Resummation of Negative Geometries

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Based on upcoming work with Lance Dixon, Shruti Paranjape, Jaroslav Trnka, Yongqun Xu, Shun-Qing Zhang

Motivation

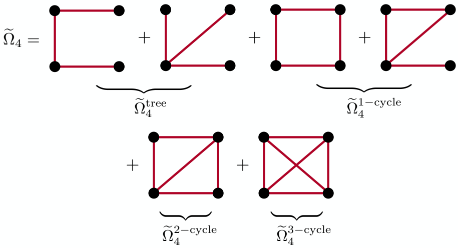
- Bern-Dixon-Smirnov ansatz gives the form of the planar $\mathcal{N} = 4$ *sYM* amplitude at 4 and 5 point

$$M = \exp\left[\sum_{l=1}^{\infty} g^l \left(f^{(l)}(\epsilon)M^{(1)}(l\epsilon) + C^{(l)}(\epsilon) + E^{(l)}(\epsilon)\right)\right] \quad (1)$$

- At $L-$loop leading IR divergence goes like $M_L \sim \frac{1}{\epsilon^{2L}}$
- Log of the amplitude better behaved $\ln M_4 = \frac{\gamma_{cusp}}{\epsilon^2} + \mathcal{O}(\frac{1}{\epsilon})$
- Can we compute γ_{cusp} for $g \gg 1$ from amplitudes?
- Need to resum perturbative series

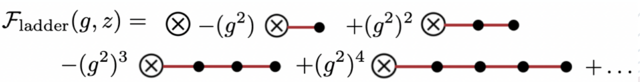
Recap: Negative Geometries

- (see Jara's talk) Integrand for $\ln M_4$ can be expanded in terms of objects called "negative geometries"
- Freeze one loop and integrate the rest - get the IR-finite object $\mathcal{F}(g, z)$
- $\mathcal{I}[\mathcal{F}(g, z)]$ computes γ_{cusp} contributions

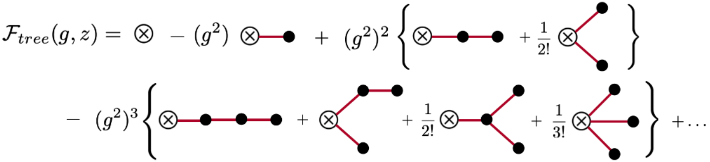


γ_{cusp} from Resummation

- Can resum certain subsets of these diagrams



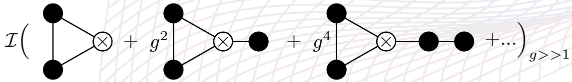
$$\gamma_{ladder,g\gg 1} = \mathcal{I}[\mathcal{F}_{ladder}(g \gg 1, z)] = 4\sqrt{2}g - \frac{4\log 2}{\pi} + \frac{4e^{-2\sqrt{2}g\pi}}{\pi} + \ldots$$



$$\gamma_{tree,g\gg 1} = \mathcal{I}[\mathcal{F}_{tree}(g \gg 1, z)] = \frac{8g}{\pi} + \frac{1}{\pi g} + \ldots$$

Resummation with 1-Cycle

- We can resum the next class of diagrams



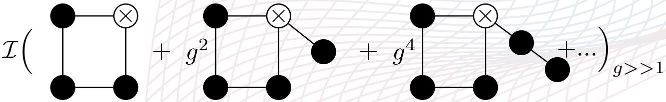
$$\mathcal{I}[\mathcal{F}_{lad}(z)\mathcal{F}_{tri}(z)]_{g>>1} \sim 10.71g^5 - 7.98g^4 + 4.86g^3 - 2.07g^2 + 0.47g$$

There are individual $\log g$ terms but they cancel out

- Leading diagram contributes g^6, resummation only suppresses $\frac{1}{g}$

Resummation with 1-Cycle

- Same happens for the next 1-cycle

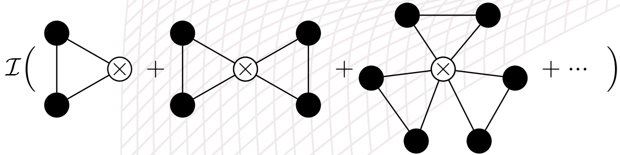


$$\mathcal{I}[\mathcal{F}_{box}(z)\mathcal{F}_{ladder}(g,z)]_{g>>1} \sim 1.88g^7 - 2.13g^6 - 0.04g^5 + 0.83g^4$$
$$-0.73g^3 + 0.36g^2 - 0.08g$$

- Leading diagram contributes g^8, resummation again suppresses $\frac{1}{g}$

Open Questions and Outlook

- How do we systematically determine which class of diagrams are good to resum?
- γ_{cusp} contribution of multi-cycle diagrams



- Hierarchy and cancellation of transcendental numbers that appear
- Resumming $\mathcal{F}(g, z)$

Thanks for Listening!