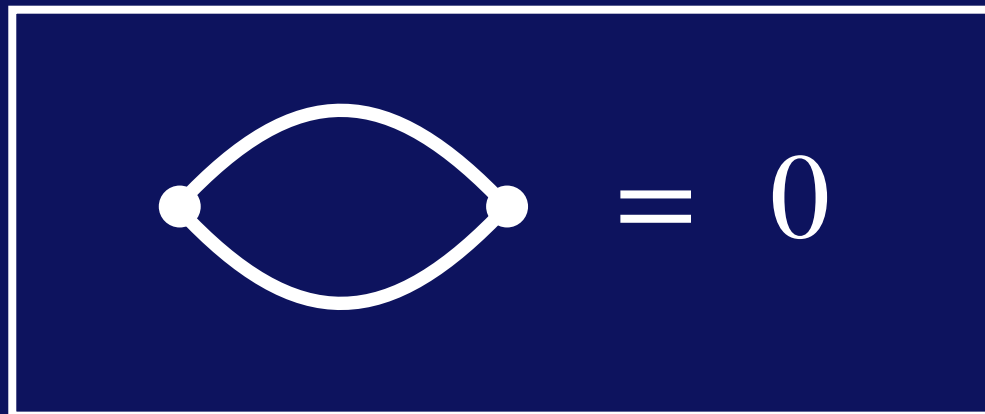


# Double Copy from Ambitwistor Worldline

Joonhwi Kim (Caltech)

CA Amps 11/08/25 @ UCLA



# Symplectic Perturbation Theory

Joon-Hwi Kim (Caltech)

CA Amps 03/18/23 @ UCSD

$$\{ \quad , \quad \} = \text{diagram 1} + \text{diagram 2} - \text{diagram 3}$$

The equation shows the symplectic bracket of two elements, represented by a pair of curly braces containing a comma, equal to the sum of two diagrams minus a third diagram. The diagrams are oriented strand configurations:

- Diagram 1:** Two strands crossing. The region to the right of the crossing is shaded with diagonal lines.
- Diagram 2:** Two strands crossing. The region to the left of the crossing is shaded with diagonal lines.
- Diagram 3:** Two strands crossing twice, forming a full twist. The regions to the far left and far right are shaded with diagonal lines. A small black dot is located at the top of the central loop.

# *Canonical*

---

$$\{x, x\} = 0$$

$$\{x, P\} = 1$$

$$\{P, P\} = 0$$

$$\int d\tau \left( P\dot{x} \right)$$

## ***Canonical***

$$\{x, x\} = 0$$

$$\{x, P\} = 1$$

$$\{P, P\} = 0$$

$$\int d\tau \left( P \dot{x} \right)$$

## ***Covariant***

$$\{x, x\} = 0$$

$$\{x, p\} = 1$$

$$\{p, p\} = F(x)$$

$$\int d\tau \left( p \dot{x} + A(x) \dot{x} \right)$$

## ***Canonical***

$$\{x, x\} = 0$$

$$\{x, P\} = 1$$

$$\{P, P\} = 0$$

$$\int d\tau \left( P \dot{x} \right)$$

$$H = (P - A(x))^2$$

## ***Covariant***

$$\{x, x\} = 0$$

$$\{x, p\} = 1$$

$$\{p, p\} = F(x)$$

$$\int d\tau \left( p \dot{x} + A(x) \dot{x} \right)$$

$$H = p^2$$

# Covariant

## **Feynman's proof of the Maxwell equations**

Freeman J. Dyson

*Institute for Advanced Study, Princeton, New Jersey 08540*

(Received 3 April 1989; accepted for publication 19 April 1989)

Feynman's proof of the Maxwell equations, discovered in 1948 but never published, is here put on record, together with some editorial comments to put the proof into its historical context.

