An overview of lattice QCD+QED progress for g-2

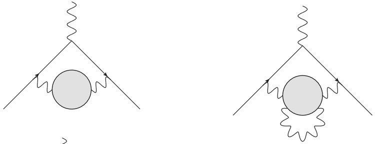
Christoph Lehner (BNL)

December 4, 2018 – Schwinger Fest 2018, UCLA

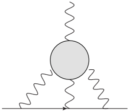
There is a tension of 3.7σ for the muon $a_\mu = (g_\mu - 2)/2$:

$$a_\mu^{\mathrm{EXP}} - a_\mu^{\mathrm{SM}} = 27.4 \underbrace{(2.7)}_{\mathrm{HVP}} \underbrace{(2.6)}_{\mathrm{HLbL}} \underbrace{(0.1)}_{\mathrm{other}} \underbrace{(6.3)}_{\mathrm{EXP}} \times 10^{-10}$$

HVP



HLbL



2019: $\delta a_\mu^{\mathrm{EXP}} \to 4.5 \times 10^{-10}$ (avg. of BNL/estimate of 2019 Fermilab result)

Targeted final uncertainty of Fermilab E989: $\delta a_\mu^{\mathrm{EXP}} \to 1.6 \times 10^{-10}$

$\Rightarrow$ by 2019 consolidate HVP/HLbL, over the next years uncertainties to $O(1 \times 10^{-10})$

There is also a tension of -2.4σ for the muon $a_e = (g_e - 2)/2$:

$$a_e^{\mathrm{EXP}} - a_e^{\mathrm{SM}} = -87\,\underbrace{(28)}_{\mathrm{EXP}}\,\underbrace{(23)}_{\alpha}\,\underbrace{(02)}_{\mathrm{SM}} \times 10^{-14}\,,$$

SM uncertainty far from dominant, however, check of five-loop QED calculation by Aoyama/Kinoshita/Nio is desirable (and a six-loop approximate answer?)

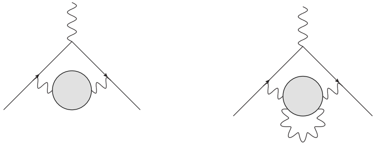
Possible future progress by lattice methods:

- Numerical Stochastic Perturbation Theory Burgio et al. 1998

  "The final goal of this project is ... to push one loop further the computation of electron's g-2"

- Diagrammatic Monte-Carlo Prokof'ev & B.V.Svistunov 1998

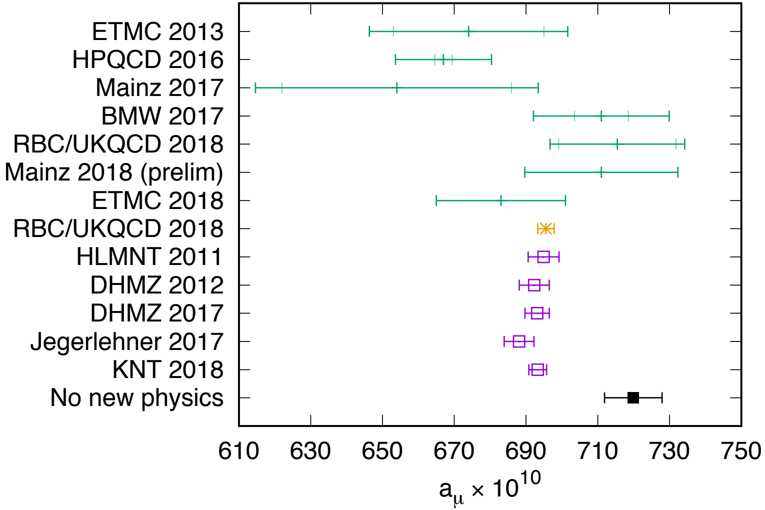
The HVP contribution to the muon g-2



Talks by Mattia Bruno (Mo/4:30), Hartmut Wittig (Tue/3:00), Vera Guelpers (Tue/4:00), Kotaroh Miura (Tue/4:30), Christine Davies (Tue/5:00), Davide Giusti (Tue/5:30), Marina Marinkovic (Wed/10:45), Aaron Meyer (Wed/11:30)

