[2510.23080]

# Double Copy from Ambitwistor Worldline


Joonhwi Kim (Caltech)



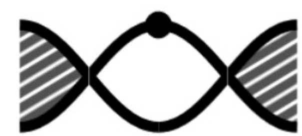
CA Amps 11/08/25 @ UCLA


 $= 0$

# Symplectic Perturbation Theory


Joon-Hwi Kim (Caltech)


CA Amps 03/18/23 @ UCSD

$\{\ ,\ \} =$  $+$  $-$ 

# Canonical

$$\{x, x\} = 0$$

$$\{x, P\} = 1$$

$$\{P, P\} = 0$$

$$\int d\tau \left( P\dot{x} \right)$$

| Canonical | Covariant |
|---|---|
| $\{x, x\} = 0$ | $\{x, x\} = 0$ |
| $\{x, P\} = 1$ | $\{x, p\} = 1$ |
| $\{P, P\} = 0$ | $\{p, p\} = F(x)$ |
| $\int d\tau \left( P\dot{x} \right)$ | $\int d\tau \left( p\dot{x} + A(x)\,\dot{x} \right)$ |

| Canonical | Covariant |
| --- | --- |
| $\{x,x\}=0$ | $\{x,x\}=0$ |
| $\{x,P\}=1$ | $\{x,p\}=1$ |
| $\{P,P\}=0$ | $\{p,p\}=F(x)$ |
| $\int d\tau \left( P\dot{x} \right)$ | $\int d\tau \left( p\dot{x} + A(x)\,\dot{x} \right)$ |
| $H=(P-A(x))^2$ | $H=p^2$ |

# Covariant

## Feynman's proof of the Maxwell equations

Freeman J. Dyson
*Institute for Advanced Study, Princeton, New Jersey 08540*

(Received 3 April 1989; accepted for publication 19 April 1989)

Feynman's proof of the Maxwell equations, discovered in 1948 but never published, is here put on record, together with some editorial comments to put the proof into its historical context.

# Covariant

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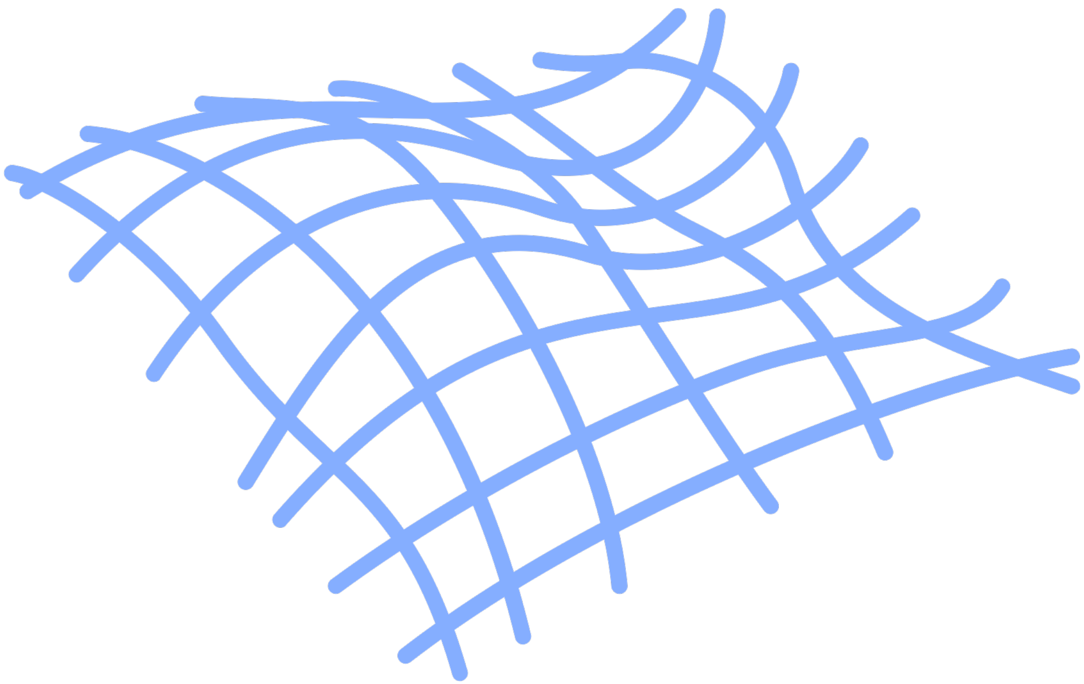
$$\{p, p\} = F(x)$$ Noncanonical

$$\{\{p_{[\mu}, p_\nu\}, p_{\rho]}\} = 0 \quad \Longrightarrow \quad \partial_{[\rho} F_{\mu\nu]} = 0$$

