Hawking Radiation Meets the Double Copy

College | Physical Sciences
Physics & Astronomy

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Mani L. Bhaumik Institute for Theoretical Physics

Based on [2412.05267], [2510.25866].
With *Donal O'Connell, Rafael Aoude*, Chris White.

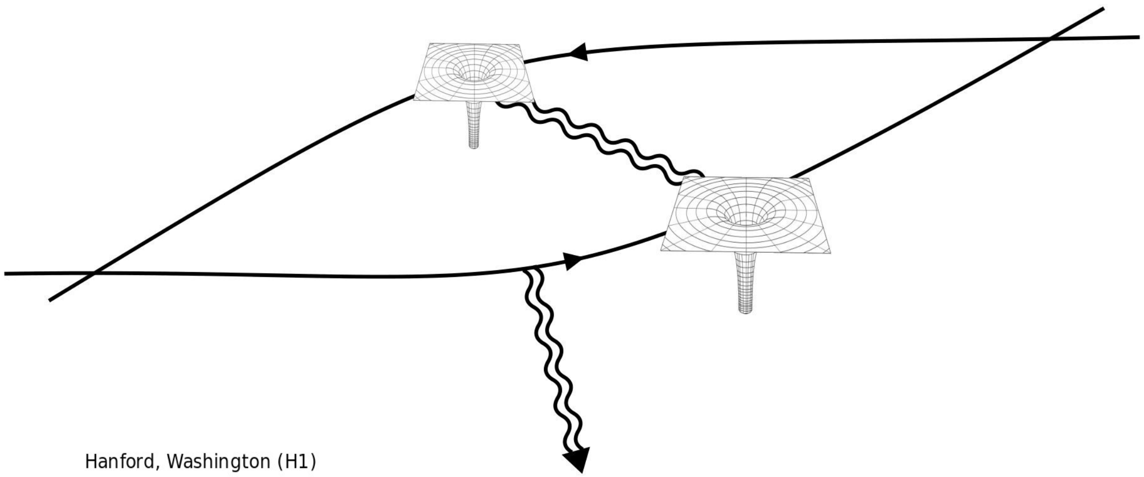
California Amplitudes 2025, 11/08/2025

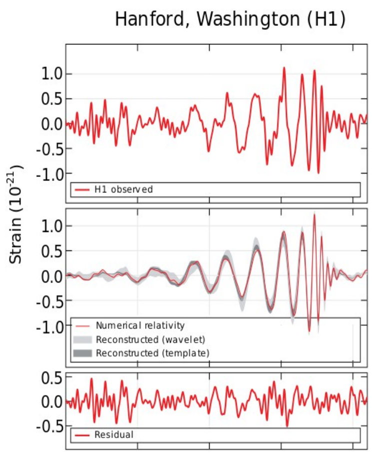
Outline

UCLA

- Introduction
- Hawking radiation from amplitudes
- Bogoliubovs, amplitudes and crossing
- Double Copy