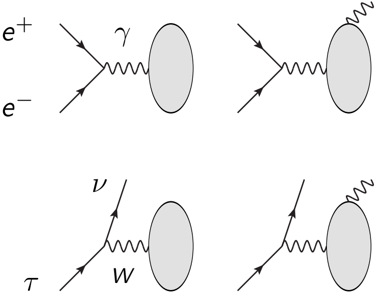
4 hours of lattice talks on HVP

Status of HVP determinations



Green: LQCD, Orange: LQCD+Dispersive, Purple: Dispersive

Dispersive method - Overview



$$e^+e^- \to \mathrm{hadrons}(\gamma)$$

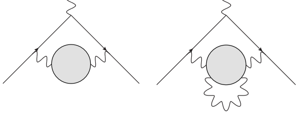
$$J_\mu = V_\mu^{I=1,I_3=0} + V_\mu^{I=0,I_3=0}$$

$$\tau \to \nu\mathrm{hadrons}(\gamma)$$

$$J_\mu = V_\mu^{I=1,I_3=\pm1} - A_\mu^{I=1,I_3=\pm1}$$

Knowledge of isospin-breaking corrections and separation of vector and axial-vector components needed to use τ decay data. This connection can be provided by lattice QCD+QED. (Talk M. Bruno)

Lattice QCD – Time-Moment Representation



Starting from the vector current $J_\mu(x) = i\sum_f Q_f \overline{\Psi}_f(x)\gamma_\mu\Psi_f(x)$ we may write

$$a_\mu^{\mathrm{HVP\ LO}} = \sum_{t=0}^{\infty} w_t C(t)$$

with

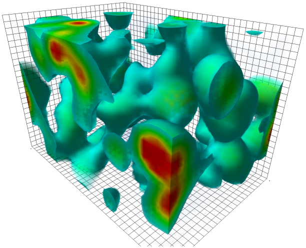
$$C(t) = \frac{1}{3}\sum_{\vec{x}}\sum_{j=0,1,2}\langle J_j(\vec{x},t)J_j(0)\rangle$$

and w_t capturing the photon and muon part of the HVP diagrams (Bernecker-Meyer 2011).

The correlator $C(t)$ is computed in lattice QCD+QED at physical pion mass with non-degenerate up and down quark masses including up, down, strange, and charm quark contributions. The missing bottom quark contributions are computed in pQCD.

Small interlude - Lattice QCD

- Simulate QFT in terms of fundamental quarks and gluons (QCD) on a supercomputer with discretized four-dimensional space-time lattice
- Hadrons are emergent phenomena of statistical average over background gluon configurations to which quarks are coupled
- In this framework draw diagrams only with respect to quarks, photons, and leptons; gluons and their effects are generated by the statistical average.



Lattice QCD action density, Leinweber, CSSM, Adelaide, 2003

Computing resources (example RBC/UKQCD)

The RBC/UKQCD $g-2$ project has used on the order of 10^9 core hours (100k years on a single core) on the Mira supercomputer at Argonne, USQCD clusters at JLab and BNL, the BNL CSI KNL cluster, and the Oakforest and Hokusai supercomputers in Japan.

We have processed on the order of 5 petabytes of QCD data related to this project.

Top 10 positions of the 49th TOP500 in June 2017[11]