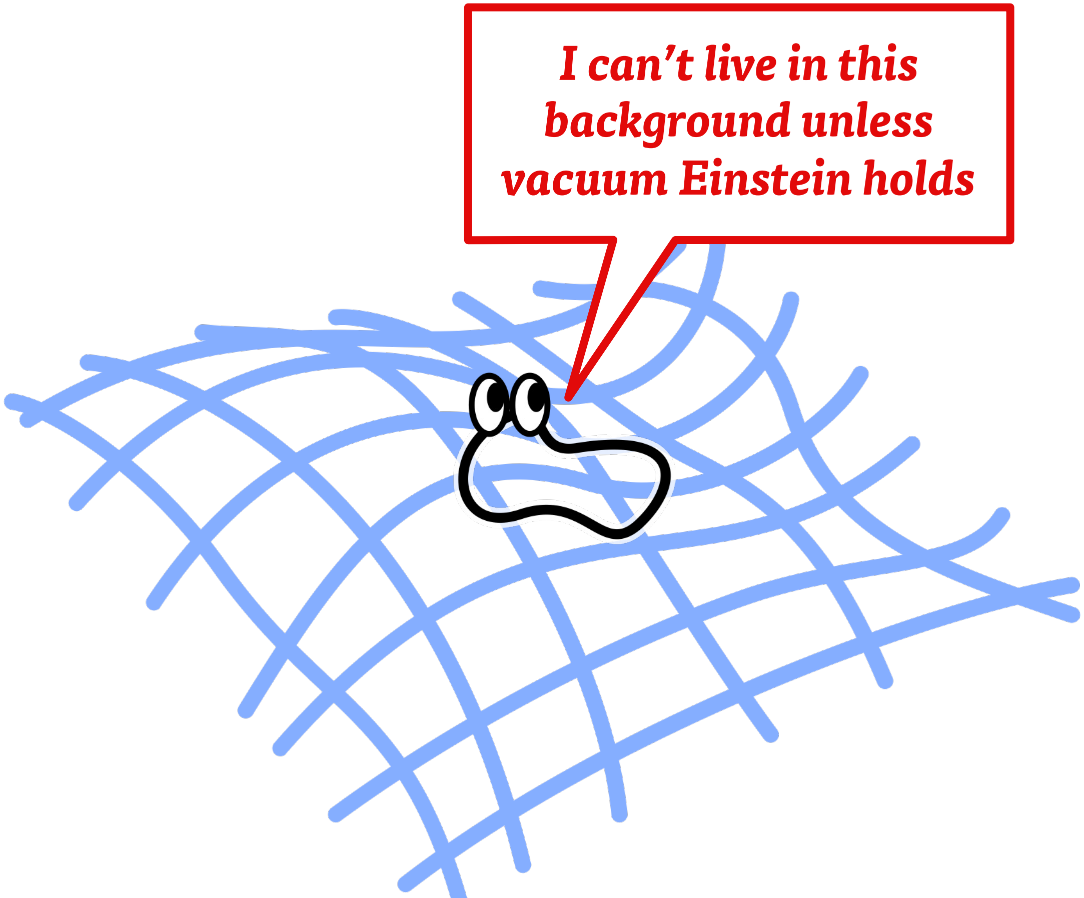
Jacobi

Field Eqn
(Magnetic)

“**Field Equations** *from*
**Consistency of Particle** *Mechanics*”

