The Double Copy of a Point Charge

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QCD meets Gravity

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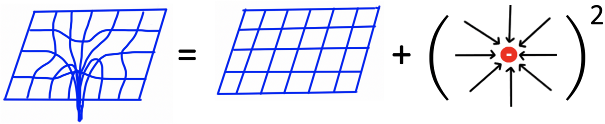
Based on arXiv:1912.02177 with

Kwangeon Kim, Kanghoon Lee, Isobel Nicholson, David Peinador Veiga

Motivation

Simple gauge theory solution: Coulomb.

Schwarzschild is natural double copy of Coulomb.



Full story? Dilaton?

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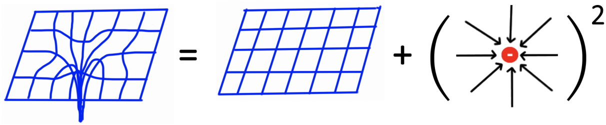
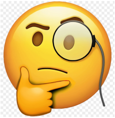


Full story? Dilaton?

Double-copy structure of Einstein equations?

double copy		**double** field theory
Gravity = YM × YM		doubled geometry $(x^\mu, \tilde{x}_\mu)$

Double copy of Coulomb: perturbative approach

Gravity $\sim$ (Yang-Mills)^2

Scattering amplitudes

[Kawai, Lewellen, Tye '86; Bern, Carrasco, Johansson '08; ...]

- YM states: $A_\mu^a = e^{ik\cdot x}\,\epsilon_\mu\,T^a$, ϵ_μ has $D-2$ dof.

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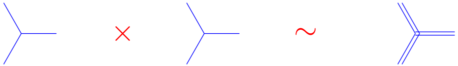
Perturbative classical solutions

- First map free solutions (linear).
- Then correct solutions in double-copy-ish perturbation theory.



Double copy for Coulomb?

Linearised "fat graviton": [Luna, RM, Nicholson, Ochirov, O'Connell, White, Westerberg 16]

$$H_{\mu\nu} = \left(h_{\mu\nu} - \frac{1}{2}\, h + P_{\mu\nu}[h] \right) + B_{\mu\nu} + P_{\mu\nu}\,[\phi]$$

(graviton + B-field + dilaton)
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Coulomb usual gauge: $\boxed{A^a_\mu = -\frac{q^a}{r}\,u_\mu}$ $\qquad u^\mu = (1,0,0,0)\,, \quad \partial_\mu q^a = 0\,.$

Natural double copy: $\boxed{H_{\mu\nu} = \frac{M}{r}\,u_\mu\,u_\nu}$ $\qquad$ both graviton and dilaton.

[also Goldberger, Ridgway 16]

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Coulomb different gauge: $A^a_\mu = \frac{q^a}{r}\, k_\mu$ $\qquad k = dt + dr\,, \quad k^2 = 0\,.$

Natural double copy: $h_{\mu\nu} = \frac{2M}{r}\, k_\mu\, k_\nu$ $\qquad$ exact Schwarzschild! Kerr-Schild double copy.

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[RM, O'Connell, White 14] $\qquad$ Kerr-Schild double copy.

Both consistent with 'convolution' idea [Anastasiou, Borsten, Duff, Hughes, Nagy 14] :

$$A^a_\mu * \mathrm{inv}(\Phi)_{a\dot{a}} * A^{\dot{a}}_\nu \quad \rightsquigarrow \quad (1/r) * \mathrm{inv}(1/r) * (1/r) = (1/r)$$

Double copy for Coulomb: JNW solution

Clue from momentum states: take polarisations ϵ_μ , $\tilde{\epsilon}_\mu$. $\epsilon \cdot k = \tilde{\epsilon} \cdot k = 0$

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Why not $\epsilon_{(\mu}\tilde{\epsilon}_{\nu)}$, $\epsilon_{[\mu}\tilde{\epsilon}_{\nu]}$, $\epsilon \cdot \tilde{\epsilon}\, \Delta_{\mu\nu}$?

$$\Delta_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu q_\nu + k_\nu q_\mu}{k \cdot q}$$