Rank	Rmax Rpeak (PFLOPS)	Name	Model	Processor	Interconnect	Vendor	Site country, year	Op s
1	93.015 125.436	*Sunway TaihuLight*	Sunway MPP	SW26010	Sunway[12]	NRCPC	National Supercomputing Center in Wuxi China, 2016[12]	Linux
2	33.863 54.902	*Tianhe-2*	TH-IVB-FEP	Xeon E5–2692, Xeon Phi 31S1P	TH Express-2	NUDT	National Supercomputing Center in Guangzhou China, 2013	Linux
3	19.590 25.326	*Piz Daint*	Cray XC50	Xeon E5-2690v3, Tesla P100	Aries	Cray	Swiss National Supercomputing Centre Switzerland, 2016	Linux
4	17.590 27.113	*Titan*	Cray XK7	Opteron 6274, Tesla K20X	Gemini	Cray	Oak Ridge National Laboratory United States, 2012	Linux based
5	17.173 20.133	*Sequoia*	Blue Gene/Q	A2	Custom	IBM	Lawrence Livermore National Laboratory United States, 2013	Linux CNK)
6	14.015 27.881	*Cori*	Cray XC40	Xeon Phi 7250	Aries	Cray	National Energy Research Scientific Computing Center United States, 2016	Linux
7	13.555 24.914	*Oakforest-PACS*	Fujitsu	Xeon Phi 7250	Intel Omni-Path	Fujitsu	Kashiwa, Joint Center for Advanced High Performance Computing Japan, 2016	Linux
8	10.510 11.280	*K computer*	Fujitsu	SPARC64 VIIIfx	Tofu	Fujitsu	Riken, Advanced Institute for Computational Science (AICS) Japan, 2011	Linux
9	8.587 10.066	*Mira*	Blue Gene/Q	A2	Custom	IBM	Argonne National Laboratory United States, 2012	Linux

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We have processed on the order of 5 petabytes of QCD data related to this project.

Next generation of runs on Summit in preparation

Top 10 positions of the 52nd TOP500 in November 2018[16]

Rank	Rmax Rpeak (PFLOPS)	Name	Model	Processor	Interconnect	Vendor	Site country, year
1 –	143.500 200.795	Summit	Power System AC922	POWER9, Tesla V100	Infiniband EDR	IBM	Oak Ridge National Laboratory United States, 2018
2 ▲	94.640 125.436	Sierra	Power System S922LC	POWER9, Tesla V100	Infiniband EDR	IBM	Lawrence Livermore National Laboratory United States, 2018
3 ▼	93.015 125.436	Sunway TaihuLight	Sunway MPP	SW26010	Sunway[17]	NRCPC	National Supercomputing Center in Wuxi China, 2016[17]
4 ▼	61.445 100.679	Tianhe-2A	TH-IVB-FEP	Xeon E5–2692 v2, Matrix-2000	TH Express-2	NUDT	National Supercomputing Center in Guangzhou China, 2013
5 ▲	21.230 27.154	Piz Daint	Cray XC50	Xeon E5-2690 v3, Tesla P100	Aries	Cray	Swiss National Supercomputing Centre Switzerland, 2016
6 ▼	20.159 41.461	Trinity	Cray XC40	Xeon E5–2698 v3, Xeon Phi 7250	Aries	Cray	Los Alamos National Laboratory United States, 2015

Diagrams – Isospin limit

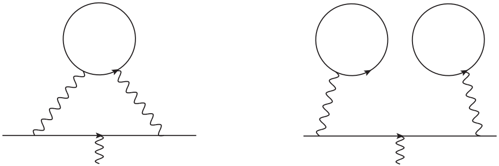
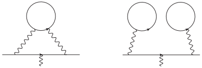
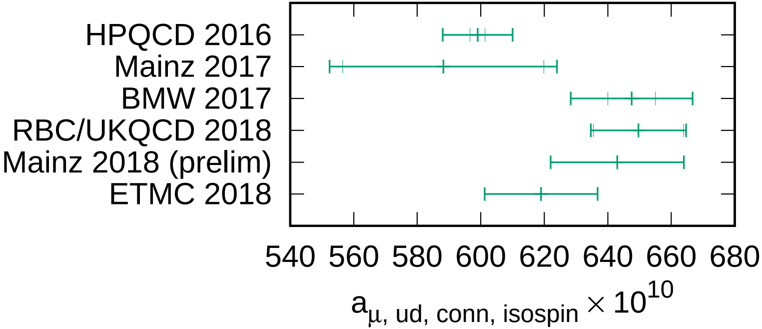
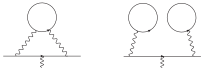