Background and motivation



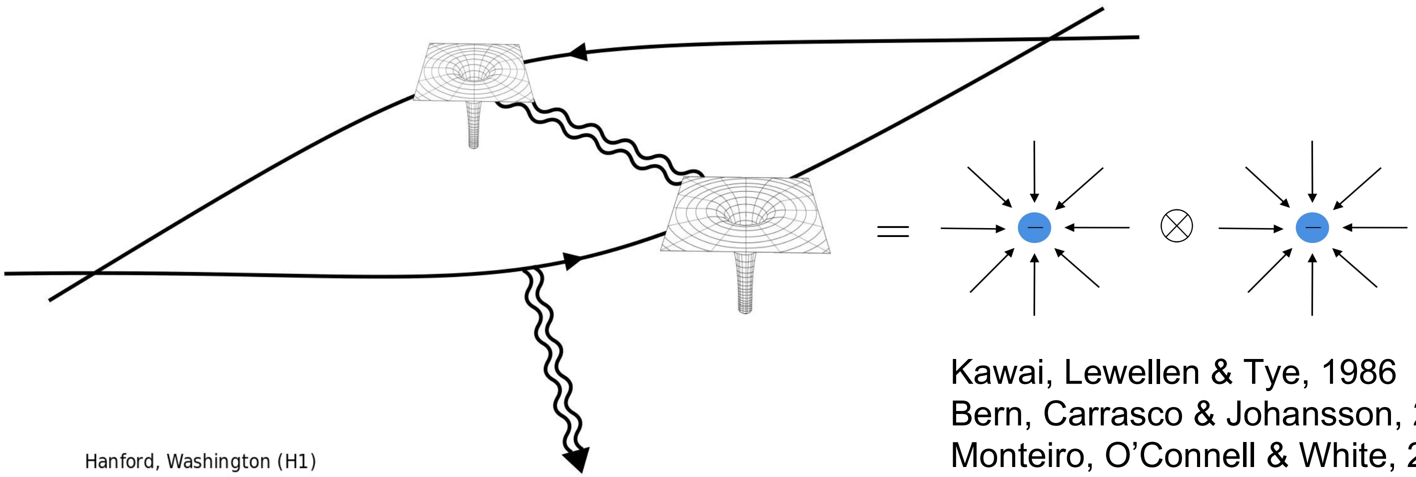


LIGO collaboration, 2016

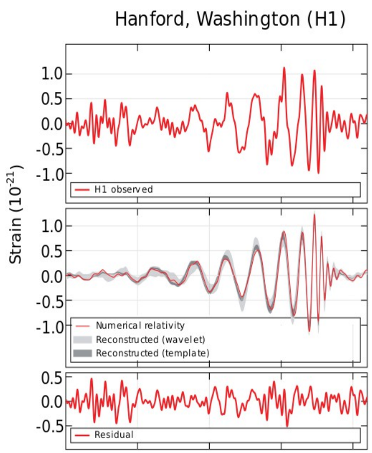
$$\Rightarrow \langle R_{\mu\nu\rho\sigma}(x)\rangle = \int_k \mathcal{A}(k)\, e^{-ik\cdot x}$$

Snowmass White Paper: Gravitational Waves and Scattering Amplitudes 2022

Background and motivation



Kawai, Lewellen & Tye, 1986
Bern, Carrasco & Johansson, 2008
Monteiro, O'Connell & White, 2014



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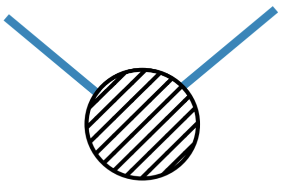
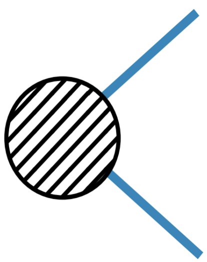
Snowmass White Paper: Gravitational Waves
and Scattering Amplitudes 2022

Scope

→ **First, reproduce Hawking radiation with amplitudes**

First we consider a scattering problem: 

Then we analytically continue: 

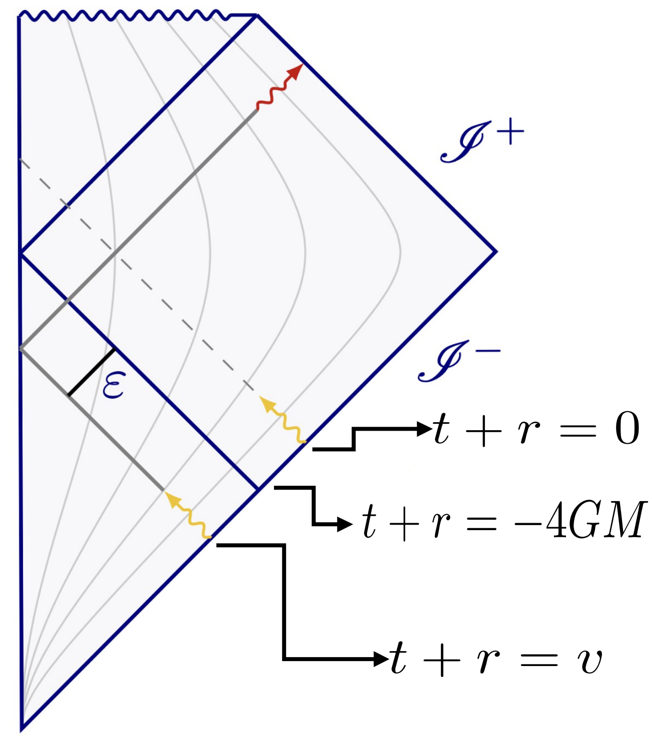
Hawking scattering: The Vaidya background

