Super-Gauss-Bonnet and other Evanescent Operators in Half-Maximal SUGRA

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QCD Meets Gravity

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Evanescent object: object that is zero in 4D, but not in other dimensions

- One-loop pure gravity is finite: only counterterm is Gauss-Bonnet, vanishes for 4D states
  
  [’t Hooft, Veltman]

- First pure gravity divergence is at two loops
  
  [Goroff, Sagnotti; van de Ven]

- This divergence has contributions from GB subdivergence and depends on choice of duality transformations
  
  [Bern, Cheung, Chi, Davies, Dixon, Nohle; Duff, van Nieuwenhuizen]
Ultraviolet properties of gravity are more complex than previously thought

- “Enhanced cancellations”: no known symmetry arguments for lack of divergence [Bern, Davies, Dennen, Huang]
- Vanishing log $\mu^2$ dependence in two-loop SUGRA, can have non-vanishing divergence
  [Bern, Cheung, Chi, Davies, Dixon, Nohle; Bern, Chi, Dixon, AE to appear]
- Half-Maximal SUGRA explored at high loop order, but limited understanding of evanescence [Bern, Davies, Dennen]

Evanescent effects might contribute to $\mathcal{N} = 4$ four-loop divergence
- $\mathcal{N} = 4 \otimes \mathcal{N} = 0$
- SUSY numerator non-zero only for box
- BCJ double copy gives

\[
\int \frac{n_{\mathcal{N}=0,\text{box}}}{\prod_i D_i}
\]

- Evaluate using standard techniques

Can find contributions to gravity evanescence with just a YM calculation
Gauge Invariant Tensor Bases

- \( T_i = a_i(s, t)(\epsilon_1 \cdot \epsilon_3)(\epsilon_2 \cdot \epsilon_4) + b_i(s, t)(\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_3)(\epsilon_3 \cdot \epsilon_4) + \ldots \)

- Organize amplitude using tensor bases
  - [Glover, Tejeda-Yeomans]
  - Map to gravity
  - Explore evanescence

- Project onto tensors using

\[
m_{ij} = \sum_{\text{states}} T_i^* T_j
\]

\[
P_i = (m^{-1})_{ij} T_j^*
\]
F Tensor Basis

- Define a set of gauge invariant tensors

\[ F^\mu_\nu i \equiv i(k^\mu i \epsilon^\nu i - k^\nu i \epsilon^\mu i) \]

\[ F^\mu_\nu F^\nu_\rho F^\rho_\sigma F^\sigma_\mu \equiv (F_i F_j F_k F_l) \quad F^\mu_\nu F^\nu_\mu F^\rho_\sigma F^\sigma_\rho \equiv (F_i F_j)(F_k F_l) \]

\[ F^4_{st} = (F_1 F_2 F_3 F_4) \quad F^4_{tu} = (F_1 F_4 F_2 F_3) \quad F^4_{us} = (F_1 F_3 F_4 F_2) \]

\[ F^{2,2}_s = (F_1 F_2)(F_3 F_4) \quad F^{2,2}_t = (F_1 F_4)(F_2 F_3) \quad F^{2,2}_u = (F_1 F_3)(F_4 F_2) \]

- Need seventh tensor to match one loop coeff.
$F^3$ Insertion

Will use color-ordered four-point tree with insertion of $F^3$ operator

\[ F^3 \equiv \frac{1}{3} \text{tr} F^\mu_\nu F^\nu_\rho F^\rho_\mu \]