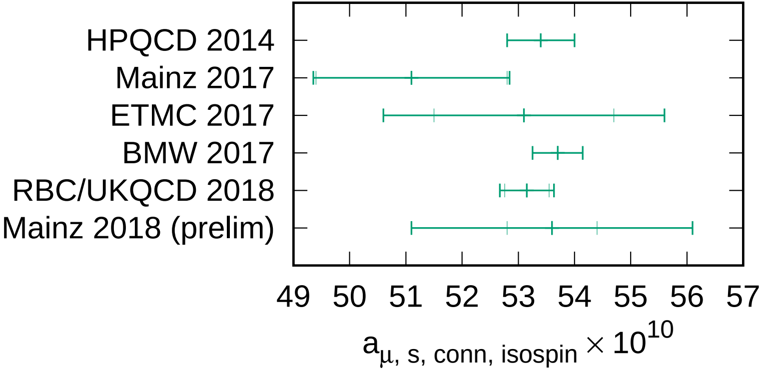
FIG. 1. Quark-connected (left) and quark-disconnected (right) diagram for the calculation of $a_\mu^{\mathrm{HVP\ LO}}$. We do not draw gluons but consider each diagram to represent all orders in QCD.



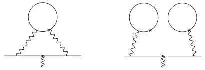


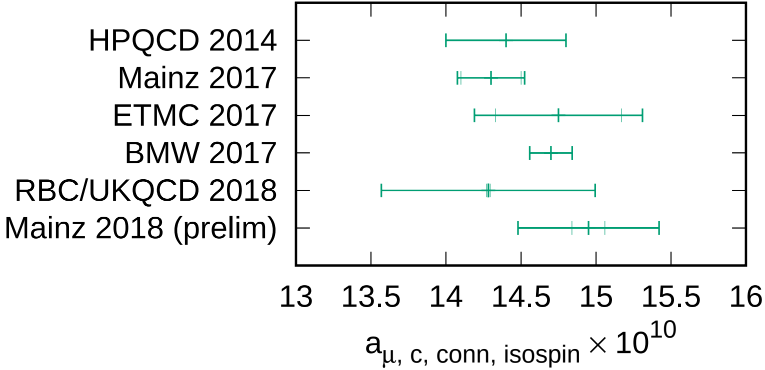
HPQCD 2016
Mainz 2017
BMW 2017
RBC/UKQCD 2018
Mainz 2018 (prelim)
ETMC 2018
540 560 580 600 620 640 660 680
$a_{\mu,\text{ ud, conn, isospin}} \times 10^{10}$

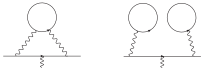


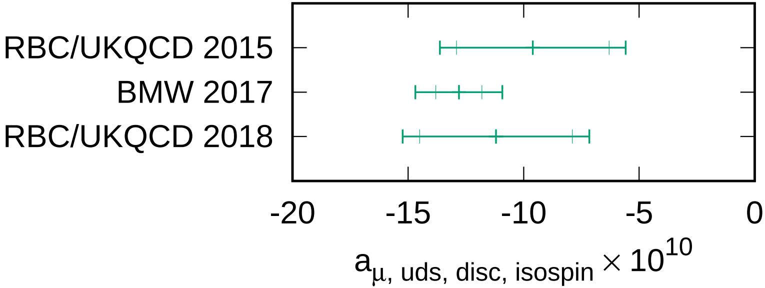


HPQCD 2014
Mainz 2017
ETMC 2017
BMW 2017
RBC/UKQCD 2018
Mainz 2018 (prelim)
49
50
51
52
53
54
55
56
57
$a_{\mu,\,s,\,\mathrm{conn,\,isospin}} \times 10^{10}$