In-falling shell of
Classical radiation

$$g_{\mu\nu} = \eta_{\mu\nu} - \frac{2GM\Theta(t+r)}{r} k_\mu k_\nu$$

$$k^\mu(x) = (1, \hat{r})$$

Kerr-Schild null vector field



Vaidya, 1966

Hawking scattering:
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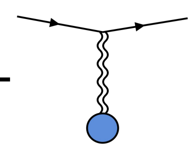
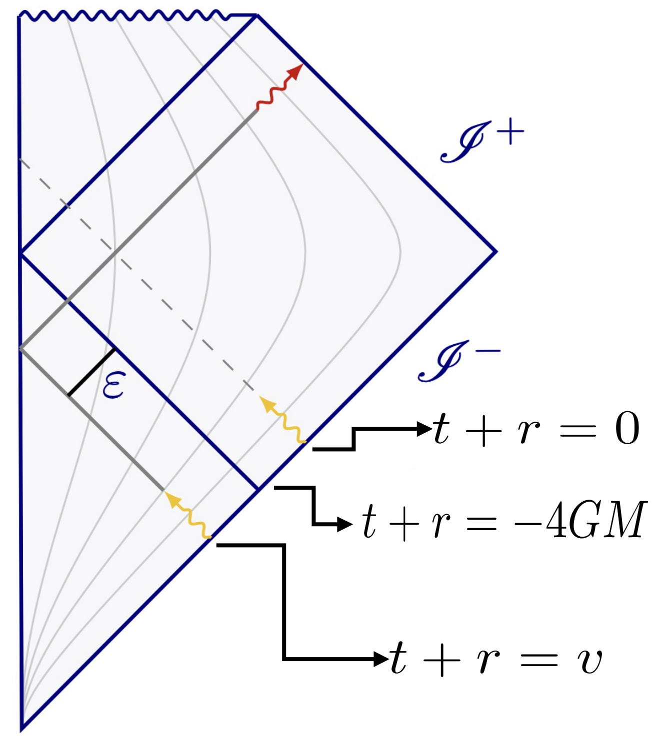
Kerr-Schild null vector field

$$\mathcal{L} = \sqrt{-g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$
$$= \mathcal{L}_{free} + \mathcal{L}_{cubic}$$



Vaidya, 1966



The KMOC state

- Consider the LO eikonal for a spherically symmetric massless scalar on a time-varying background:

$$|\psi\rangle = \int d\Phi(p)\varphi(p)|p\rangle = \int d\Phi(p) \int dv\, \varphi(v)e^{iEv}|p\rangle$$

$$= \int d\Phi(p)|p\rangle \int dv\, \varphi(v)e^{ib\cdot p}$$

On-shell Lorentz invariant phase space of a massless quantum scalar

$$b^\mu = (v,0,0,0)$$

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On-shell Lorentz invariant phase space of a massless quantum scalar

$$b^\mu = (v, 0, 0, 0)$$

$\Rightarrow$ *Now* $p^2 = 0$ *but still* $p \gg q$ *: Geometric-optics approximation!*

Scattering and the eikonal

Time evolution is
given by the
S-matrix!

$$\langle p'|S|\psi\rangle = \int d\Phi(p) \int dv\, \varphi(v) e^{ib\cdot p} \langle p'|S|p\rangle$$

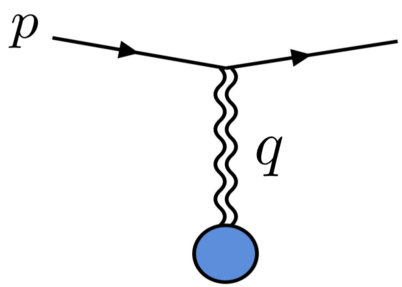
Computable using
Feynman diagrams
and QFT techniques

Kosower, Maybee, O'Connell 2018
Sergola, Cristofoli, O'Connell, Ross, Moynihan,
Gonzo, White, 2021
Sergola, O'Connell, Aoude, 2024