# Covariant

## Feynman's proof of the Maxwell equations

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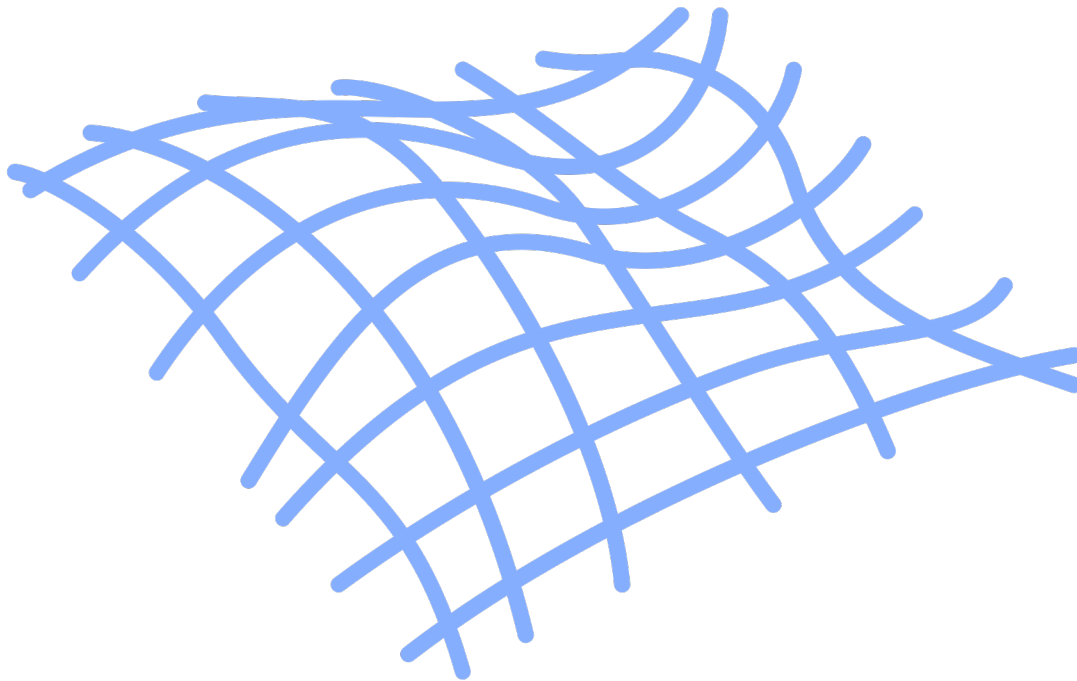
$$\{p, p\} = F(x) \quad \text{Noncanonical}$$

$$\{\{p_{[\mu}, p_{\nu]}, p_{\rho]}\} = 0 \quad \implies \quad \partial_{[\rho} F_{\mu\nu]} = 0$$