Actual tensor will be

\[ T^F_3 \equiv stA^{\text{tree}}_{F^3}(1, 2, 3, 4) \]
Helicity basis

Basis of tensors that can be reduced to helicity configurations in 4D

<table>
<thead>
<tr>
<th>Tensor</th>
<th>Helicities</th>
<th>[m]</th>
<th>Mapping from Other Basis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_0$</td>
<td>$- - ++$</td>
<td>8</td>
<td>$- i/2 (F_{st}^4 + F_{tu}^4 + F_{us}^4) + i/8 (F_{s}^{2,2} + F_{u}^{2,2} + F_{t}^{2,2})$</td>
</tr>
<tr>
<td>$T_1$</td>
<td>$++ + +$</td>
<td>8</td>
<td>$- iF_{st}^4 + i/4 (F_{s}^{2,2} + F_{u}^{2,2} + F_{t}^{2,2})$</td>
</tr>
<tr>
<td>$T_3$</td>
<td>$- + + +$</td>
<td>10</td>
<td>$i/4 T^F_3 - i/4 ((s + t)F_{st}^4 + (s - t)F_{us}^4 + (t - s)F_{tu}^4)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$+ i/16 (s + t)(F_{s}^{2,2} + F_{u}^{2,2} + F_{t}^{2,2})$</td>
</tr>
<tr>
<td>$T_4$</td>
<td>$- - ++$</td>
<td>8</td>
<td>$i/2 (F_{us}^4 - F_{tu}^4) + i/4 (F_{u}^{2,2} - F_{s}^{2,2})$</td>
</tr>
<tr>
<td>$T_5$</td>
<td>$- + - -$</td>
<td>8</td>
<td>$- iF_{st}^4 + i/4 (F_{s}^{2,2} - F_{u}^{2,2} + F_{t}^{2,2})$</td>
</tr>
<tr>
<td>$T_6$</td>
<td>Evanescent</td>
<td>10</td>
<td>$i/2 ((s + t)F_{st}^4 - (3s + t)F_{us}^4 - (s + 3t)F_{tu}^4)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$+ 3i/8 (s + t)(F_{s}^{2,2} + F_{u}^{2,2} + F_{t}^{2,2})$</td>
</tr>
<tr>
<td>$T_7$</td>
<td>Evanescent</td>
<td>10</td>
<td>$- i/2 ((s - t)F_{st}^4 + (s + t)F_{us}^4 - (s + t)F_{tu}^4)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$+ i/8 (s - t)(F_{s}^{2,2} + F_{u}^{2,2} + F_{t}^{2,2})$</td>
</tr>
</tbody>
</table>

Use this basis to examine evanescent properties of gravity
Results of Projections

F basis projects nicely:

\[
\begin{align*}
\frac{T^{F^3}_{stu}}{stu} & - F^4_{st} \left( \frac{4}{3st} + \text{logs} + \pi^2 \text{ terms} \right) \\
& - F^{2,2}_s \left( \frac{1}{s^2} + \text{logs} + \pi^2 \text{ terms} \right) + \text{perms}
\end{align*}
\]

Notably the \( F^3 \) contribution has no transcendental part

Helicity basis projection not as clean
Mapping to Gravity

\[ s t A^{\text{tree}}(1, 2, 3, 4) \otimes X \]

- **Special Combination**

\[
16 s t A^{\text{tree}} = t_8 \mu \nu \rho \sigma \alpha \beta \gamma \delta F^{\mu \nu} F^{\rho \sigma} F^{\alpha \beta} F^{\gamma \delta} = t_8 F^4
\]

\[
= 8(F^4_{st} + F^4_{tu} + F^4_{us}) - 2(F^2_{s} + F^2_{t} + F^2_{u})
\]

- Can map linearized \( F_i \) to linearized \( R_i \)

\[
F_{i \mu \nu} F_{i \rho \sigma} \rightarrow -2R_{i \mu \nu \rho \sigma}
\]

- **Gravity mappings**

\[
t_8 F^4 t_8 F^4 \rightarrow t_8 t_8 R^4
\]

\[
t_8 F^4(F_i F_j F_k F_l) \rightarrow t_8(R_i R_j R_k R_l)
\]

\[
t_8 F^4(F_i F_j)(F_k F_l) \rightarrow t_8(R_i R_j)(R_k R_l)
\]
Looking at $F^3$

- $F^3$ tensor doesn’t map to gravity as simply as $F^4$
- KLT maps $F^3 \rightarrow R^2$ [Broedel, Dixon; Bjerrum-Bohr; He, Zhang]

\[ s\ t \ A^{\text{tree}}(1, 2, 3, 4) \ T^{F^3} = s\ u \ A^{\text{tree}}(1, 2, 4, 3) s\ t \ A_{F^3}^{\text{tree}}(1, 2, 3, 4) \rightarrow s\ t\ u \ \mathcal{M}_{R^2} \]

- $R^2$ contribution to one-loop amplitude is trivial up to loop factors!

\[ \frac{T^{F^3}}{stu} \rightarrow \mathcal{M}_{R^2} \]
Beginnings of Super-GB

- On YM side, $F^3$ is not supersymmetrizable, so only contribute from pure YM
- $\mathcal{N} = 4$ is highest SUGRA that can contain GB
- Helicity basis confirms GB is evanescent [He, Zhang]
- Ongoing work to get off-shell Super-GB
- Connection to string predictions [Green, Rudra]
Conclusions

- Evanescent operators present at one loop even in SUSY gravity
- GB contribution is trivial for one-loop $\mathcal{N} = 4$ SUGRA, no divergence and no log or $\pi^2$
- Potential effects at higher loops
- Re-examine four-loop $\mathcal{N} = 4$ divergence for evanescent contributions
Questions?