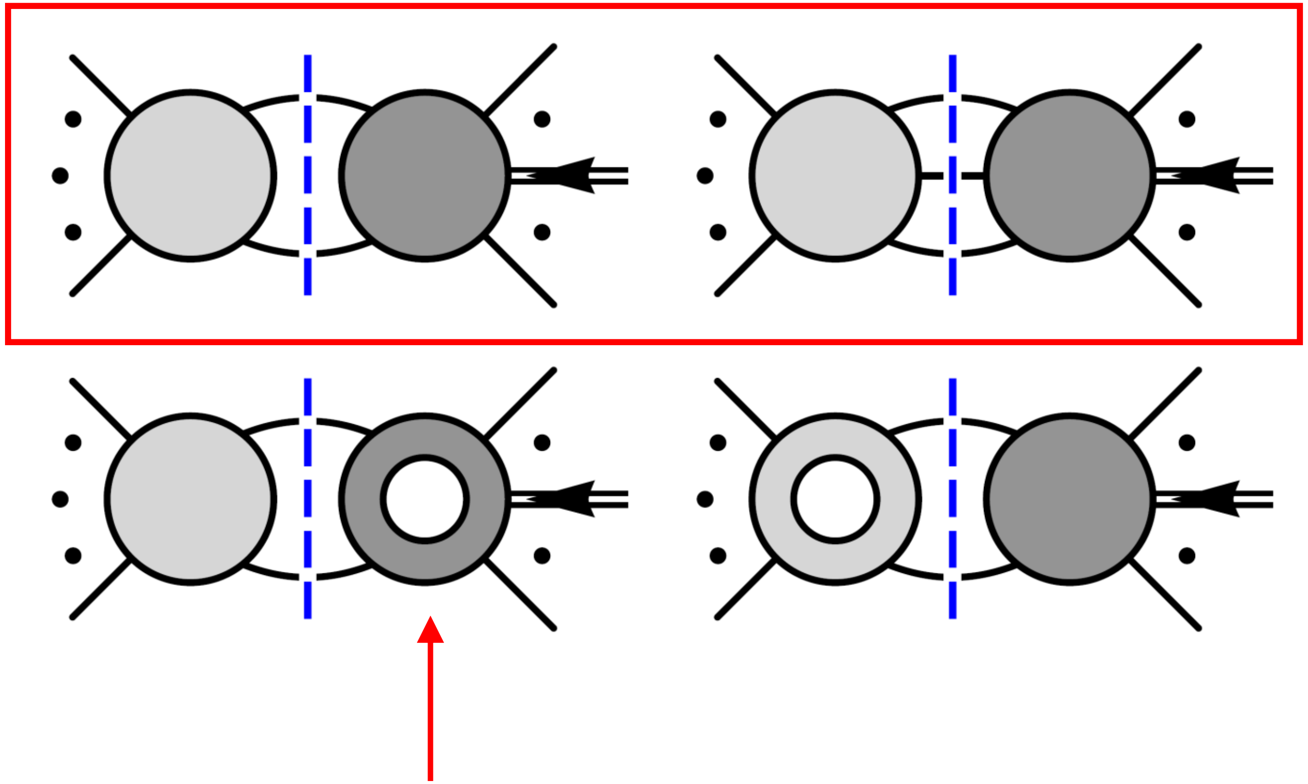
$$l(\mathcal{O}_l) - L + 2 - \delta_{L_F,0} \leq l(\mathcal{O}_s)$$