*Jacobi*

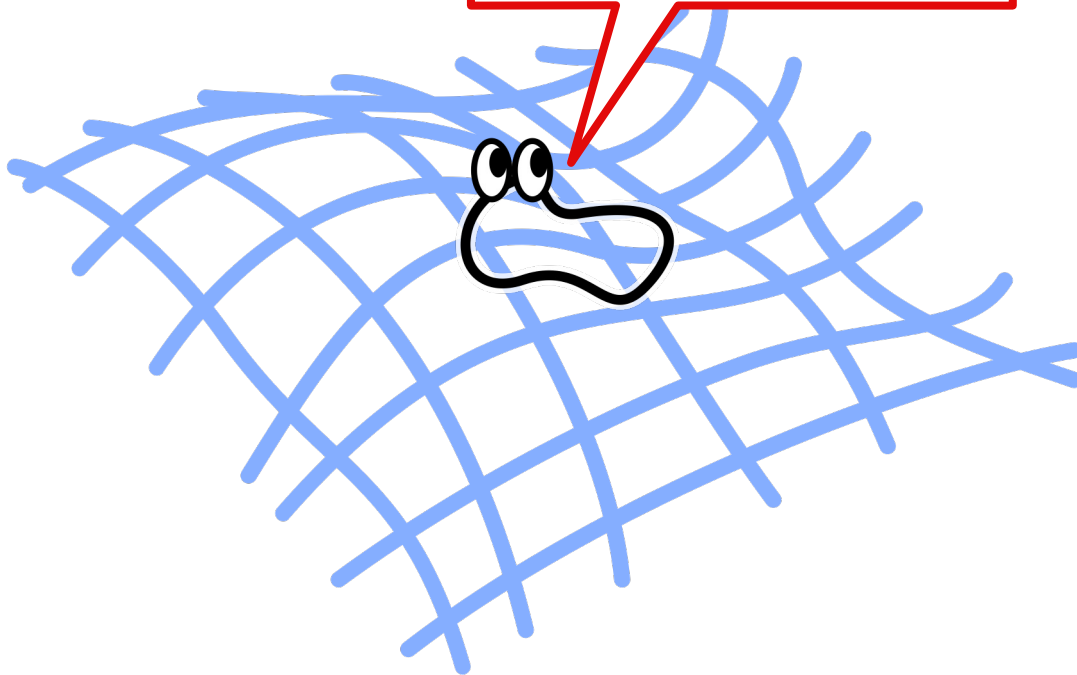
*Field Eqn  
(Magnetic)*

# ***“Field Equations from Consistency of Particle Mechanics”***



# ***“Field Equations from Consistency of Particle Mechanics”***

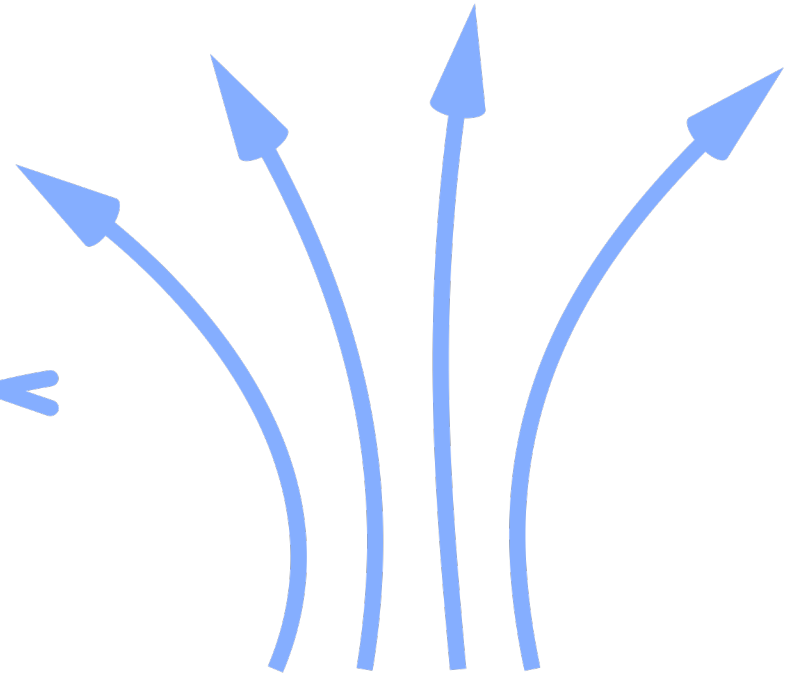
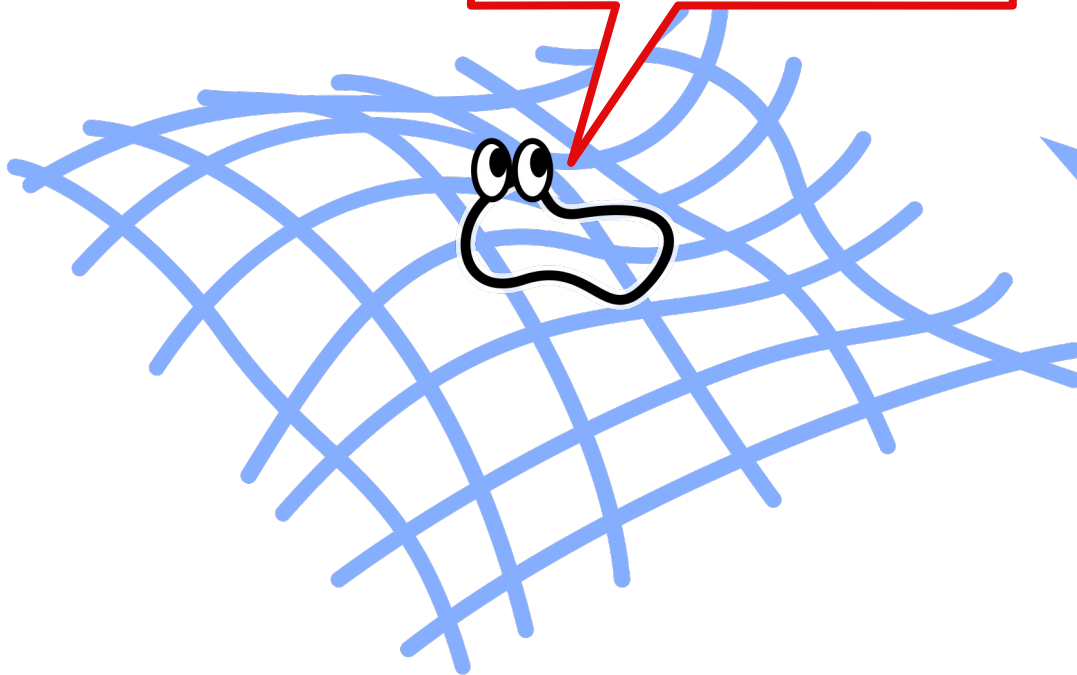
***I can't live in this  
background unless  
vacuum Einstein holds***