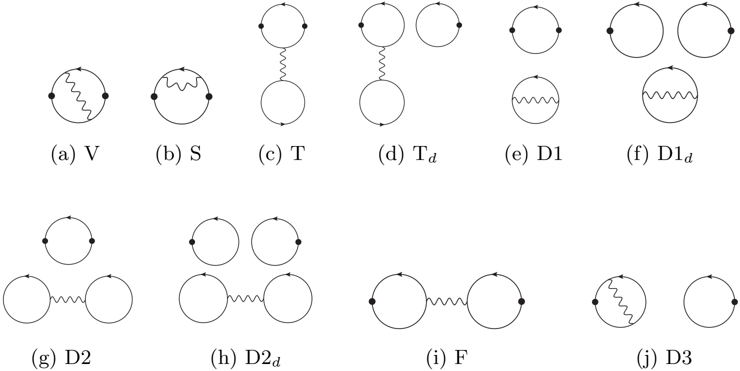

HPQCD 2014
Mainz 2017
ETMC 2017
BMW 2017
RBC/UKQCD 2018
Mainz 2018 (prelim)
13
13.5
14
14.5
15
15.5
16
$a_{\mu,\ c,\ conn,\ isospin} \times 10^{10}$



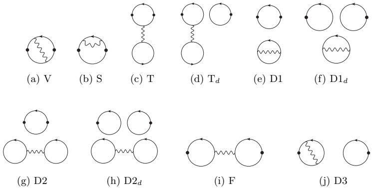


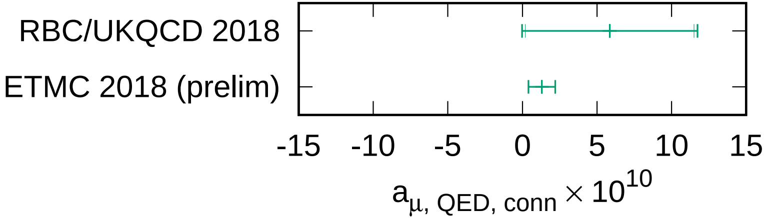
RBC/UKQCD 2015
BMW 2017
RBC/UKQCD 2018
-20
-15
-10
-5
0
$a_{\mu,\,\text{uds, disc, isospin}} \times 10^{10}$

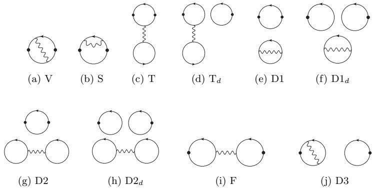
Diagrams – QED corrections

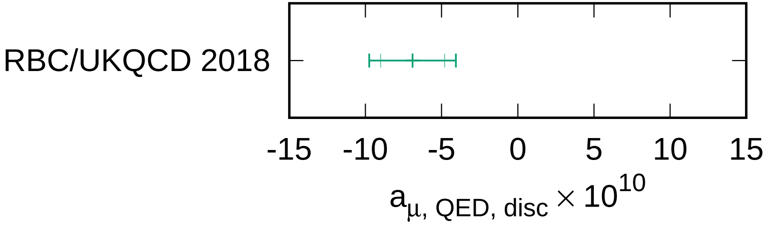


For diagram F we enforce exchange of gluons between the quark loops as otherwise a cut through a single photon line would be possible. This single-photon contribution is counted as part of the HVP NLO and not included for the HVP LO.

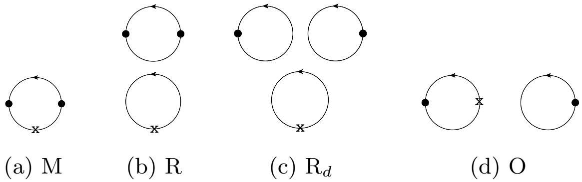

(a) V
(b) S
(c) T
(d) T_d
(e) D1
(f) $D1_d$
(g) D2
(h) $D2_d$
(i) F
(j) D3


RBC/UKQCD 2018
ETMC 2018 (prelim)
-15
-10
-5
0
5
10
15
$a_{\mu,\text{ QED, conn}} \times 10^{10}$

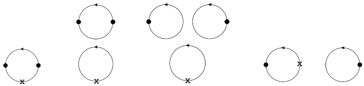

(a) V
(b) S
(c) T
(d) T$_d$
(e) D1
(f) D1$_d$
(g) D2
(h) D2$_d$
(i) F
(j) D3


RBC/UKQCD 2018
-15
-10
-5
0
5
10
15
$a_{\mu,\,\mathrm{QED,\,disc}} \times 10^{10}$