$$\boxed{L > l(\mathcal{O}_l) - l(\mathcal{O}_s)}$$

First order of renormalization can often be calculated using only 4-d methods

$$L > l(\mathcal{O}_l) - l(\mathcal{O}_s)$$

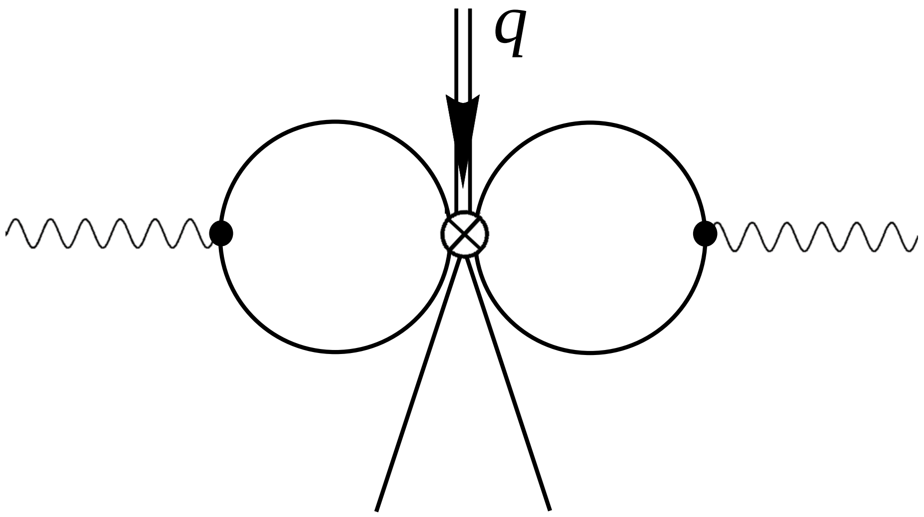
$$L > \ell(\mathcal{O}_\ell) - \ell(\mathcal{O}_s) + (1 - \delta_{L_F,0})$$



Requires d-dimensional information

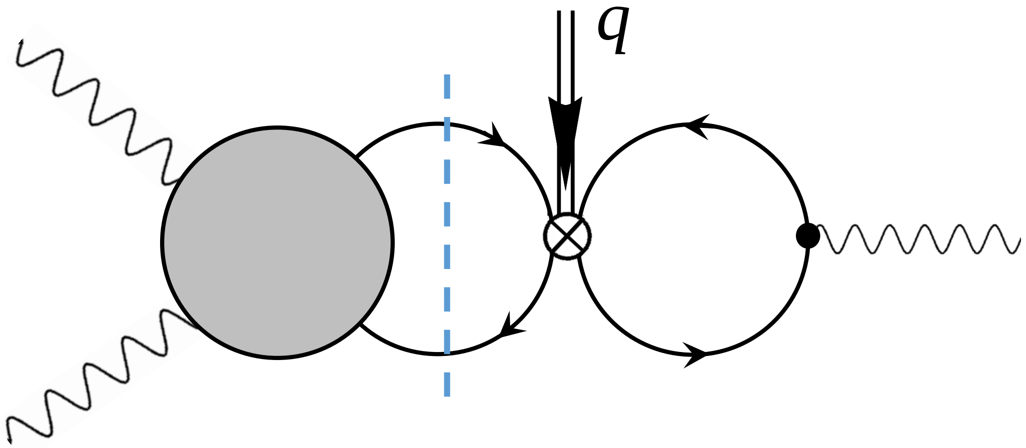
Two-loop Examples

Renormalization of $\phi^2 F^2$ by ϕ^6



$$\ell(\phi^6) - \ell(\phi^2 F^2) = 2 \not> L$$

Renormalization of F^3 by ψ^4



Scaleless bubble evaluates to zero

Results for Dimension 5-7 Operators

d=5

	$F^2\phi$	$F\psi^2$	$\phi^2\psi^2$	ϕ^5
$F^2\phi$			(2)	$\times_2$
$F\psi^2$			$\times_1$	$\times_3$
$\phi^2\psi^2$				(2)
ϕ^5				

d=6

	F^3	ϕ^2F^2	$F\phi\psi^2$	$D^2\phi^4$	$D\phi^2\psi^2$	ψ^4	$\phi^3\psi^2$	ϕ^6
F^3		$\times_1$	(2)	$\times_2$	$\times_2$	$\times_2$	$\times_3$	$\times_3$
ϕ^2F^2							(2)	$\times_2$
$F\phi\psi^2$							$\times_1$	$\times_3$
$D^2\phi^4$							$\times_1$	$\times_2$
$D\phi^2\psi^2$							$\times_1$	(3)
ψ^4							(2)	(4)
$\phi^3\psi^2$								(2)
ϕ^6								

Zero by helicity rules of Cheung and Shen

$\times_1$ Overlap of zeros of Cheung and Shen and our work

$\times_2$ New anomalous dimension zeros

(Bern, Parra-Martinez, ES 1910.05831)