*No Weyl anomaly on the worldsheet*

# ***“Field Equations from Consistency of Particle Mechanics”***

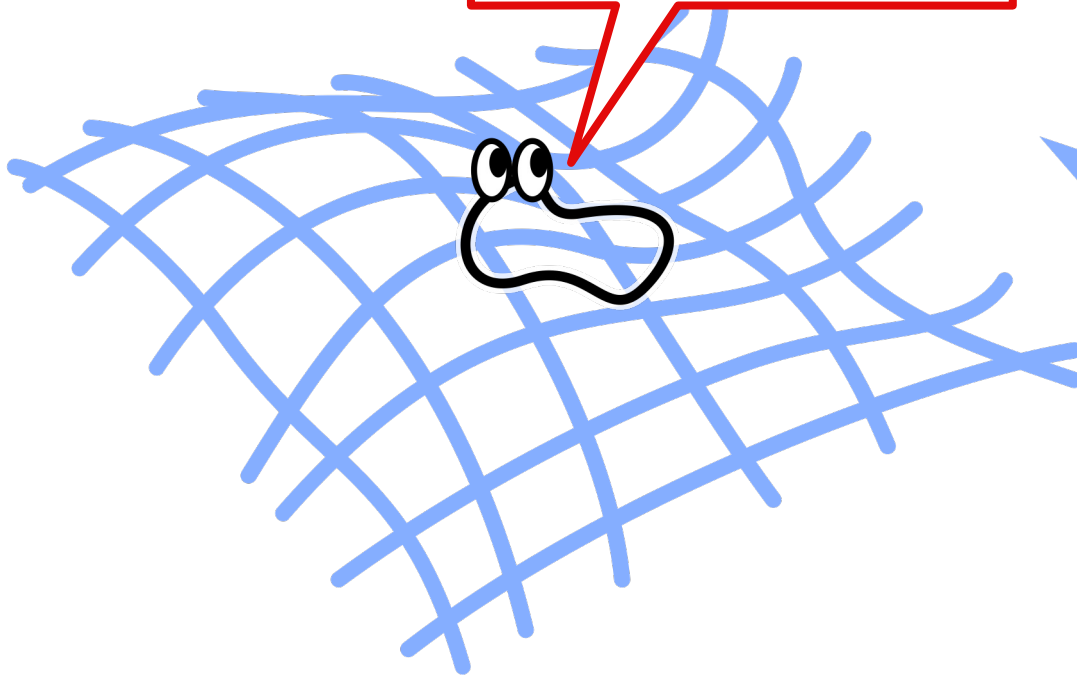
***I can't live in this  
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*No Weyl anomaly on the worldsheet*