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General: graviton + B-field + dilaton.

$$\varepsilon_{\mu\nu} = C^{(h)} \left(\epsilon_{(\mu}\tilde{\epsilon}_{\nu)} - \frac{\Delta_{\mu\nu}}{D-2}\, \epsilon \cdot \tilde{\epsilon} \right) + C^{(B)}\, \epsilon_{[\mu}\tilde{\epsilon}_{\nu]} + C^{(\phi)}\, \frac{\Delta_{\mu\nu}}{D-2}\, \epsilon \cdot \tilde{\epsilon} \,.$$

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Linearised (Coulomb)^2: no B-field, $M\sim C^{(h)}$ graviton, $Y\sim C^{(\phi)}$ dilaton.

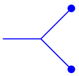
$$\boxed{H_{\mu\nu} = M\left(\frac{u_\mu u_\nu}{r} - P_{\mu\nu}\left[\frac{u^2}{r}\right]\right) + Y\,P_{\mu\nu}\left[\frac{u^2}{r}\right]} \qquad P_{\mu\nu}\left[\tfrac{u^2}{r}\right] = \tfrac{-1}{2\,r}(\eta_{\mu\nu} - q_\mu l_\nu - q_\nu l_\mu)$$

$Y=0$: linearised Schwarzschild solution.

Any Y: linearised JNW solution [Janis, Newman, Winicour '68].

Perturbative construction

Starting point: $H^{(0)}_{\mu\nu}$ is linearised solution, $H^{(1)}_{\mu\nu}$ is first non-linear correction.

$H^{(1)} =$ 

Gauge theory field A^a_μ

$$A^{(1)a\mu}(-p_1) = \frac{i}{2p_1^2} \int d^D p_2 d^D p_3 \delta^D(p_1+p_2+p_3) \boxed{f^{abc}\ V^{\mu\beta\gamma}}\ A^{(0)b}_\beta(p_2) A^{(0)c}_\gamma(p_3)$$

YM vertex $V(p_1,p_2,p_3)^{\mu\beta\gamma} = (p_1-p_2)^\gamma \eta^{\mu\beta} + (p_2-p_3)^\mu \eta^{\beta\gamma} + (p_3-p_1)^\beta \eta^{\gamma\mu}$

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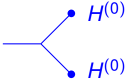
$H^{(1)} =$ 

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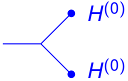
$H^{(1)} =$ 

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Case $Y = M$ [Luna, RM, Nicholson, Ochirov, O'Connell, White, Westerberg 16].

Case $Y \neq M$ [Kim, Lee, RM, Nicholson, Veiga 19].

Comparison to exact solution messy (gauge choices, field redefinitions).

General point charge: JNW solution

Unique static, spherically symmetric, asymptotically flat solution of Einstein + minimally coupled scalar.

Two parameters (M, Y) or (ρ_0, γ). Found by Janis, Newman, Winicour '68:

$$ds^2 = -\left(1 - \frac{\rho_0}{\rho}\right)^{\gamma} dt^2 + \left(1 - \frac{\rho_0}{\rho}\right)^{-\gamma} d\rho^2 + \left(1 - \frac{\rho_0}{\rho}\right)^{1-\gamma} \rho^2 d\Omega^2$$

$$\phi = \frac{Y}{\rho_0} \log\left(1 - \frac{\rho_0}{\rho}\right) \qquad \rho_0 = 2\sqrt{M^2 + Y^2} \qquad \gamma = \frac{M}{\sqrt{M^2 + Y^2}}$$

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Solution above is in Einstein frame. In string frame, $\quad g^S_{\mu\nu} = e^{2\phi}\, g^E_{\mu\nu}\,.$

Double copy of Coulomb:
exact map with double field theory

Kerr-Schild double copy

[RM, O'Connell, White 14] [with Luna, Nicholson 15-18]

"Exact perturbation" $$g_{\mu\nu} = \eta_{\mu\nu} + \phi\, k_\mu k_\nu$$

where k_μ is null and geodesic wrt $\eta_{\mu\nu}$ <u>and</u> $g_{\mu\nu}$. $(k^\mu = g^{\mu\nu} k_\nu = \eta^{\mu\nu} k_\nu)$

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Stationary vacuum case (take $k_0 = 1$):

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Why abelian? Exact linear Einstein $\rightsquigarrow$ exact linear YM, i.e., Maxwell.

