A SURPRISING OBSERVATION FOR 5-LOOP INTEGRAND IN $\mathcal{N}=8$ SUPERGRAVITY

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Dec 6th, QCD Meets Gravity Workshop
work in progress
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MOTIVATION

- More examples for loop amplitudes

- UV property of $\mathcal{N}=8$ SUGRA

- Critical dimension: the lowest dimension where UV divergence first appears

\[ D_c = 4 + \frac{6}{L}, \quad L=2,3,4 \]

- $D_c = \frac{26}{5}$ in 5-loop?
ANSATZ APPROACH

• if it works, simplest way to do it: BCJ with master diagrams

• various ansatz for master diagrams w/ parameters 1k ~120k

• attempts for high power of loop momenta in the same spirit of 1511.06652 [Mogull, O’Connell]
  2-loop 5 gluons YM

• not succeeded yet, despite considerable effort
ANSATZ APPROACH

• building ansatz with or without BCJ involves guesswork

• large linear system needed to be solved

LESSON:
We want method without guesswork & no large linear system
CONTACT TERM APPROACH

• **a constructive way** to build the integrand, no guesswork, no large linear system

• contact terms approach: add contact terms to **incomplete gravity amp** to satisfy every cut

![Diagram of cut and contact]

• if amplitude $\sim \frac{stuM_4^{tree}s^2f(l)}{l_5^2l_6^2...l_{20}^2}$, $[f(l)]= [M]^{12}$

  hundred thousand cuts, through $N^6$-max needed to be checked

• However, **miracle** happens so contacts are either relatively **simple** or **zero** and have a new structure.
CONTACT TERM APPROACH

• starting with the known 5-loop SYM amps

\[
\sum_{i} \int dl_5 \cdots dl_9 \frac{n_i^{\text{SYM}}}{\prod_{\alpha_i} P_{\alpha_i}} \quad \Rightarrow \quad \hat{M} = \sum_{i} \int dl_5 \cdots dl_9 \frac{(n_i^{\text{SYM}})^2}{\prod_{\alpha_i} P_{\alpha_i}}
\]

max-cut works by construction

\[N^1\text{max-cut}\]

works by BCJ

\[N^2\text{max-cut}\]

does not work in general

add contact term to make it work
**CONTACT TERM APPROACH**

- no contact terms in **max-cut** and **N-max-cut**
- non-zero contacts start to appear at **N²-max-cut**
- how to obtain a contact

\[
\hat{M} = \hat{M} \bigg|_{\text{cut}} + \text{cut}
\]

define an off-shell contact from the on-shell result

\[
\hat{M} = \hat{M} + \text{cut} + \ldots
\]

- \( \hat{M} \) satisfies all **N²-max-cuts**
- iteratively check cuts and construct amp through **N⁶-max-cut**
CONTACT TERM APPROACH

\[\begin{align*}
N^2 & \quad (stuM_4^{\text{tree}})g(s,t)f_1(l) & [f_1(l)] & = [M]^8 \\
N^3 & \quad (stuM_4^{\text{tree}})g(s,t)f_2(l) & [f_2(l)] & = [M]^6 \\
N^4 & \quad (stuM_4^{\text{tree}})g(s,t)f_3(l) & [f_3(l)] & = [M]^4 \\
N^5 & \quad (stuM_4^{\text{tree}})g(s,t)f_4(l) & [f_4(l)] & = [M]^2 \\
N^6 & \quad (stuM_4^{\text{tree}})g(s,t)f_5(l) & [f_5(l)] & = [M]^0 \\
\end{align*}\]

\[g(l) = a_1 s^2 + a_2 st + a_3 t^2\]

**Computational complexity of cuts**

- analytically
- numerically
CONTACT TERM APPROACH

- easy and fast way to obtain SUGRA cut by applying KLT to known 5-loop SYM cuts

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**HOW TO OBTAIN SUGRA CUT**

- SUGRA cuts are expressed by color-ordered SYM cuts

\[
\text{(SUGRA cut)} = \sum_{N=8 \text{ states}} M_{n_1}^{\text{tree}} M_{n_2}^{\text{tree}} \ldots M_{n_k}^{\text{tree}}
\]

1. **KLT relation**

\[
M_n^{\text{tree}} = \sum_{i,j} K_{ij} (A_n^{\text{tree}})_i (\tilde{A}_n^{\text{tree}})_j
\]

2. **(state sum of N=8)**

\[
= (\text{state sum of N=4}) \times (\text{state sum of N=4})
\]

(\text{gravity cut})

\[
= \sum_{i_1 \ldots i_k} \sum_{j_1 \ldots j_k} \tilde{K}_{i_1 \ldots i_k} \left[ \sum_{N=4 \text{ states}} (A_{n_1}^{\text{tree}} \ldots A_{n_k}^{\text{tree}})_{i_1 \ldots i_k} \right] \sum_{N=4 \text{ states}} (\tilde{A}_{n_1}^{\text{tree}} \ldots \tilde{A}_{n_k}^{\text{tree}})_{j_1 \ldots j_k}
\]

**color-ordered SYM cuts can be obtained from known 5-loop SYM amps**

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MIRACLES OF SIMPLICITY

(gravity cut) - (incomplete amp on cut) \(\rightarrow\) contact

- Most of contacts are zero
  
  \(N^2\) and \(N^3\) contacts most complicated.

- File sizes for non-zero cuts are mostly few kB

- Contacts for cuts w/ only three or four-point blob always factorized

- Sample results for contact at \(N^2\) level

\[
\left[ 2s^3 - 2s^2u - 3su^2 + 4s^2(2k_1 \cdot l_6) + \ldots \right] \left[ s^2u + 2su^2 + 3u^3/2 - s^2(2k_1 \cdot l_6) + \ldots \right]
\]
MIRACLES OF SIMPLICITY

- nontrivial, $\text{KLT} - \sum_i \frac{(n_i^{\text{SYM}})^2}{p_i^2}$. Why should it factorize?

- Clearly there is a double copy-like formula for contacts.

- For 5 or higher point blobs, double copy-like formula does not imply factorization but still simple.

\[-s^6 - 2s^5u - 2s^4u^2 - 2s^5(2k_1 \cdot l_5) - 4s^4u(2k_1 \cdot l_5) - 4s^4(2k_1 \cdot l_5)^2 + \ldots\]
SUMMARY

- use any known SYM representation to build the integrand

- Naive double copy is almost right up to contact terms

- contact terms miraculously seem to also obey double copy-like formulae
SUMMARY

• miracle happens in our interesting 5-loop case
  (gravity cut) - (incomplete amp on cut) → contact

• What are these formulae? How to derive them?

• See J.J.’s talk