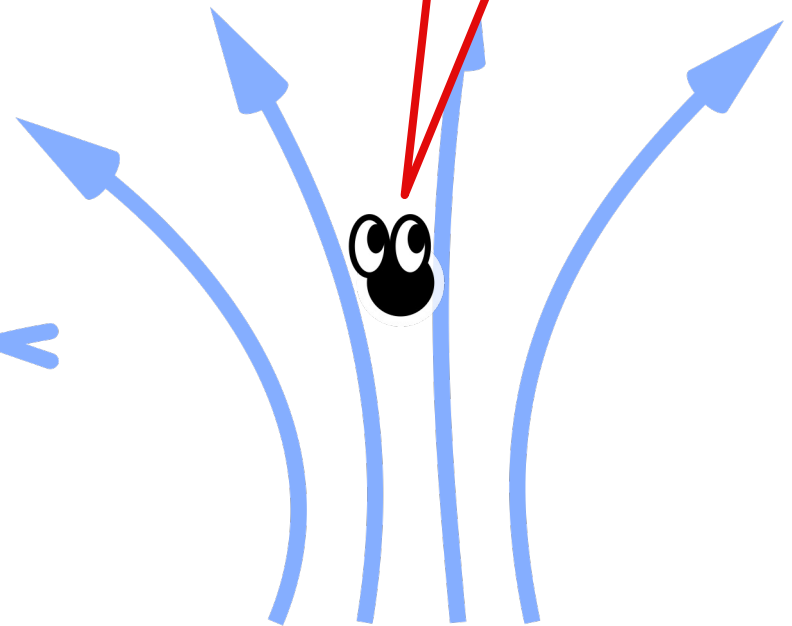
# ***“Field Equations from Consistency of Particle Mechanics”***