Hawking scattering: LO

$$h_{\mu\nu} = \frac{2GM\Theta(t+r)}{r} k_\mu k_\nu$$

$$\langle p+q | iT_{\mathrm{LO}} | p \rangle =$$



$$= \tilde{h}^{\mu\nu}(q) p_\mu (p+q)_\nu$$

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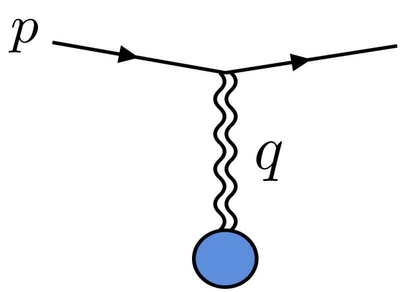
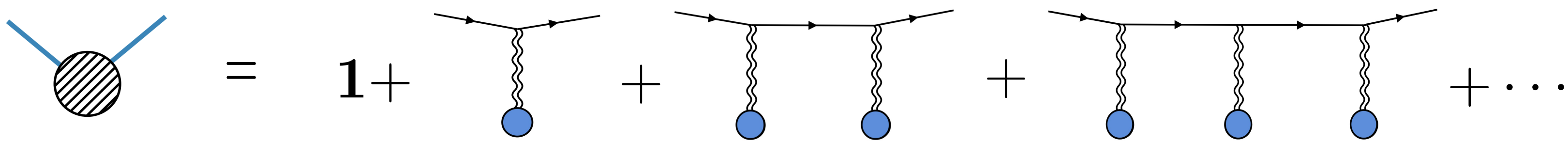


$$= \tilde{h}^{\mu\nu}(q) p_\mu (p+q)_\nu$$

$$\Rightarrow \langle p | S - 1 | \psi \rangle = \int dv\, \varphi(v) e^{ib\cdot p} \left(-4iGME \log(-v/\mu)\right)$$

LO Eikonal resummation



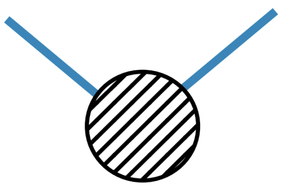
$$\Rightarrow \langle p|S|\psi\rangle = \int dv\, \varphi(v) e^{ib\cdot p} e^{-4iGME\log(-v/\mu)}$$

Higher loops resum into an exponential in the leading eikonal/geometric-optics approximation $p \gg q, \ell$

Sergola, O'Connell, Aoude, 2024

Computing the resummed amplitude

We choose the spherical initial state $\varphi(v) = e^{-iE_0 v}$



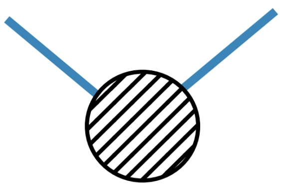
$$= \int_{-\infty}^{\infty} dv\, e^{iv(E-E_0)} e^{-4iGME\log(-v/\mu)}$$

$$\Rightarrow \int_{-\infty}^{0} dv\, e^{iv(E-E_0)} e^{-4iGME\log(-v/\mu)}$$

Cannot integrate over all v
Because of the logarithm cut:
not an inclusive result!

Computing the resummed amplitude

We choose the spherical initial state $\varphi(v) = e^{-iE_0 v}$



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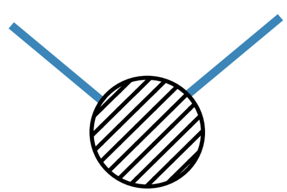
$\Rightarrow \Bigg|$  $\Bigg|^2 = N \dfrac{e^{8\pi GME}}{e^{8\pi GME} - 1}$

Looks familiar..

Bogoliubov and amplitudes

Our EOM $\partial^2\phi = -\partial_\mu h^{\mu\nu}(x)\partial_\nu\phi$ can be resolved

- Onto a past basis: $\phi(x) = \int d\Phi(p)\,(a(p)P(x,p) + h.c.)$

$$P(x,p) \xrightarrow{t\to-\infty} e^{-ip\cdot x}$$

- Or a future one: $\phi(x) = \int d\Phi(p)\,(b(p)F(x,p) + h.c.)$

$$F(x,p) \xrightarrow{t\to\infty} e^{-ip\cdot x}$$

Bogoliubov and amplitudes

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$$||$$

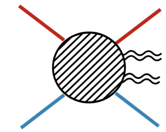
- Or a future one: $\phi(x) = \int d\Phi(p)\,\left(b(p)F(x,p) + h.c.\right)$

$$F(x,p) \xrightarrow[t \to \infty]{} e^{-ip\cdot x}$$

$$\Rightarrow b(k) = S^\dagger a(p) S = \int d\Phi(p)\,\big(A(p,k)a(p) + B(p,k)a^\dagger(p)\big)$$

Bogoliubov and amplitudes

$$S = \exp\left[\int d\Phi(p,k)\xi(p,k)a^\dagger(p)a^\dagger(k)\right] = \exp\left[\quad\right]$$



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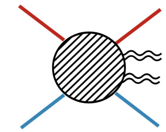

