# Gravity Tree Amplitudes at Infinity

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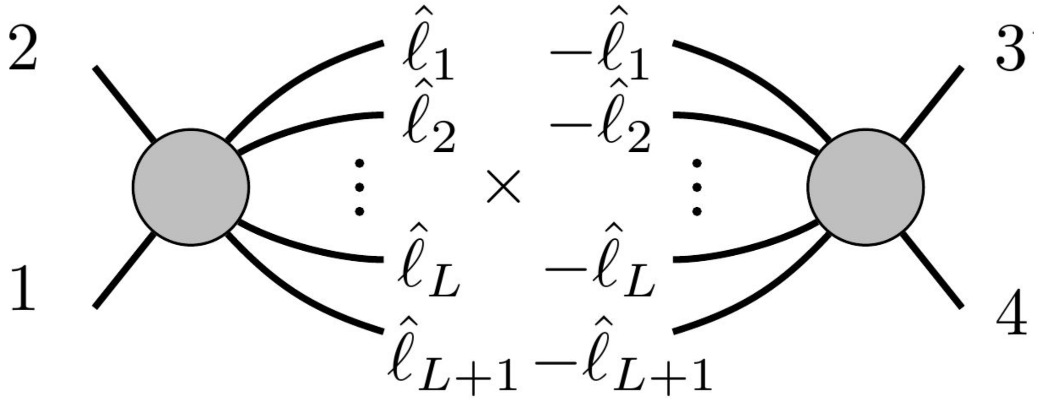
# Cut integrands, chiral shift

- Take an L-loop cut 4-point amplitude and deform momenta by:

$$\tilde{\lambda}_{\ell_k} \longrightarrow \hat{\tilde{\lambda}}_{\ell_k} = \tilde{\lambda}_{\ell_k} + z\,\alpha_k\,\tilde{\eta} \qquad \sum_{k=1}^{L+1} \alpha_k\,\lambda_{\ell_k} = 0$$

- Integrand scaling is improved in d = 4




$$\sim \frac{1}{z^5} \quad (\mathcal{N} = 8)$$

(Edison, Hermann, Parra-Martinez, Trnka 2019)

# 'Poles' at infinity

- Study leading behavior of amplitudes at infinity
- Not usual $p_i^2 = 0$ poles, limit is ambiguous
  - No true factorization
- Shift is a prescription to take the limit
- Different z scalings

# Leading 6pt term from chiral shift

- Example of a leading term at O($z^2$)

$$\mathcal{M}_6(1^-2^+\hat{3}^-\hat{4}^-\hat{5}^+\hat{6}^+) \sim z^2 \frac{[2\eta]^8}{[12]^2[1\eta]^2[2\eta]^2} \frac{\langle 34\rangle^8}{\langle 35\rangle\langle 46\rangle\langle 3\tilde{\eta}]\langle 4\tilde{\eta}]\langle 5\tilde{\eta}]\langle 6\tilde{\eta}]} \times \left( \frac{[\chi_{36}\eta][\chi_{45}\eta]}{\langle 36\rangle\langle 45\rangle} + \frac{[\chi_{34}\eta][\chi_{56}\eta]}{\langle 34\rangle\langle 56\rangle} \right) + \mathcal{O}(z^2)$$

- Unshifted legs do not mix with shifted

## Pole for *n*-point amplitude

- Pattern continues for *n*-point amplitude:

$$\mathcal{M}_n(1^-2^+\hat{3}^+\hat{4}^-5^+\ldots\ \hat{n}^+) \sim (-1)^n z^{n-4} s_{12}\mathcal{M}_3(1^-2^+\eta^+) \\ \times \mathcal{M}_{n-1}(3^-4^-5^+\ldots n^+\omega^+) + \mathcal{O}(z^{n-5})$$

- Similar formulas exist for other helicity choices

# Other patterns

- More legs unshifted or different helicities have different forms:

$$\mathcal{M}_6(1^-2^-3^+\hat{4}^+\hat{5}^+\hat{6}^+) \sim -z^3 \frac{\langle 12\rangle^8[1n][2n][3\eta]\alpha_4\alpha_5\alpha_6}{\langle 14\rangle\langle 15\rangle\langle 16\rangle\langle 24\rangle\langle 25\rangle\langle 26\rangle\langle 34\rangle\langle 35\rangle\langle 36\rangle}$$

- Leading behavior would vanish due to helicities

# Behavior of YM Amplitudes

- Near-identical ‘factorization’ to the GR case

$$\mathcal{A}(1^-2^+\hat{3}^-\hat{4}^-\hat{5}^+\hat{6}^+) \sim -z^0 \frac{[2\eta]^4}{[12][1\eta][2\eta]} \frac{\langle 34\rangle^4}{\langle 3\tilde{\eta}]\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 6\tilde{\eta}]}$$

- Similar *n*-point formula

$$\mathcal{A}(1^-2^+\hat{3}^-\hat{4}^-\hat{5}^+\cdots\hat{n}^+) \sim -z^0\mathcal{A}(1^-2^+\eta^+)\mathcal{A}(3^-4^-5^+\cdots n^+\omega^+)$$

# Outlook and goals

- Understand UV behavior on cut using tree amplitudes at infinity
- Catalogue patterns present at infinity
- Write $n$-point formulas for all possible helicities

Thank you for your time!