***I can't live in this  
background unless  
vacuum Einstein holds***



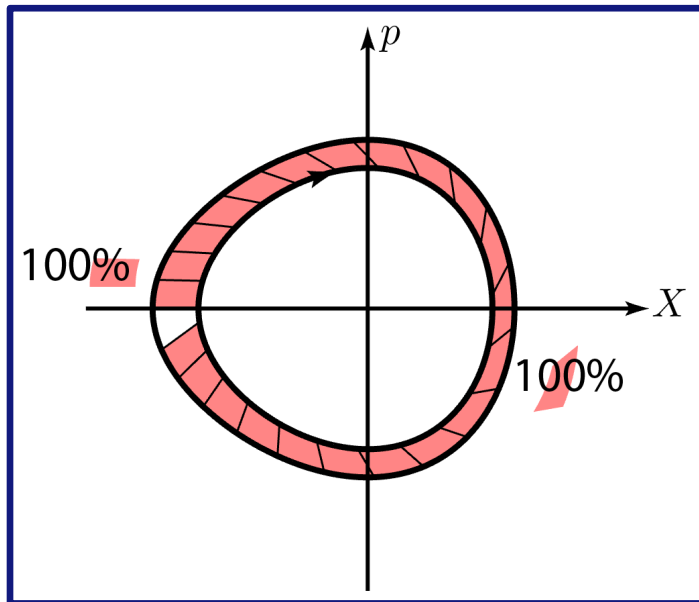
*No Weyl anomaly on the worldsheet*

***I can't live in this  
background unless  
 $dF=0$  holds***



*Conservation of phase space area*

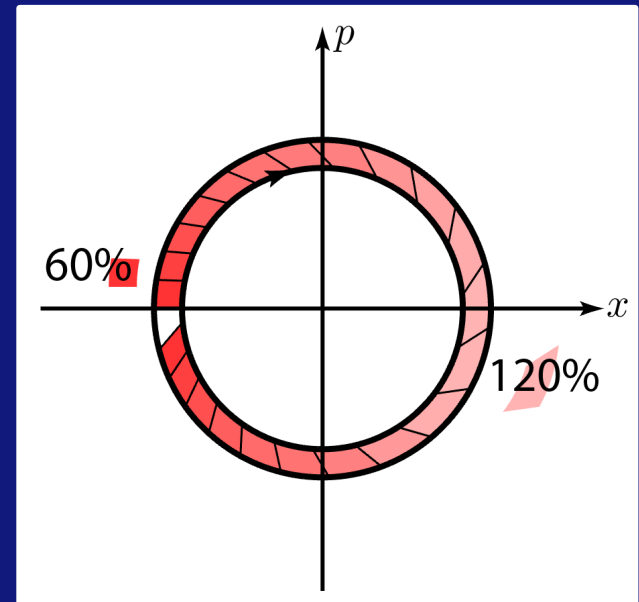
# Canonical



**Unitarity** (✓)

**Gauge Invariance** (✗)

# Covariant



**Unitarity** (✗)

**Gauge Invariance** (✓)

# Covariant

## Feynman's proof of the Maxwell equations

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$$\{p, p\} = F(x) \quad \text{Noncanonical}$$

$$\{\{p_{[\mu}, p_{\nu]}, p_{\rho]}\} = 0 \quad \implies \quad \partial_{[\rho} F_{\mu\nu]} = 0$$

*Jacobi*

*Field Eqn  
(Magnetic)*

# Covariant

## A Connection Between the Einstein and Yang-Mills Equations

L. J. Mason<sup>\*,\*\*</sup> and E. T. Newman<sup>\*\*</sup>

Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA 15260, USA

Communications in  
**Mathematical  
Physics**

© Springer-Verlag 1989

$$\{p, p\} = F(x) \quad \text{Noncanonical}$$

$$\{\{p_\mu, p_\rho\}, p^\rho\} = 0 \quad \implies \quad \partial^\rho F_{\mu\rho} = 0$$

Mason-Newman  
“Postulate”

Field Eqn  
(Electric)

# Known Fact:

Field Equations from Bosonic Worldline

[Feynman '48] [Mason-Newman '89]

# Known Fact:

Field Equations from Bosonic Worldline

[Feynman '48] [Mason-Newman '89]

# New Fact:

Field Equations from **SUSY** Worldline

[Joonhwi 2510.23080]

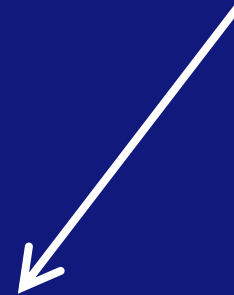
# Covariant

*Feynman '48*

$$\{\{p_{[\mu}, p_{\nu]}, p_{\rho]}\} = 0$$

*Mason-Newman '89*

$$\{\{p_{\mu}, p_{\rho}\}, p^{\rho}\} = 0$$



$$\{\{p_{\mu}, p_{\nu}\}, p_{\rho}\} \psi^{\mu} \psi^{\nu} \psi^{\rho} = 0$$

$$\{\{p_{\mu}, p_{\nu}\}, p_{\rho}\} \psi^{\mu} \{\psi^{\nu}, \psi^{\rho}\} = 0$$

*Joonhwi '25*

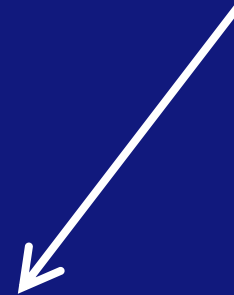
# Covariant

*Feynman '48*

$$\{\{p_{[\mu}, p_{\nu]}, p_{\rho]}\} = 0$$

*Mason-Newman '89*

$$\{\{p_{\mu}, p_{\rho}\}, p^{\rho}\} = 0$$



*Some Eqn about  $\mathbf{Q} = \mathbf{p}\psi$ ,  
computing  $\mathbf{PB}^2$*

*Joonhwi '25*

# Covariant

arXiv > hep-th > arXiv:2510.23080

Search.