Results for Dimension 5-7 Operators

Zero by helicity rules of Cheung and Shen

$\times_1$ Overlap of zeros of Cheung and Shen and our work

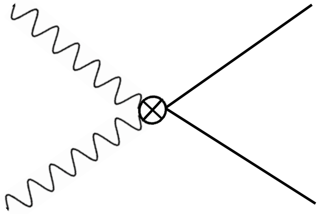
$\times_2$ New anomalous dimension zeros

d=7

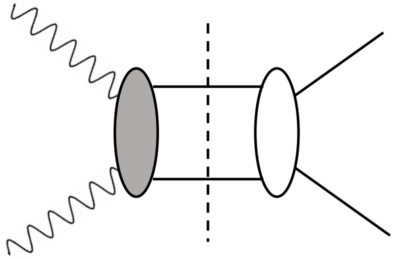
	ϕ^3F^2	$D^2\phi^5$	$D\phi^3\psi^2$	$\phi\psi^4$	$F\phi^2\psi^2$	$\phi^4\psi^2$	ϕ^7
$F^3\phi$	$\times_1$	$\times_2$	$\times_2$	$\times_2$	(2)	$\times_3$	$\times_3$
$D^2F\phi^3$	$\times_1$	$\times_1$	$\times_1$	$\times_2$	$\times_1$	$\times_2$	$\times_3$
$DF\phi\psi^2$	(2)	$\times_2$	$\times_1$	$\times_1$	$\times_1$	$\times_2$	$\times_4$
$F^2\psi^2$	(2)	(3)	(2)	(2)	$\times_1$	$\times_2$	$\times_4$
$D^2\phi^2\psi^2$	(2)	(2)	$\times_1$	$\times_1$	$\times_1$	$\times_2$	(4)
$D\psi^4$	(3)	(3)	(2)	$\times_1$	(2)	(3)	(5)
ϕ^3F^2						(2)	$\times_2$
$D^2\phi^5$						$\times_1$	$\times_2$
$D\phi^3\psi^2$						$\times_1$	(3)
$\phi\psi^4$						(2)	(4)
$F\phi^2\psi^2$						$\times_1$	$\times_3$
$\phi^4\psi^2$							(2)

(Bern, Parra-Martinez, ES 1910.05831)

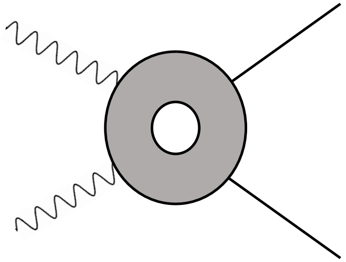
Non-zero Entries Require 1-loop Amplitudes



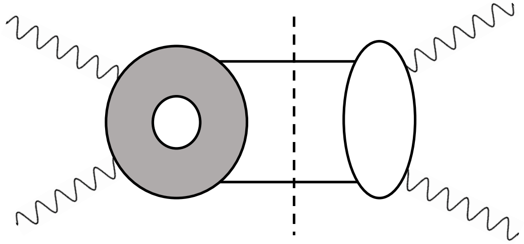
1. Write down tree amplitudes



2. Construct cuts between trees



3. Integrate and merge cuts
    $\Rightarrow$ d-dimensional 1-loop amplitudes



4. Construct two-loop cuts

5. Extract 2-loop anomalous dimension

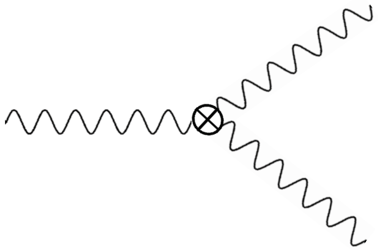
$$\gamma_{ij}^{(2)}$$

The Dimension 6 Operators

$$\mathcal{L}^{(4)} = -\frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu} + D_\mu\phi\, D^\mu\overline{\phi} - \lambda\,(\phi\overline{\phi})^2 + i\,\overline{\psi}\not{D}\psi$$

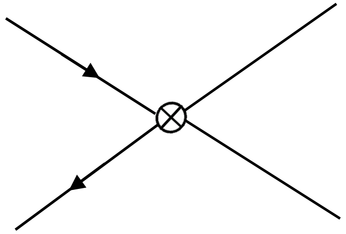
$$\mathcal{L} = \mathcal{L}^{(4)} + \frac{1}{\Lambda^2}\sum_k C_i^{(6)}\mathcal{O}_i^{(6)}$$

$$f^{ABC}F^A_{\mu\nu}F^B_{\nu\rho}F^C_{\rho\mu}$$



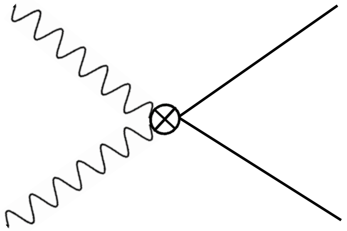
$$(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)(\overline{\psi}\gamma^\mu\psi)$$

$$(\phi^\dagger i\overleftrightarrow{D}^A_\mu\phi)(\overline{\psi}T^A\gamma^\mu\psi)$$