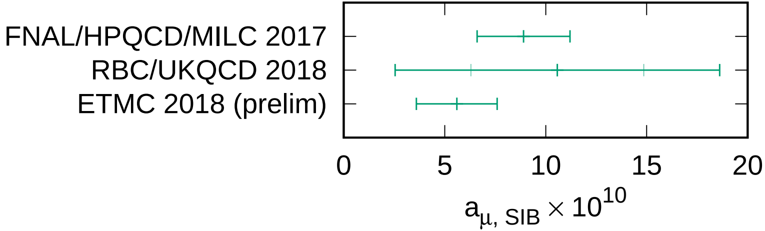
Diagrams – Strong isospin breaking



For the HVP R is negligible since $\Delta m_u \approx -\Delta m_d$ and O is SU(3) and $1/N_c$ suppressed.


x
x
x
x

(a) M (b) R (c) R_d (d) O


FNAL/HPQCD/MILC 2017
RBC/UKQCD 2018
ETMC 2018 (prelim)
0
5
10
15
20
$a_{\mu,\,\mathrm{SIB}} \times 10^{10}$

Lattice & Dispersive Analysis

Regions of precision (R-ratio data here is from **Fred Jegerlehner** 2017)

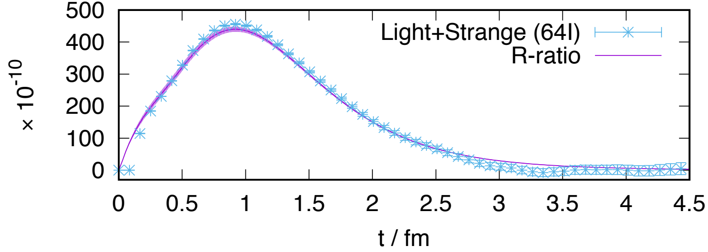


FIG. 4. Comparison of $w_t C(t)$ obtained using R-ratio data [1] and lattice data on our 64I ensemble.

The precision of lattice data deteriorates exponentially as we go to large t, however, is precise at intermediate distances. The R-ratio is very precise at long distances.

Note: in this plot a direct comparison of R-ratio and lattice data is not appropriate. Continuum limit, infinite-volume corrections, charm contributions, and IB corrections are missing from lattice data shown here.

Window method (implemented in RBC/UKQCD 2018)

We therefore also consider a window method. Following Meyer-Bernecker 2011 and smearing over t to define the continuum limit we write

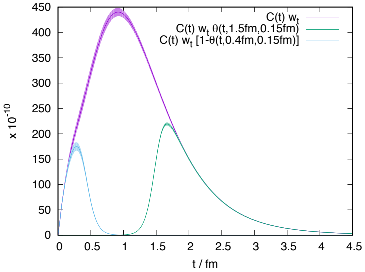
$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

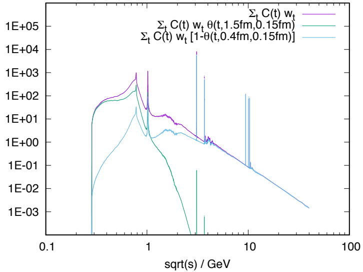
with

$$\begin{aligned} a_\mu^{\mathrm{SD}} &= \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)] \,, \\ a_\mu^{\mathrm{W}} &= \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)] \,, \\ a_\mu^{\mathrm{LD}} &= \sum_t C(t) w_t \Theta(t, t_1, \Delta) \,, \\ \Theta(t, t', \Delta) &= [1 + \tanh[(t - t')/\Delta]]/2 \,. \end{aligned}$$

In this version of the calculation, we use $C(t) = \frac{1}{12\pi^2} \int_0^\infty d(\sqrt{s}) R(s) s e^{-\sqrt{s}t}$ with $R(s) = \frac{3s}{4\pi\alpha^2} \sigma(s, e^+e^- \to \mathrm{had})$ to compute a_μ^{SD} and a_μ^{LD}.

How does this translate to the time-like region?