Help

## High Energy Physics – Theory

*[Submitted on 27 Oct 2025]*

# Double copy and the double Poisson bracket

Joon-Hwi Kim

We derive first-order and second-order field equations from ambitwistor spaces as phase spaces of massless particles. In particular, the second-order field equations of Yang–Mills theory and general relativity are formulated in a unified form

$$\{\{H, H\}\}_{\nabla} = 0$$

# Covariant



A Feynman diagram enclosed in a white rectangular box. It consists of two external lines, each labeled with the letter  $H$ , connected by a loop. The loop is formed by two curved lines meeting at two vertices, represented by small black dots. To the right of the diagram is an equals sign followed by a zero, indicating that the value of this diagram is zero.

$$H \text{ --- } \text{loop} \text{ --- } H = 0$$

$$H_{\text{YM}} = p^2 + \bar{\theta}\theta F(x) \psi\psi$$

$$H_{\text{GR}} = p^2 + \bar{\psi}\psi R(x) \bar{\psi}\psi$$

$$\{\{H, H\}\}_{\nabla} = 0$$

# *Canonical*

---

$$\{\{H, H\}\}_{\nabla} = 0$$



$$\{\{\hat{H}, \hat{H}\}\} = 0$$

# Canonical

---

$$\{\{H, H\}\}_{\nabla} = 0$$



$$\{\{\hat{H}, \hat{H}\}\} = 0$$

$$\hat{H} = P^2 + V$$

# Canonical

---

$$\{\{H, H\}\}_{\nabla} = 0$$



$$\{\{\hat{H}, \hat{H}\}\} = 0$$

$$\hat{H} = P^2 + V$$

$$\{\{P^2, V\}\} = \{\{V, V\}\}$$

# Canonical

---

$$\{\{H, H\}\}_{\nabla} = 0$$



$$\{\{\hat{H}, \hat{H}\}\} = 0$$

$$\hat{H} = P^2 + V$$

**Manifest BCJ**  $\square V = \{\{V, V\}\}$

# Canonical

*“CCK completion” “Un-CCK” “Tower Construction”*

$$\mathcal{A}^{\text{FS}}(y) = y F(x) + yy DF(x) + \dots$$

$$\Box V = \{ \{ V, V \} \}$$

$$\Rightarrow \Box_y \mathcal{A}^{\text{FS}}(y) = \dots$$

# Canonical

“CCK completion” “Un-CCK” “Tower Construction”

$$\mathcal{A}^{\text{FS}}(y) = y F(x) + yy DF(x) + \dots$$

$$\square V = \{ \{V, V\} \}$$

$$\Rightarrow \square_y \mathcal{A}^{\text{FS}}(y) = \dots$$

$$\Rightarrow \underbrace{DF = 0 \quad \text{CCK} = 0 \quad D(\text{CCK}) = 0 \quad \dots}_{\text{Giant “Stringy” Multiplet}}$$

Giant “Stringy” Multiplet

# **Known Fact:**

**KLT** from Ambitwistor **Worksheet**

[Adamo, Casali, Skinner '15] [Adamo, Casali, Nekovar '18]

# **New Fact:**

**BCJ** from Ambitwistor **Worldline**



# Symplectic Perturbation Theory

Joon-Hwi Kim (Caltech)

CA Amps 03/18/23 @ UCSD

$$\{ \ , \ } = \text{diagram 1} + \text{diagram 2} - \text{diagram 3}$$

The equation shows the bracket operation on two strands. The first two terms are positive and represent the two possible resolutions of a crossing: the first with the top-left and bottom-right regions shaded, and the second with the top-right and bottom-left regions shaded. The third term is subtracted and represents a more complex resolution involving a loop. Blue arrows point from the bottom towards the diagrams.

