UCDAVIS
DEPARTMENT OF PHYSICS AND ASTRONOMY

Non-planar MHV on-shell Diagrams

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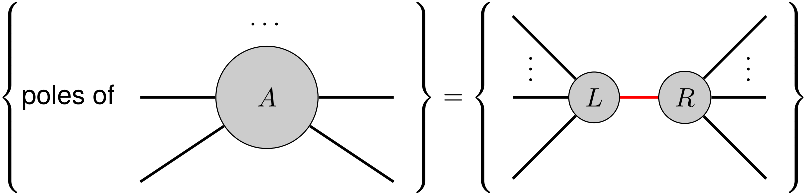
Based on ongoing work with U. Oktem, M. Sherman-Bennett, J. Trnka

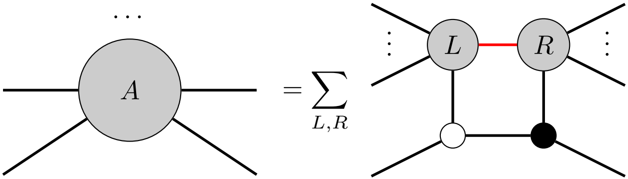
BCFW Recursion in terms of on-shell diagrams

Tree amplitudes are uniquely identified by poles coming from factorization channels



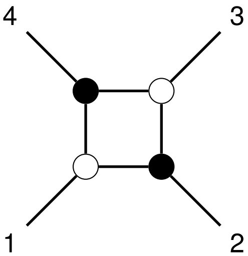
BCFW Recursion in terms of on-shell diagrams

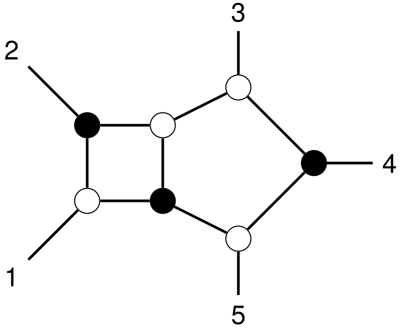
In planar $\mathcal{N} = 4$ SYM, this amounts to a recursive description



(BCFW bridge implements complex shift of adjacent momenta)

BCFW Recursion in terms of on-shell diagrams





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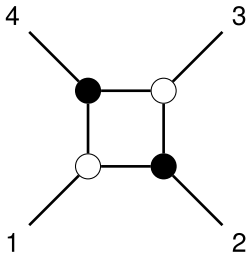

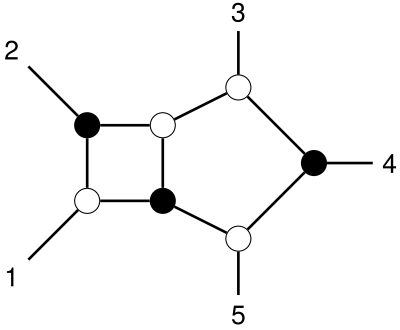

What can we say about the non-planar generalization of these diagrams?

(e.g. those coming from non-adjacent BCFW bridges)

Pole structure made geometric at MHV

$$\text{kinematic data as spinors} \sim \begin{pmatrix} x_1 & \cdots & x_n \\ y_1 & \cdots & y_n \end{pmatrix} \sim 2\text{-plane spanned by } \vec{x} \text{ and } \vec{y}$$

$$\text{Use minors } (ij) = \det\begin{pmatrix} x_i & x_j \\ y_i & y_j \end{pmatrix} \text{ as coordinates.}$$

Pole structure made geometric at MHV

$$\{\text{space of possible logarithmic forms}\} = \operatorname{span}\left\{ \overbrace{\frac{1}{(\sigma_1\sigma_2)(\sigma_2\sigma_3)\cdots(\sigma_n\sigma_1)}}^{\text{Parke-Taylor factor}} \forall \sigma \in S_n \right\}$$

$$\{\text{set of } 2\text{-planes in } n\text{-dimensions}\} = \bigcup_{\pm_i, \sigma\in S_n} \overbrace{\{\pm_i \pm_j (\sigma_i\sigma_j) \geq 0\}}^{\text{Positive Region}}$$

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Parke-Taylor factors correspond to Positive Regions (via residues $\leftrightarrow$ boundary)	$\Longrightarrow$	Sums of Parke-Taylor factors (e.g. on-shell functions) correspond to some union of PR

Combinatorial/topological properties probe underlying functional structure

Geometries corresponding to MHV diagrams

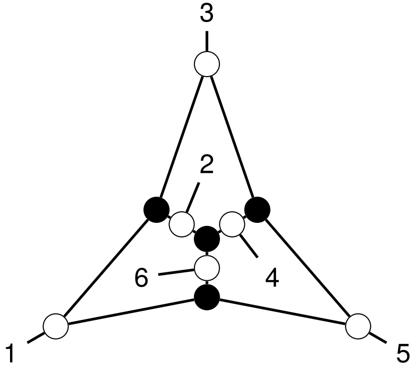
Internally planar: diagram is planar once external edges are removed
Irreducible: diagram does not contain non-trivial subdiagram
(square in numerator does not factorize)

$$\frac{[(53)(61)(24)+(34)(15)(62)]^2}{(12)(23)(31)(34)(45)(53)(56)(61)(15)(24)(46)(62)}$$

$$\updownarrow$$

$PR(1,4,2,5,3,6)$

$\cup$ (combinations of $1\leftrightarrow 4, 2\leftrightarrow 5, 3\leftrightarrow 6$)



Geometries corresponding to MHV diagrams

$$\begin{array}{ccc}
\boxed{\text{Planar}} & & \\
\cap & & \\
\boxed{\begin{array}{l}\text{Internally planar}\\ \text{Irreducible}\end{array}} & \subset & \boxed{\text{Internally planar}} \\
\cap & & \cap \\
\boxed{\text{Irreducible}} & \subset & \boxed{\text{Arbitrary MHV}}
\end{array}$$

Geometries corresponding to MHV diagrams

$\exists$ diagrams s.t. $\forall$ geometries $\exists$ spurious higher-codimension boundaries

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$\exists$ diagrams s.t. $\forall$ geometries $\exists$ spurious codim-1 boundaries

Geometries corresponding to MHV diagrams

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$$\begin{tikzcd}
\boxed{\text{Planar}} \arrow[d, phantom, "\cap"] & \\
\boxed{\begin{array}{l}\text{Internally planar}\\ \text{Irreducible}\end{array}} \arrow[r, phantom, "\subset"] \arrow[d, phantom, "\cap"] & \boxed{\text{Internally planar}} \arrow[d, phantom, "\cap"] \\
\boxed{\text{Irreducible}} \arrow[r, phantom, "\subset"] & \boxed{\text{Arbitrary MHV}}
\end{tikzcd}$$

$\forall$ diagrams $\exists$ geometries with well-behaved positroid boundaries

$\exists$ diagrams s.t. $\forall$ geometries $\exists$ spurious codim-1 boundaries

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$\forall$ diagrams $\exists$ geometries connected in codim-1

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