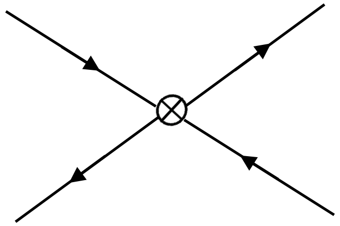


$$(\phi^\dagger\phi)F^A_{\mu\nu}F^A_{\mu\nu}$$

$$d^{ABC}(\phi^\dagger T^A\phi)F^B_{\mu\nu}F^C_{\mu\nu}$$

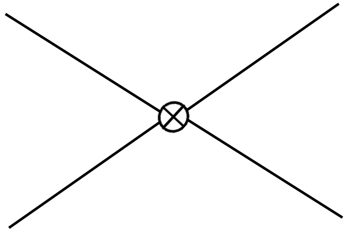


$$(\overline{\psi}\gamma_\mu\psi)(\overline{\psi}\gamma^\mu\psi)$$

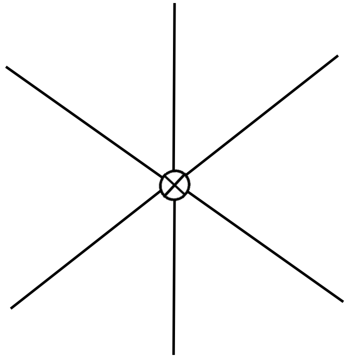


$$(\phi^\dagger D^\mu\phi)^*(\phi^\dagger D_\mu\phi)$$

$$(\phi^\dagger\phi)\Box(\phi^\dagger\phi)$$



$$(\phi^\dagger\phi)^3$$



Results

- All four point SM EFT amplitudes with dimension 6 operators
- Verifying the anomalous dimension calculations using on-shell techniques
- Non-zero rational terms everywhere
  $\Rightarrow$ 2-loop anomalous dimensions likely generally non-zero

One loop anomalous dimensions γ_{ij}

$\emptyset$ indicates there are no contributing one-loop diagrams.

	F^3	$\phi^2 F^2$	d^{abc}	$D^2\phi^4$	$\square$	$\phi^2\psi^2 D(1)$	$\phi^2\psi^2 D(2)$	4ψ
F^3		0	0	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\phi^2 F^2$				0	0	0	0	$\emptyset$
d^{abc}				0	0	0	0	$\emptyset$
$D^2\phi^4$	0	0	0			0		$\emptyset$
$\square$	0	0	0			0		$\emptyset$
$\phi^2\psi^2 D(1)$	0	0	0	0	0	0	0	0
$\phi^2\psi^2 D(2)$	0	0	0	0		0		
4ψ	0	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	0		

$$A^{(1)}_{F^3}(ssss)_1 = \frac{1}{12} g^3(-N(2s+t)+s+2t)$$

$$A^{(1)}_{\phi^2 F^2}(ssss)_1 = \frac{g^2\left(N^2-1\right)s}{N}$$

$$\dot{C}_\square = \left(\frac{g^2\left(18N^2 - 2N(N_s+9) + N_s - 9\right)}{3N} + 8\lambda(N+1)\right) C_\square + \frac{(N-2)\left(3g^2(N+1)+4\lambda N\right)}{2N} C_{D^2\phi^4}$$

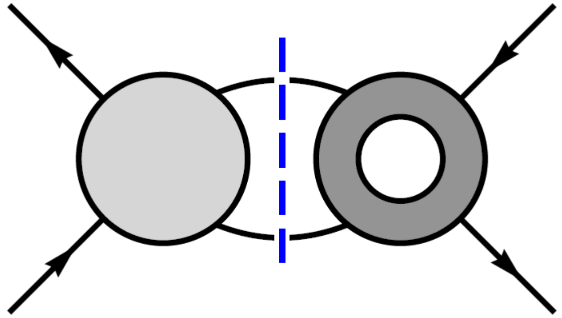
(Bern, Parra-Martinez, ES 2001.XXXXX)