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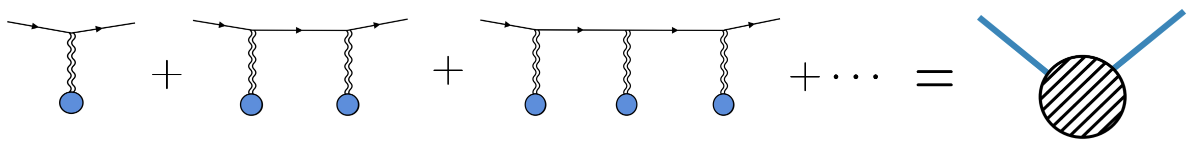
- $A(p,k) = \langle\Omega|S^\dagger a(p) S a^\dagger(k)|\Omega\rangle = \int dv r^2 F(v, E_p) i\partial_v P(v, -E_k)$

$= \mathbf{1} +$



Solve the EOM with Born iterations

Bogoliubov and amplitudes

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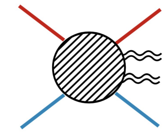

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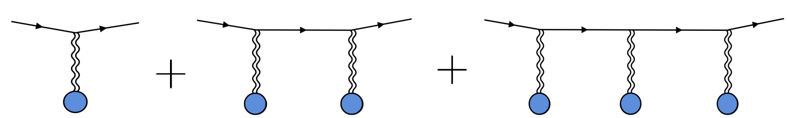
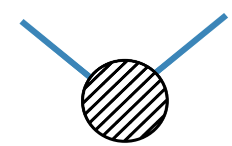
$$= \mathbf{1} + \cdots + \cdots + \cdots + \cdots =$$

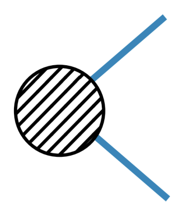




Solve the EOM with Born iterations

- $B(p,k) = \langle \Omega | a(k) S^\dagger a(p) S | \Omega \rangle = \int dv r^2 F(v, E_p) i\partial_v P(v, E_k)$

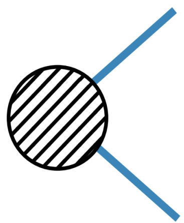
$$=$$



Crossing and spectrum

- The number of particles in the future is

$$\langle \Omega | S^\dagger a^\dagger(p) a(p) S | \Omega \rangle = \int d\Phi(k) |B(p,k)|^2$$

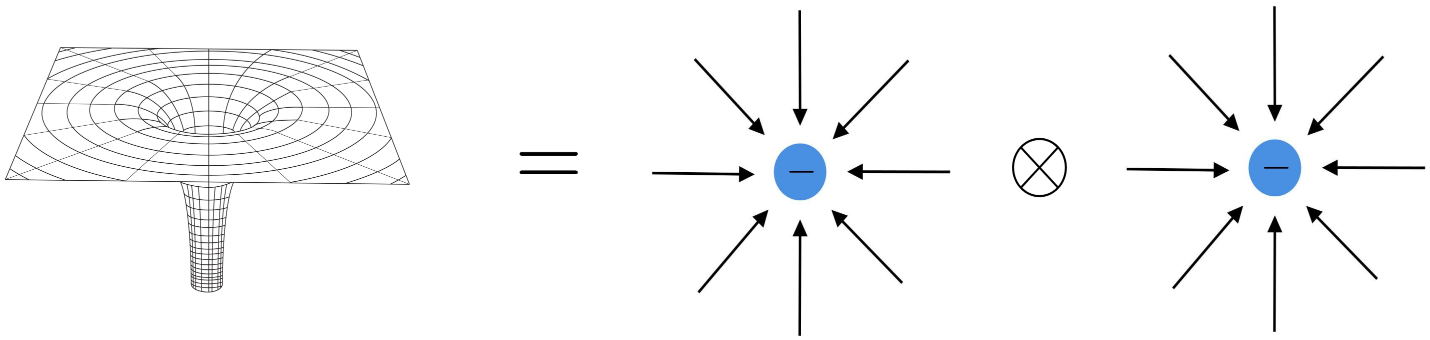
$B(p,k) =$  $= \int_{-\infty}^{0} dv\, e^{iv(E+E_0)} e^{-4iGM \log(-v/\mu)}$