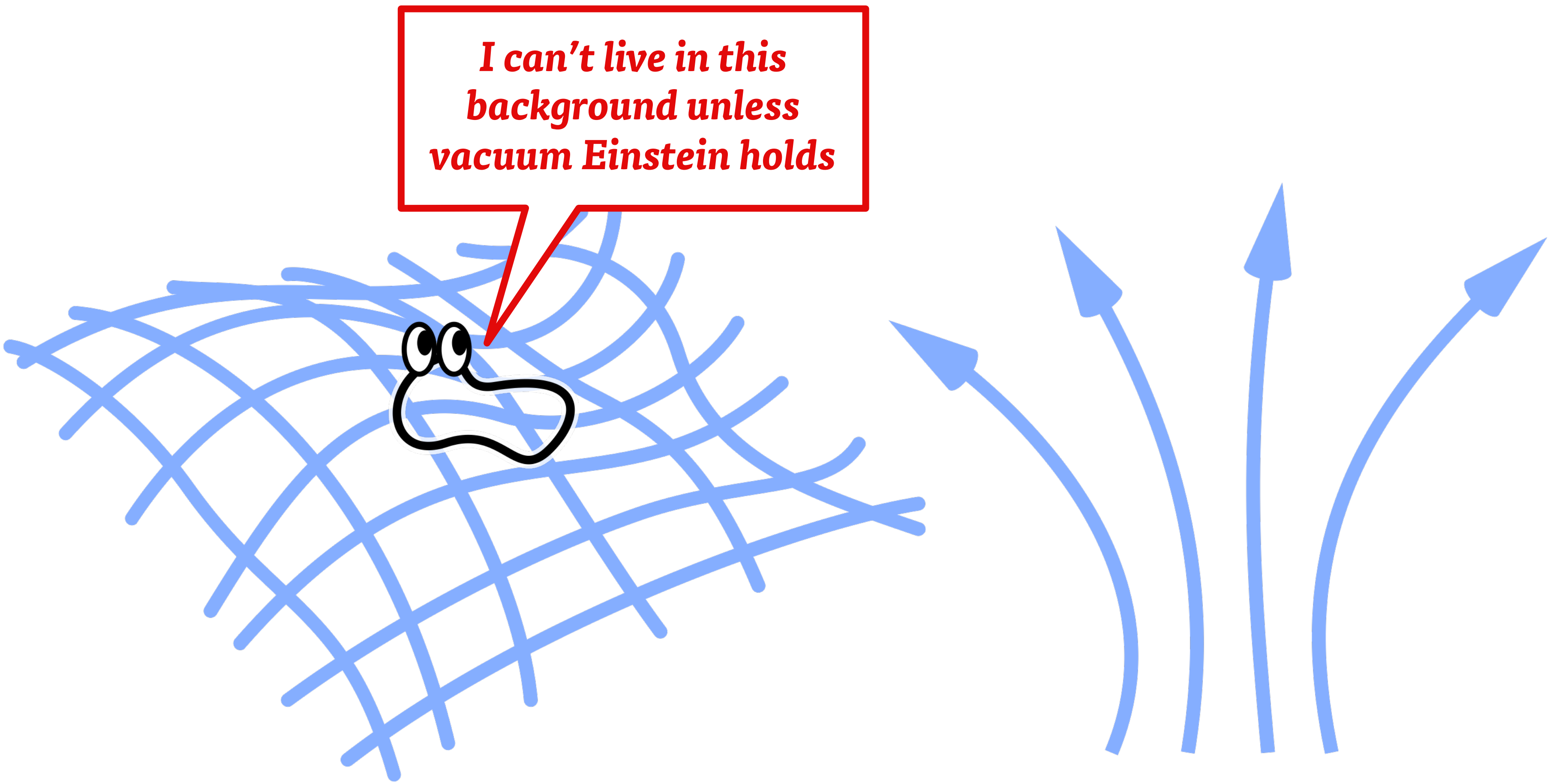


# “**Field Equations** *from* **Consistency of Particle** *Mechanics*”



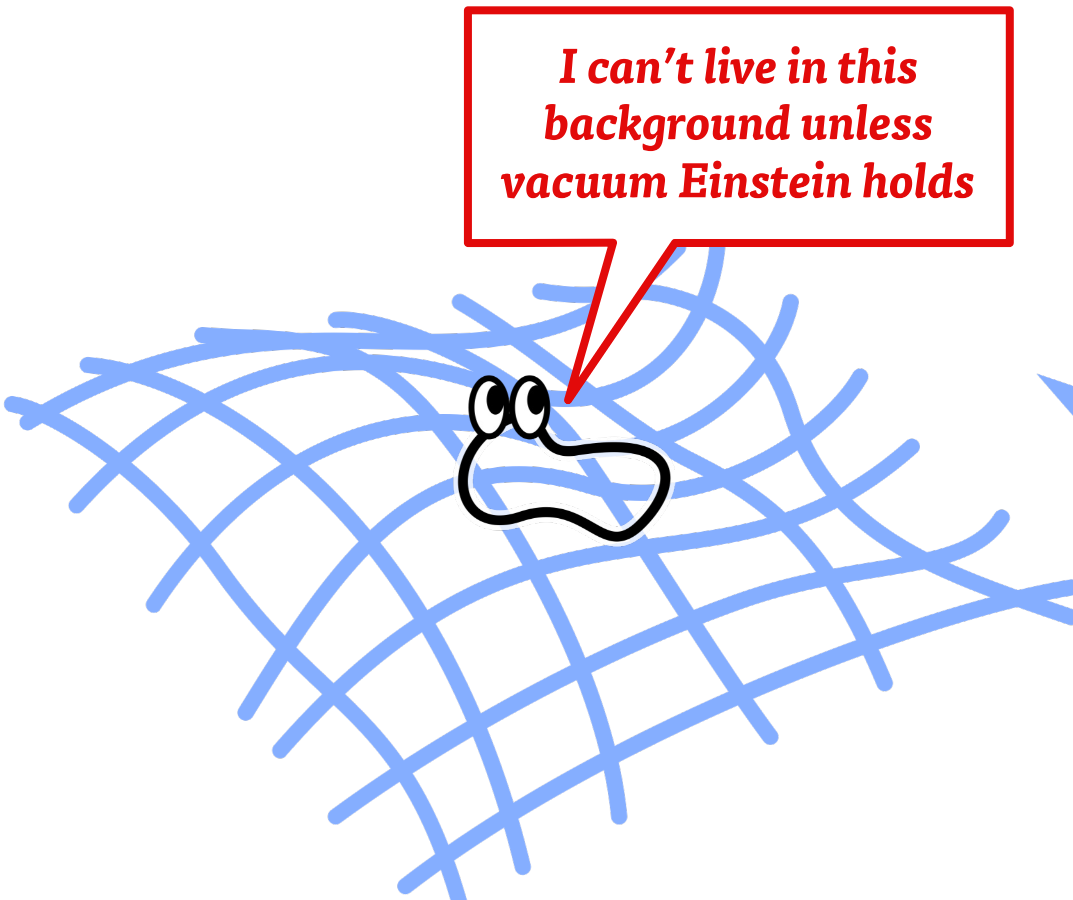

*No Weyl anomaly on the worldsheet*

# “**Field Equations** *from* **Consistency of Particle** *Mechanics*”

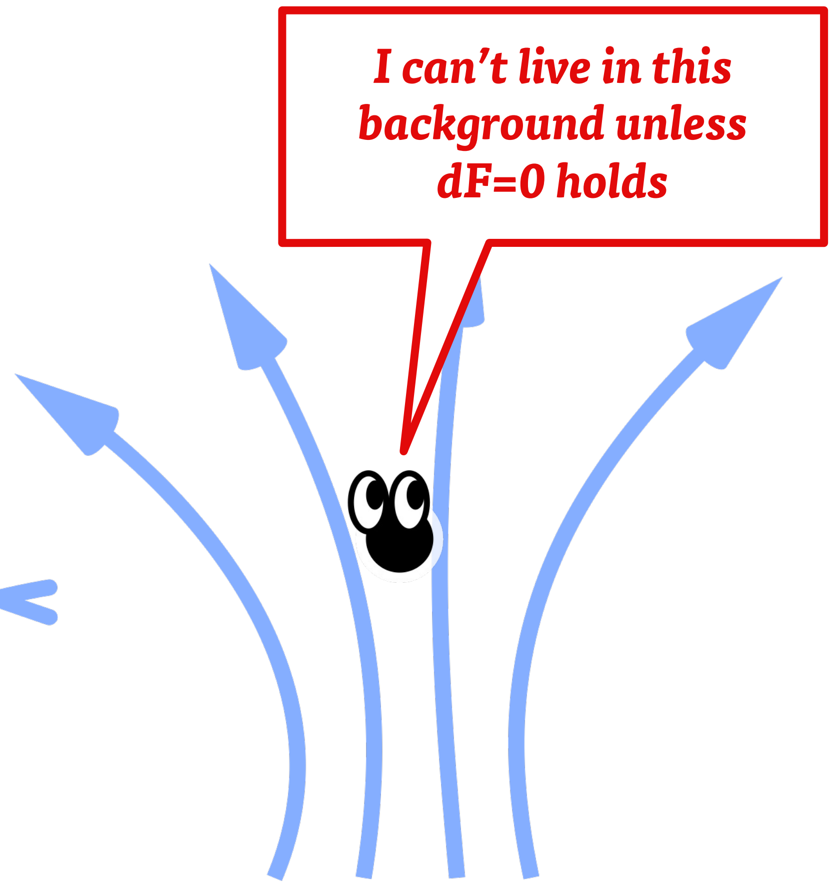



*No Weyl anomaly on the worldsheet*

# “**Field Equations** *from* **Consistency of Particle** *Mechanics*”

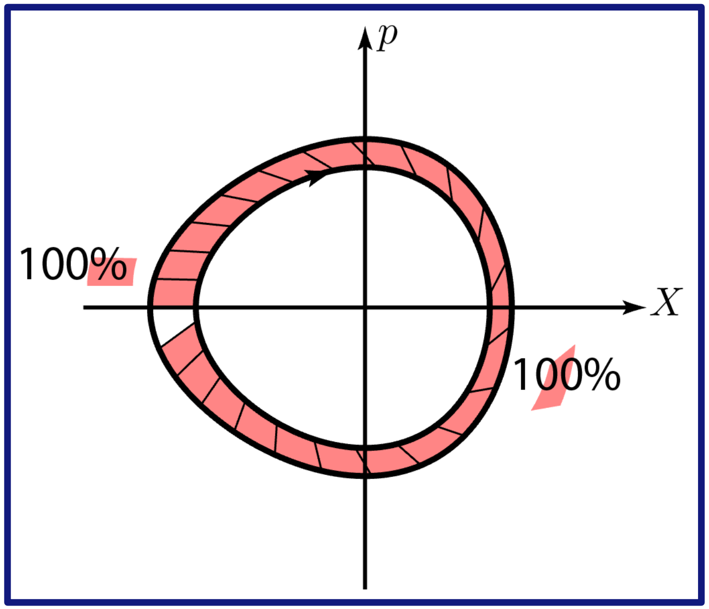



*No Weyl anomaly on the worldsheet*