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Rest of the talk Vacuum here. Kerr-Schild-type ansatz for NS-NS gravity?

Why is this double copy?

Double Field Theory

[Siegel '93] [Hull, Zwiebach '09 + Hohm '10]

For our purposes: fancy formulation of NS-NS gravity.

Motivation: low-energy effective theory of closed string exhibiting T-duality.

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Motivation: low-energy effective theory of closed string exhibiting T-duality.

- Doubled space $X_M = (x^\mu, \tilde{x}_\mu)$, $\dim = 2D$.
  String on torus: quantised momenta, winding. Mixed by T-duality.
  DFT idea: $(x^\mu, \tilde{x}_\mu)$ conjugate to (momenta, winding). T-duality: $O(D,D)$.

$\Lambda^M{}_N \in O(D,D):\ (\Lambda)^T(\mathcal{J})(\Lambda) = (\mathcal{J})$. $\quad \mathcal{J}_{MN} = \begin{pmatrix} 0 & \delta^\mu{}_\nu \\ \delta_\mu{}^\nu & 0 \end{pmatrix}$ is $O(D,D)$ metric.

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- Doubled space $X_M = (x^\mu, \tilde{x}_\mu)$, $\dim = 2D$.
  String on torus: quantised momenta, winding. Mixed by T-duality.
  DFT idea: $(x^\mu, \tilde{x}_\mu)$ conjugate to (momenta, winding). T-duality: $O(D, D)$.

$$\Lambda^M{}_N \in O(D,D): \; (\Lambda)^T(\mathcal{J})(\Lambda) = (\mathcal{J}). \quad \mathcal{J}_{MN} = \begin{pmatrix} 0 & \delta^\mu{}_\nu \\ \delta_\mu{}^\nu & 0 \end{pmatrix} \text{ is } O(D,D) \text{ metric.}$$

- T-duality manifest: $O(D, D)$ covariance.
  Section condition, e.g., $\partial/\partial\tilde{x}_\mu = 0$: correct dof, breaks covariance.

Double Field Theory

[Siegel '93] [Hull, Zwiebach '09 + Hohm '10]

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NS-NS fields packaged as tensor and scalar wrt to $O(D,D)$.

- Generalised metric: $\mathcal{H}_{MN} = \begin{pmatrix} g^{\mu\nu} & -g^{\mu\rho}B_{\rho\nu} \\ B_{\mu\rho}g^{\rho\nu} & g_{\mu\nu} - B_{\mu\rho}g^{\rho\sigma}B_{\sigma\nu} \end{pmatrix} \in O(D,D)$.
- DFT dilaton d: $e^{-2d} = \sqrt{-g}\, e^{-2\phi}$.

Kerr-Schild-inspired ansatz

Recall Kerr-Schild ansatz: $g_{\mu\nu} = \eta_{\mu\nu} + \varphi\, k_\mu k_\nu$ k_μ null and geodesic.

DFT version: take $\mathcal{H}_{0MN} = \mathcal{H}_{MN}\,(g_{\mu\nu} = \eta_{\mu\nu}, B_{\mu\nu} = 0)\,,$ [Lee 18] [Cho, Lee 19] [Kim, Lee, RM, Nicholson, Veiga 19]

$$\mathcal{H}_{MN} = \mathcal{H}_{0MN} + \varphi(K_M \bar{K}_N + K_N \bar{K}_M) - \frac{1}{2}\varphi^2 \bar{K}^2 K_M K_N$$

$$K_M = \frac{1}{\sqrt{2}}\begin{pmatrix} k^\mu \\ \eta_{\mu\nu} k^\nu \end{pmatrix} \qquad \bar{K}_M = \frac{1}{\sqrt{2}}\begin{pmatrix} \bar{k}^\mu \\ -\eta_{\mu\nu} \bar{k}^\nu \end{pmatrix}$$

where k_μ and $\bar{k}_\mu$ are null and satisfy diff. contraints (or $\bar{k}_\mu$ not null).