$$|B(p,k)|^2 = N \frac{1}{e^{8\pi GME} - 1}$$

Hawking spectrum with $T = \dfrac{1}{8\pi GM}$

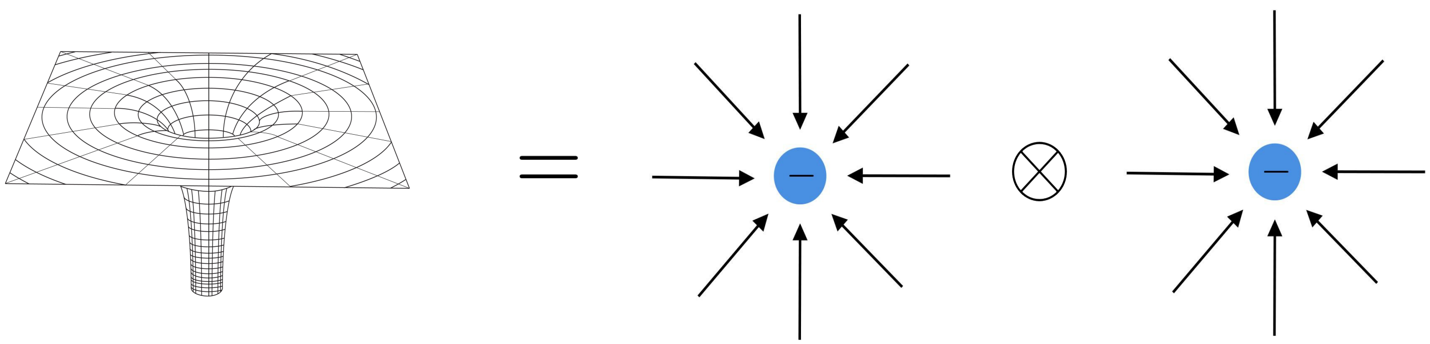
Hawking, 1975
Aoude, O'Connell, **Sergola** 2024

The double copy



Kawai, Lewellen & Tye, 1986
Bern, Carrasco & Johansson, 2008
Monteiro, O'Connell & White, 2014

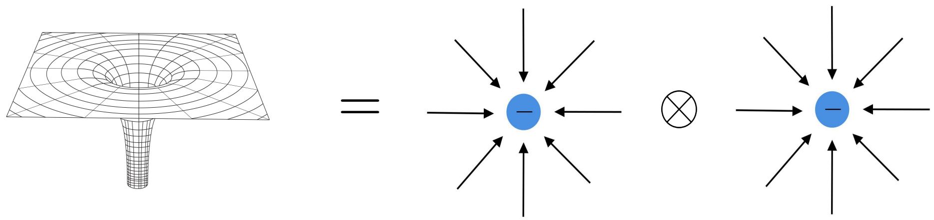
The double copy



$\Rightarrow$ We will use the classical (Kerr-Schild) double copy

Kawai, Lewellen & Tye, 1986
Bern, Carrasco & Johansson, 2008
Monteiro, O'Connell & White, 2014

Kerr-Schild double copy



$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \phi(x) k_\mu(x) k_\nu(x)$$

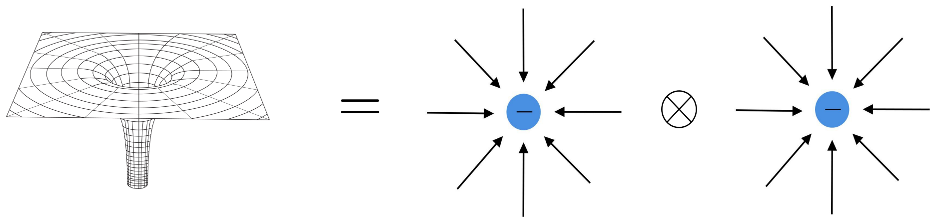
$$k^\nu \partial_\nu k^\mu = 0, \quad k^2 = 0$$

$$A^\mu(x) = \phi(x) k^\mu(x)$$

$$R^{\mu\nu}(x) = 0 \Rightarrow \underbrace{\partial_\mu(\partial^\mu(\phi k^\nu) - \partial^\nu(\phi k^\mu))}_{\partial_\mu F^{\mu\nu}(x) = 0} = 0$$

Monteiro, O’Connell & White, 2014
Luna, Monteiro, Nicholson, White & O'Connell, 2016
Sergola, Peinador Veiga, Monteiro & O'Connell 2021

The classical double copy



$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \phi(x) k_\mu(x) k_\nu(x)$$

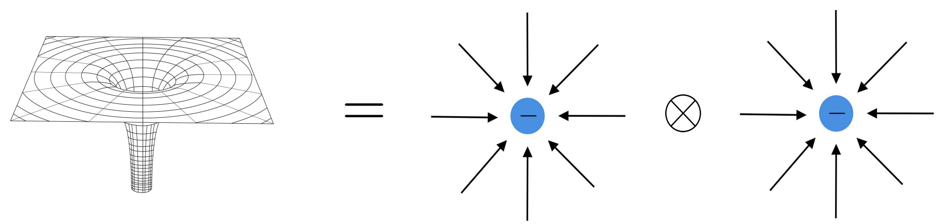
$$k^\nu \partial_\nu k^\mu = 0, \quad k^2 = 0$$

$$\phi(x) = \frac{2GM}{r}, \quad k^\mu = \left(1, \frac{\vec{x}}{r}\right) \Rightarrow ds^2 = g^{Sch.}_{\mu\nu}(x) dx^\mu dx^\nu$$

$$\Rightarrow A^{Coul.}_\mu = \frac{Q}{4\pi r} k_\mu$$

Coulomb $\Longleftrightarrow$ Schwarzschild!