*Conservation of phase space area*

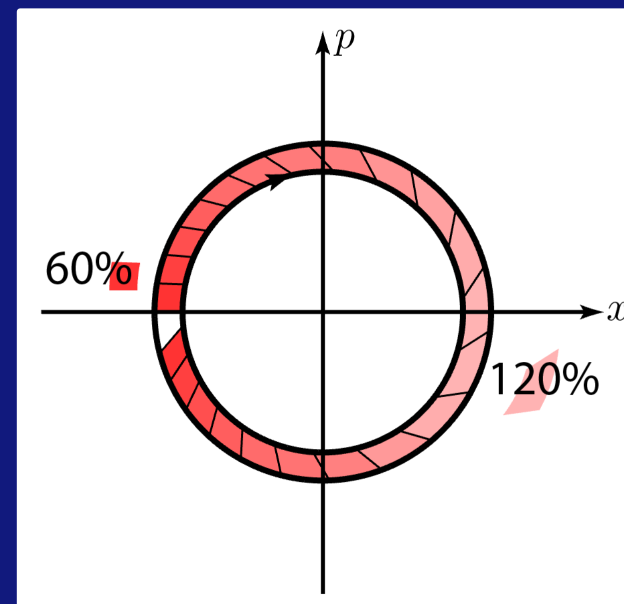
# Canonical




**Unitarity** (✓)

*Gauge Invariance* (X)

# Covariant




*Unitarity* (X)

***Gauge Invariance*** (✓)

# Covariant

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$$\{\{p_{[\mu}, p_\nu\}, p_{\rho]}\} = 0 \quad \Longrightarrow \quad \partial_{[\rho} F_{\mu\nu]} = 0$$

Jacobi

Field Eqn
(Magnetic)

# Covariant

## A Connection Between the Einstein and Yang-Mills Equations

Communications in Mathematical Physics


L. J. Mason*,** and E. T. Newman**

Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA 15260, USA

$$\{p, p\} = F(x)$$ *Noncanonical*

$$\{\{p_\mu, p_\rho\}, p^\rho\} = 0 \quad \Longrightarrow \quad \partial^\rho F_{\mu\rho} = 0$$

*Mason-Newman "Postulate"*

*Field Eqn (Electric)*

# Known Fact:

Field Equations from Bosonic Worldline

[Feynman ‘48] [Mason-Newman ‘89]

# Known Fact:

Field Equations from Bosonic Worldline

[Feynman ‘48] [Mason-Newman ‘89]

# New Fact:

Field Equations from **SUSY** Worldline

[Joonhwi 2510.23080]