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$$g_{\mu\nu} = \eta_{\mu\nu} - \frac{\varphi}{1 + \frac{\varphi}{2}(k \cdot \bar{k})}\, k_{(\mu} \bar{k}_{\nu)}\,,$$

$$B_{\mu\nu} = \frac{\varphi}{1 + \frac{\varphi}{2}(k \cdot \bar{k})}\, k_{[\mu} \bar{k}_{\nu]}\,.$$

First examples of exact double copy with dilaton and B-field. [Lee 18]

JNW solution: fits ansatz, B-field is pure gauge.

Double Field Theory versus Double Copy

Generalised metric $\mathcal{H}^M{}_N$ induces chirality:

$$P_M{}^N = \frac{1}{2}\left(\delta_M{}^N + \mathcal{H}_M{}^N\right), \qquad \bar{P}_M{}^N = \frac{1}{2}\left(\delta_M{}^N - \mathcal{H}_M{}^N\right).$$

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$=$ left and right moving sectors! (pullback to worldsheet)

Kerr-Schild-like ansatz

$$\boxed{\mathcal{H}_{MN} = \mathcal{H}_{0MN} + \varphi(K_M\bar{K}_N + K_N\bar{K}_M) + \dots}$$

Satisfy definite chiralities: $(P_0)_M{}^N K_N = K_M\,, \qquad (\bar{P}_0)_M{}^N \bar{K}_N = \bar{K}_M\,.$

Double-copy interpretation:

$$\boxed{K_M \rightsquigarrow A_\mu \qquad \bar{K}_M \rightsquigarrow \bar{A}_\mu}$$

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Usual DFT basis: $(x^\mu, 0)$, $(0, \tilde{x}_\mu)$. Mixed by $O(D, D)$.

Double-copy basis: $\frac{1}{2}(x^\mu + \tilde{x}^\mu, x_\mu + \tilde{x}_\mu)$, $\frac{1}{2}(x^\mu - \tilde{x}^\mu, \tilde{x}_\mu - x_\mu)$.

DFT equations of motion

'Generalised diffeomorphisms' $\longrightarrow$ curvature tensors: $\mathcal{R}_{MN}$, $\mathcal{R}$.

With section condition ($\partial/\partial\tilde{x}_\mu = 0$), NS-NS equations of motion:

$\mathcal{R}_{(\mu\nu)} = 0$ for metric $g_{\mu\nu}$

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$$4e^{-2d}\,\mathcal{R}_{\mu 0} = \partial^\nu F_{\nu\mu} = 0\,, \quad F = dA\,, \quad A_\mu = e^{-2d}\varphi\, k_\mu + C_\mu\,,$$
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C_μ , $\bar{C}_\mu$ absorb non-linearities, non-local.

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Kerr-Schild-like $(g_{\mu\nu}, B_{\mu\nu}, d)$ $\sim$ ('left-moving' A_μ) $\times$ ('right-moving' $\bar{A}_\mu$)

JNW $\sim$ ('left-moving' Coulomb) $\times$ ('right-moving' Coulomb)

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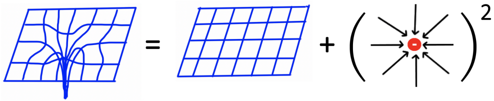
- Double copy of classical solutions possible.



- Perturbative double copy: generic but messy.
- Exact double copy: fully non-linear but not generic.

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- **Double** field theory convenient setting for **double** copy.

  KLT interpretation of Kerr-Schild-type double copy

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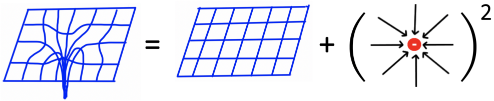

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Much more to explore

- Larger classes of solutions, duality transf., asymptotic symmetries, . . .
  [Luna et al 15; Luna et al 18] [Godazgar et al; Huang et al; Alawadhi et al; Banerjee et al 19]
- **Aim:** general formulation of fully non-linear double copy.