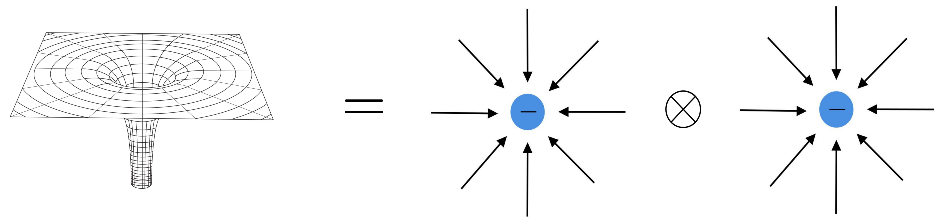
The single copy of the Vaidya black hole



$$h_{\mu\nu} = \frac{2GM\Theta(t+r)}{r} k_\mu k_\nu \quad \Leftrightarrow \quad A^\mu(x) = \frac{Q\Theta(t+r)}{4\pi r} k^\mu$$

Vaidya $\Longleftrightarrow$ root-Vaidya!

The single copy of the Vaidya black hole



$$h_{\mu\nu} = \frac{2GM\Theta(t+r)}{r} k_\mu k_\nu \quad \Leftrightarrow \quad A^\mu(x) = \frac{Q\Theta(t+r)}{4\pi r} k^\mu$$

Vaidya $\Longleftrightarrow$ root-Vaidya!

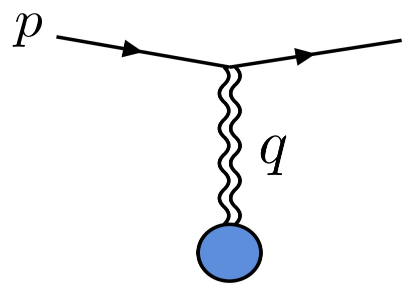
$\Longrightarrow$ We scatter a massless charged scalar on this EM background

Root-Vaidya scattering

UCLA

Ilderton, Lindved & Rajeev 2025
Carrasco & Chen 2025
Sergola, Aoude, White & O'Connell 2025

$$\langle p+q|iT_{\mathrm{LO}}|p\rangle =$$



$$= \tilde{A}^{\mu}(q)(2p+q)_{\mu}$$

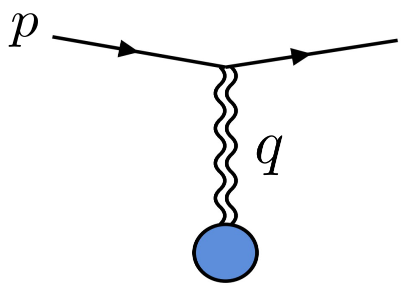
Now a massless scalar minimally coupled to the EM root-Vaidya background

$$\langle p'|S|\psi\rangle = \int d\Phi(p) \int dv\, \varphi(v) e^{ib\cdot p} \langle p'|S|p\rangle$$

Root-Vaidya scattering

Ilderton, Lindved & Rajeev 2025
Carrasco & Chen 2025
Sergola, Aoude, White & O'Connell 2025

$$\langle p+q|iT_{\mathrm{LO}}|p\rangle =$$



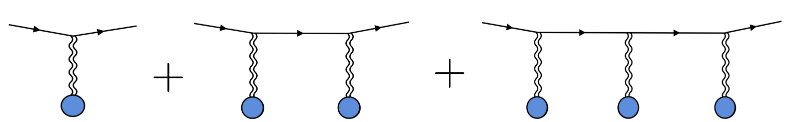
$$= Q\tilde{A}^{\mu}(q)(2p+q)_{\mu}$$

$$\Rightarrow \langle p|S-1|\psi\rangle = \int dv\, \varphi(v) e^{ib\cdot p}\left(2i\alpha\log(-v/\mu)\right)$$

$$\alpha = \frac{QQ_B}{4\pi}$$

Exponentiates just like in GR:

