

Resummation of Negative Geometries

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*Based on upcoming work with Lance Dixon, Shruti Paranjape,
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Motivation

- Bern-Dixon-Smirnov ansatz gives the form of the planar $\mathcal{N} = 4$ sYM amplitude at 4 and 5 point

$$M = \exp \left[\sum_{l=1}^{\infty} g^l \left(f^{(l)}(\epsilon) M^{(1)}(l\epsilon) + C^{(l)}(\epsilon) + E^{(l)}(\epsilon) \right) \right] \quad (1)$$

- At L -loop leading IR divergence goes like $M_L \sim \frac{1}{\epsilon^{2L}}$
- Log of the amplitude better behaved $\ln M_4 = \frac{\gamma_{cusp}}{\epsilon^2} + \mathcal{O}(\frac{1}{\epsilon})$
- Can we compute γ_{cusp} for $g \gg 1$ from amplitudes?
- Need to resum perturbative series



Recap: Negative Geometries

- ▶ (see Jara's talk) Integrand for $\ln M_4$ can be expanded in terms of objects called "negative geometries"
- ▶ Freeze one loop and integrate the rest - get the IR-finite object $\mathcal{F}(g, z)$
- ▶ $\mathcal{I}[\mathcal{F}(g, z)]$ computes γ_{cusp} contributions

$$\begin{aligned}
 \tilde{\Omega}_4 = & \underbrace{\text{[Diagram 1]} + \text{[Diagram 2]}}_{\tilde{\Omega}_4^{\text{tree}}} + \underbrace{\text{[Diagram 3]} + \text{[Diagram 4]}}_{\tilde{\Omega}_4^{1\text{-cycle}}} \\
 & + \underbrace{\text{[Diagram 5]}}_{\tilde{\Omega}_4^{2\text{-cycle}}} + \underbrace{\text{[Diagram 6]}}_{\tilde{\Omega}_4^{3\text{-cycle}}}
 \end{aligned}$$

The diagrams represent different topologies of four-point functions (squares) with red edges and black vertices. Diagram 1 is a square with a vertical diagonal. Diagram 2 is a square with a horizontal diagonal. Diagram 3 is a square with a vertical diagonal. Diagram 4 is a square with a horizontal diagonal. Diagram 5 is a square with both diagonals. Diagram 6 is a square with both diagonals and an additional internal edge forming a smaller square.



γ_{cusp} from Resummation

Can resum certain subsets of these diagrams

$$\mathcal{F}_{\text{ladder}}(g, z) = \bigotimes - (g^2) \bigotimes \text{---} \bullet + (g^2)^2 \bigotimes \text{---} \bullet \text{---} \bullet - (g^2)^3 \bigotimes \text{---} \bullet \text{---} \bullet \text{---} \bullet + (g^2)^4 \bigotimes \text{---} \bullet \text{---} \bullet \text{---} \bullet \text{---} \bullet + \dots$$

$$\gamma_{\text{ladder}, g \gg 1} = \mathcal{I}[\mathcal{F}_{\text{ladder}}(g \gg 1, z)] = 4\sqrt{2}g - \frac{4 \log 2}{\pi} + \frac{4e^{-2\sqrt{2}g\pi}}{\pi} + \dots$$

$$\mathcal{F}_{\text{tree}}(g, z) = \bigotimes - (g^2) \bigotimes \text{---} \bullet + (g^2)^2 \left\{ \bigotimes \text{---} \bullet \text{---} \bullet + \frac{1}{2!} \bigotimes \begin{array}{c} \diagup \diagdown \\ \bullet \end{array} \right\} - (g^2)^3 \left\{ \bigotimes \text{---} \bullet \text{---} \bullet \text{---} \bullet + \bigotimes \begin{array}{c} \diagup \diagdown \\ \bullet \end{array} \text{---} \bullet + \frac{1}{2!} \bigotimes \begin{array}{c} \diagup \diagdown \\ \bullet \end{array} \begin{array}{c} \diagup \diagdown \\ \bullet \end{array} + \frac{1}{3!} \bigotimes \begin{array}{c} \diagup \diagdown \diagup \diagdown \\ \bullet \end{array} \right\} + \dots$$

$$\gamma_{\text{tree}, g \gg 1} = \mathcal{I}[\mathcal{F}_{\text{tree}}(g \gg 1, z)] = \frac{8g}{\pi} + \frac{1}{\pi g} + \dots$$



Resummation with 1-Cycle

- We can resum the next class of diagrams

$$\mathcal{I}\left(\text{triangle} \otimes + g^2 \text{triangle} \otimes \bullet + g^4 \text{triangle} \otimes \bullet \bullet + \dots \right)_{g \gg 1}$$

$$\mathcal{I}[\mathcal{F}_{lad}(z)\mathcal{F}_{tri}(z)]_{g \gg 1} \sim 10.71g^5 - 7.98g^4 + 4.86g^3 - 2.07g^2 + 0.47g$$

There are individual $\log g$ terms but they cancel out

- Leading diagram contributes g^6 , resummation only suppresses $\frac{1}{g}$



Resummation with 1-Cycle

- Same happens for the next 1-cycle

$$\mathcal{I}\left(\begin{array}{c} \bullet \text{---} \circ \\ | \quad | \\ \bullet \text{---} \bullet \end{array} + g^2 \begin{array}{c} \bullet \text{---} \circ \\ | \quad | \quad \bullet \\ \bullet \text{---} \bullet \end{array} + g^4 \begin{array}{c} \bullet \text{---} \circ \\ | \quad | \quad \bullet \text{---} \bullet \\ \bullet \text{---} \bullet \end{array} + \dots \right)_{g \gg 1}$$

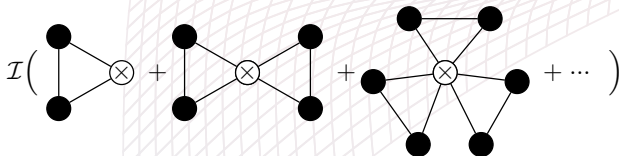
$$\mathcal{I}[\mathcal{F}_{\text{box}}(z)\mathcal{F}_{\text{ladder}}(g, z)]_{g \gg 1} \sim 1.88g^7 - 2.13g^6 - 0.04g^5 + 0.83g^4 - 0.73g^3 + 0.36g^2 - 0.08g$$

- Leading diagram contributes g^8 , resummation again suppresses $\frac{1}{g}$



Open Questions and Outlook

- ▶ How do we systematically determine which class of diagrams are good to resum?
- ▶ γ_{cusp} contribution of multi-cycle diagrams



- ▶ Hierarchy and cancellation of transcendental numbers that appear
- ▶ Resumming $\mathcal{F}(g, z)$



Thanks for Listening!