# Covariant

Feynman '48

$$\{\{p_{[\mu}, p_\nu\}, p_{\rho]}\} = 0$$

Mason-Newman '89

$$\{\{p_\mu, p_\rho\}, p^\rho\} = 0$$

$$\{\{p_\mu, p_\nu\}, p_\rho\}\, \psi^\mu \psi^\nu \psi^\rho \;=\; 0$$

$$\{\{p_\mu, p_\nu\}, p_\rho\}\, \psi^\mu\, \{\psi^\nu, \psi^\rho\} \;=\; 0$$

Joonhwi '25

# Covariant

*Feynman '48*

$$\{\{p_{[\mu}, p_\nu\}, p_{\rho]}\} = 0$$

*Mason-Newman '89*

$$\{\{p_\mu, p_\rho\}, p^\rho\} = 0$$

*Some Eqn about* $\mathbf{Q = p\psi}$*,*
*computing* **PB²**

*Joonhwi '25*

# Covariant

arXiv > hep-th > arXiv:2510.23080

Search...

Help

High Energy Physics – Theory

[Submitted on 27 Oct 2025]

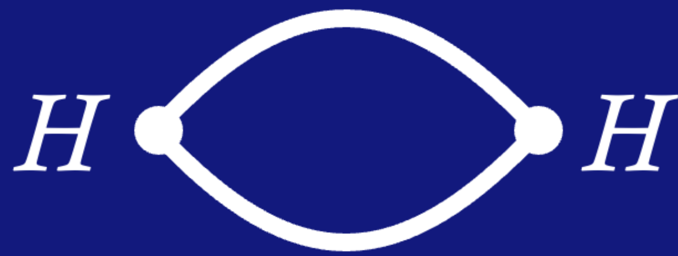
## Double copy and the double Poisson bracket

Joon-Hwi Kim

We derive first-order and second-order field equations from ambitwistor spaces as phase spaces of massless particles. In particular, the second-order field equations of Yang-Mills theory and general relativity are formulated in a unified form

$$\{\{H, H\}\}_{\nabla} = 0$$

# Covariant




$$= 0$$

$$H_{\mathrm{YM}} = p^2 + \bar{\theta}\theta\, F(x)\, \psi\psi$$

$$H_{\mathrm{GR}} = p^2 + \bar{\psi}\psi\, R(x)\, \bar{\psi}\psi$$

$$\{\{H, H\}\}_{\nabla} = 0$$

# Canonical

$$\{\{H, H\}\}_{\nabla} = 0$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$\{\{\widehat{H}, \widehat{H}\}\} = 0$$

# Canonical

$$\{\{H, H\}\}_{\nabla} = 0$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$\{\{\widehat{H}, \widehat{H}\}\} = 0$$

$$\widehat{H} = P^2 + V$$

# Canonical

$$\{\{H, H\}\}_{\nabla} = 0$$

$$\big\downarrow \quad \big\downarrow \quad \big\downarrow$$

$$\{\{\widehat{H}, \widehat{H}\}\} = 0$$

$$\widehat{H} = P^2 + V$$

$$\{\{P^2, V\}\} = \{\{V, V\}\}$$

# Canonical

$$\{\{H, H\}\}_{\nabla} = 0$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$\{\{\widehat{H}, \widehat{H}\}\} = 0$$

$$\widehat{H} = P^2 + V$$

**Manifest BCJ** $\quad \Box V = \{\{V, V\}\}$

# Canonical

**"CCK completion" "Un-CCK" "Tower Construction"**

$$\mathscr{A}^{\text{FS}}(y) = y\,F(x) + yy\,DF(x) + \cdots$$

$$\Box\, V = \{\{V, V\}\}$$

$$\Longrightarrow\ \Box_y\, \mathscr{A}^{\text{FS}}(y) = \cdots$$

# Canonical

**"CCK completion" "Un-CCK" "Tower Construction"**

$$\mathscr{A}^{\mathrm{FS}}(y) = y\,F(x) + yy\,DF(x) + \cdots$$

$$\Box\, V = \{\{V, V\}\}$$

$$\Longrightarrow\ \Box_y\, \mathscr{A}^{\mathrm{FS}}(y) = \cdots$$

$$\Longrightarrow\ \underbrace{DF = 0 \quad \mathrm{CCK} = 0 \quad D(\mathrm{CCK}) = 0 \quad \cdots}_{\textit{Giant "Stringy" Multiplet}}$$

# Known Fact:

**KLT** from Ambitwistor **Worldsheet**

[Adamo, Casali, Skinner '15] [Adamo, Casali, Nekovar '18]

# New Fact:

**BCJ** from Ambitwistor **Worldline**

# Symplectic Perturbation Theory


Joon-Hwi Kim (Caltech)


CA Amps 03/18/23 @ UCSD