Generalizations of kinematics for loop-integrands

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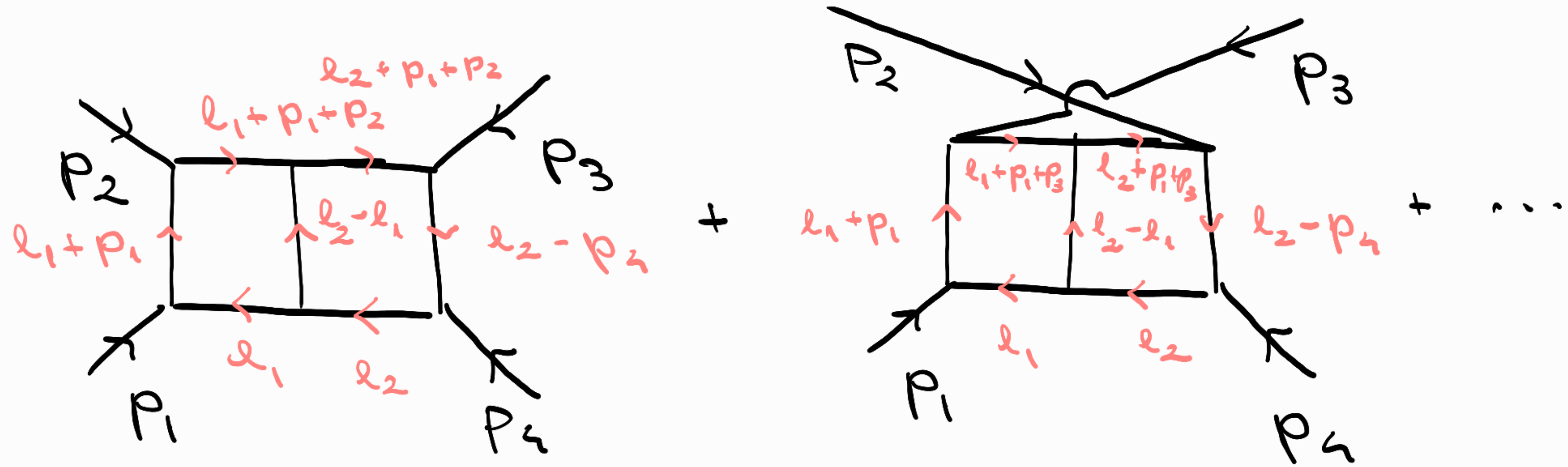
Nov 08, 2025

Loop-level integrands are ambiguous in non-planar theories

$I(\ell_1,\ell_2) =$ 

If we try to define $I(\ell_1,\ell_2)$ and construct it recursively by cuts, we face a problem: which cut we access depends on the labelling of the diagram.

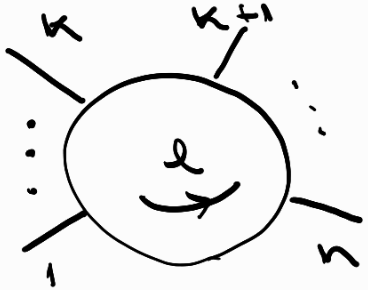
For example, the meaning of

$$\ell_1^2 = (\ell_1+p_1+p_2)^2 = 0$$

can change between diagrams.

Even for planar theories, integrands have pathologies.

E.g. We might want to recover one-loop integrands recursively using unitarity.

$I_n =$ 

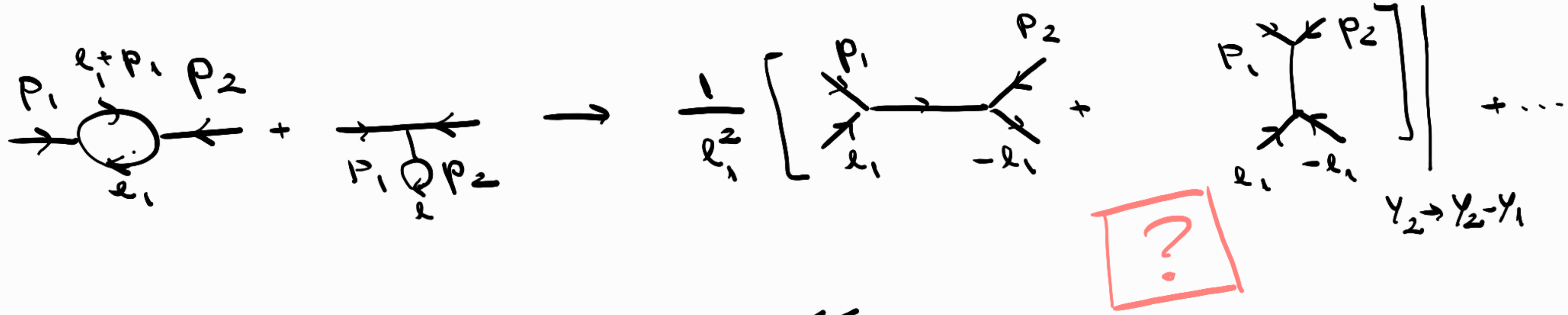
$$Y_i = (\ell + p_1 + \dots + p_{i-1})^2$$

$$X_{ij} = (p_i + \dots + p_{j-1})^2$$

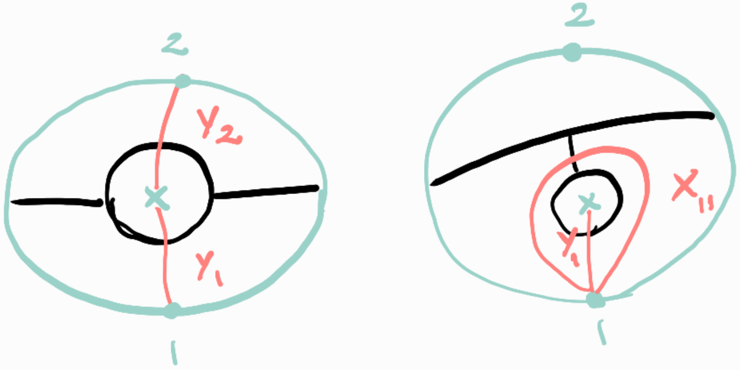
$$I_n(z) = I_n(Y_i - z)$$

$$\Rightarrow \quad I_n = \oint dz \, \frac{I_n(z)}{z} = \sum_{i=1}^{n} \frac{1}{Y_i} \operatorname*{Res}_{Y_i=0} I_n \Big|_{Y_j \to Y_j - Y_i}$$

$$= \sum_{i=1}^{n} \frac{1}{Y_i} \hat{A}^{\text{tree}}_{n+2}(+,1,\dots,n,-) \Big|_{Y_j \to Y_j - Y_i}$$



Surfaceology generalizes what we mean by kinematical data.



$$\to \frac{1}{\ell_1^2}\left[\frac{\hat{N}_u}{Y_2}+\frac{\hat{N}_s}{X_{11}}\right]\Bigg|_{Y_2\to Y_2-Y_1}$$

While Y_2 and X_{11} are zero in usual momentum-space assignments, in the surface picture they are non-zero.

Can surfaceology be used to define the non-planar integrand of $N=4$ SYM?

$\Rightarrow$ Each order in the $\frac{1}{N}$ expansion corresponds to a fixed-genus surface.

Within this genus, we can consistently define the loop integrand using kinematics (curves) on that surface.

The question remains: can we relate the curves defined at different genus?

Thank You!

Other work (pure spinors, SUGRA, manifest symmetry, worldline, strings, physical operators, amplitudes, amplitudes...)

arXiv 2508.19601

arXiv 2508.19748

arXiv 2508.19744