Two-loop Anomalous Dimensions

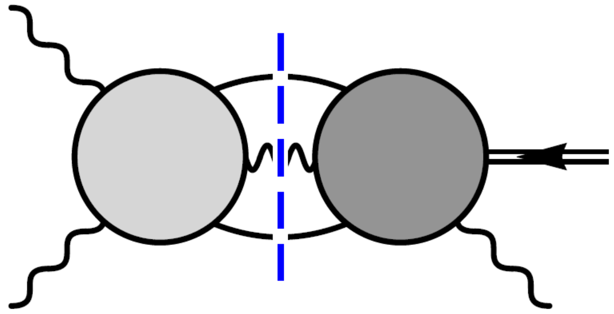
We can now calculate any 2-loop anomalous dimension of the SM EFT

- Simple cases: calculation of only one type of cut required

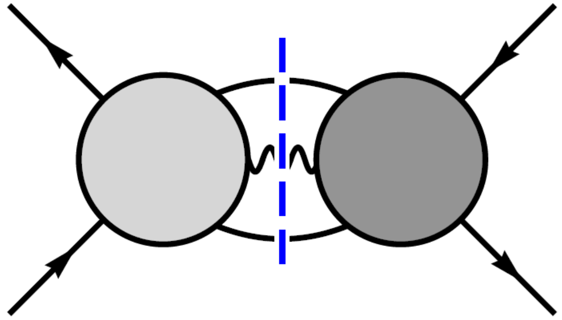
For a general entry in the matrix, IR dependence must be accounted for



$D^2\phi^4 \rightarrow \psi^4$



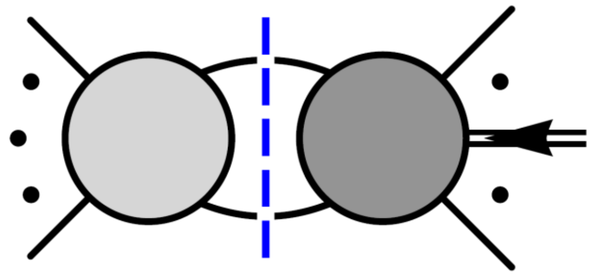
$\phi^2F^2 \rightarrow F^3$



$\phi^2F^2 \rightarrow \psi^4$

Summary and Outlook

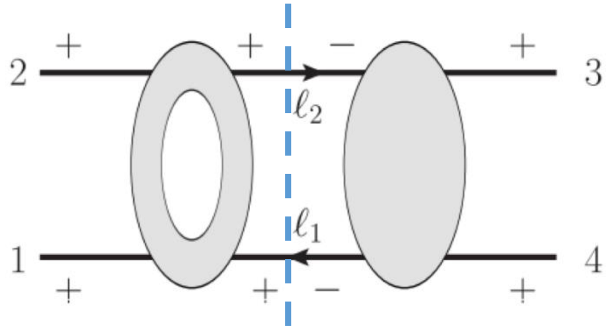
On-shell methods can be used to efficiently calculate anomalous dimensions

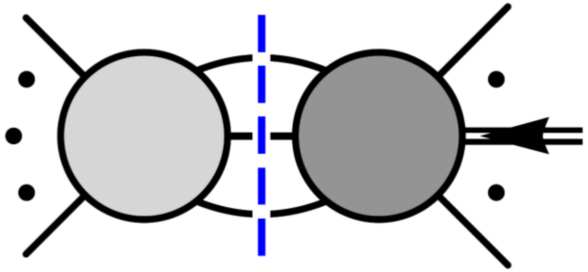


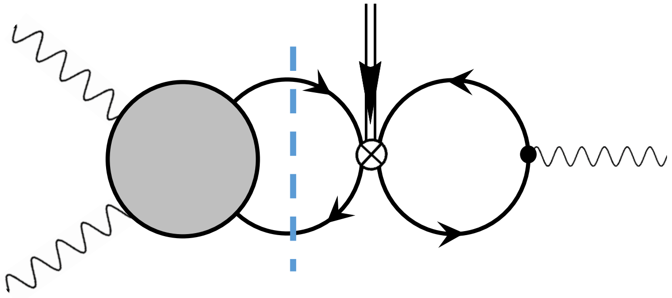
Unitarity cuts in kinematic variables yield renormalization scale dependence

On shell methods can be used in gravity as well, and double copy should help

- Example: F^3 double copies to R^3







Thanks!

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