$\mathbf{1}+$

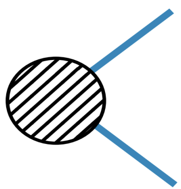


$$+\cdots = e^{2i\alpha\log(-v/\mu)}$$

The single copy particle distribution

- The number of particles in the future is

$$\langle \Omega | S^\dagger a^\dagger(p) a(p) S | \Omega \rangle = \int d\Phi(k) |B(p,k)|^2$$

$B(p,k) =$  $= \int_{-\infty}^{0} dv\, e^{iv(E+E_0)} e^{2i\alpha \log(-v/\mu)}$

$$|B(p,k)|^2 = N \frac{1}{e^{-4\pi\alpha} - 1}$$

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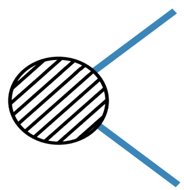
$B(p,k) =$  $= \int_{-\infty}^{0} dv\, e^{iv(E+E_0)} e^{2i\alpha \log(-v/\mu)}$

$$|B(p,k)|^2 = N \frac{1}{e^{-4\pi\alpha} - 1} \Leftrightarrow N \frac{1}{e^{8\pi GME} - 1}$$

$$-\alpha \leftrightarrow 2GME$$

The single copy particle distribution

- Compare with

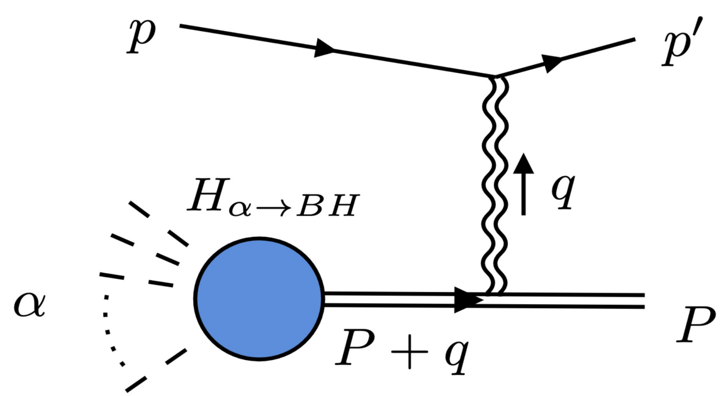
$$n(\epsilon) = N \frac{1}{e^{(\epsilon-\mu_c)/K_B T} - 1} \leftrightarrow N \frac{1}{e^{-4\pi\alpha} - 1}$$

Iso, Umetsu, Wilczek 2006

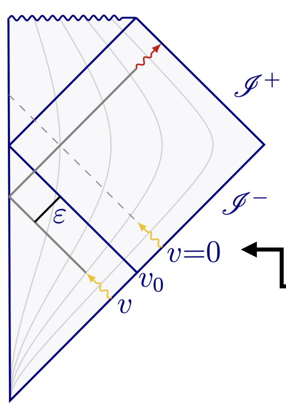
Some work in progress..

UCLA

So far we have rephrased Hawking/root-Hawking's derivation using amplitudes, but maybe we can do more!



$$\sim \frac{GM\Theta(v)}{4\pi r}$$



Coherent state which describes the in-falling classical radiation shell at $t{+}r{=}v{=}0$

Conclusions

- Amplitudes can describe Hawking radiation too!
- Nice Double Copy relation which defines root-Vaidya

Next:

- Corrections?
- More complicated spacetimes (Kerr-RN)?
- Observables, see for instance [2509.12111]
- Background independent calculation?
- Back reaction on the metric? Page curve?

Thank you!

NLO eikonal computation

We also computed sub leading
corrections to the Hawking phase in

$$B(E,E_0)=\int dv\, e^{iv(E+E_0)}e^{i(\chi_{(0)}+\chi_{(1)}+\cdots)}$$

We find $\chi_{(0)}=-4GM\log(-v/\mu),\quad \chi_{(1)}=-\dfrac{16G^2M^2E}{v}$

$$e^{-4i\frac{GME}{\hbar}\log(-(v+4GM)/\mu)}\approx 1-4i\frac{GME}{\hbar}\log(-(v+4GM)/\mu)$$
$$+\left(4i\frac{GME}{\hbar}\log(-(v+4GM)/\mu)\right)^2$$
$$-\frac{16G^2M^2E}{\hbar v}+\cdots$$

Simple translation which results
in a pure phase
For the Bogoliubov coefficient