Most of $\pi\pi$ peak is captured by window from $t_0 = 0.4$ fm to $t_1 = 1.5$ fm, so replacing this region with lattice data reduces the dependence on BaBar versus KLOE data sets.

Error budget from RBC/UKQCD 2018 (**Fred's alphaQED17 results used for window result)**

$$715.4(16.3)_{\mathrm{S}}(3.0)_{\mathrm{C}}(7.8)_{\mathrm{V}}(1.9)_{\mathrm{A}}(0.4)_{\mathrm{Z}}(1.7)_{\mathrm{E}}(2.3)_{\mathrm{L}}$$
$$(1.5)_{\mathrm{E48}}(0.1)_{\mathrm{E64}}(0.3)_{\mathrm{b}}(0.2)_{\mathrm{c}}(1.1)_{\overline{\mathrm{S}}}(0.3)_{\overline{\mathrm{Q}}}(0.0)_{\mathrm{M}}$$

$$692.5(1.4)_{\mathrm{S}}(0.2)_{\mathrm{C}}(0.2)_{\mathrm{V}}(0.3)_{\mathrm{A}}(0.2)_{\mathrm{Z}}(0.0)_{\mathrm{E}}(0.0)_{\mathrm{E48}}$$
$$(0.0)_{\mathrm{b}}(0.1)_{\mathrm{c}}(0.0)_{\overline{\mathrm{S}}}(0.0)_{\overline{\mathrm{Q}}}(0.0)_{\mathrm{M}}(0.7)_{\mathrm{RST}}(2.1)_{\mathrm{RSY}}$$

a_μ	$692.5(1.4)_{\mathrm{S}}(0.2)_{\mathrm{C}}(0.2)_{\mathrm{V}}(0.3)_{\mathrm{A}}(0.2)_{\mathrm{Z}}(0.0)_{\mathrm{E}}(0.0)_{\mathrm{E48}}$ $(0.0)_{\mathrm{b}}(0.1)_{\mathrm{c}}(0.0)_{\overline{\mathrm{S}}}(0.0)_{\overline{\mathrm{Q}}}(0.0)_{\mathrm{M}}(0.7)_{\mathrm{RST}}(2.1)_{\mathrm{RSY}}$	$715.4(16.3)_{\mathrm{S}}(3.0)_{\mathrm{C}}(7.8)_{\mathrm{V}}(1.9)_{\mathrm{A}}(0.4)_{\mathrm{Z}}(1.7)_{\mathrm{E}}(2.3)_{\mathrm{L}}$ $(1.5)_{\mathrm{E48}}(0.1)_{\mathrm{E64}}(0.3)_{\mathrm{b}}(0.2)_{\mathrm{c}}(1.1)_{\overline{\mathrm{S}}}(0.3)_{\overline{\mathrm{Q}}}(0.0)_{\mathrm{M}}$

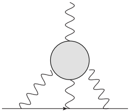
TABLE I. Individual and summed contributions to a_μ multiplied by 10^{10}. The left column lists results for the window method with $t_0 = 0.4$ fm and $t_1 = 1$ fm. The right column shows results for the pure first-principles lattice calculation. The respective uncertainties are defined in the main text.

For the pure lattice number the dominant errors are (S) statistics, (V) finite-volume errors, and (C) the continuum limit extrapolation uncertainty.
For the window method there are additional R-ratio systematic (RSY) and R-ratio statistical (RST) errors.

HVP - Thoughts

- This is a vibrant field with a lot of progress over the last 1-2 years!
- Lattice efforts by many groups, results at physical pion mass, QED, SIB corrections available. New methods to reduce statistical and systematic errors.
- Intermediate target: consolidate error at $O(3 \times 10^{-10})$ from first principles
- In early 2018, the lattice uncertainty is still $O(15 \times 10^{-10})$
- A target of $O(6 \times 10^{-10})$ seems realistic for early 2019 but requires a focused effort (estimate based on RBC/UKQCD unpublished progress)
- In a few years, new spacelike measurements from MUonE experiment (t-channel scattering) may be available (see M.Marinkovic's talk)

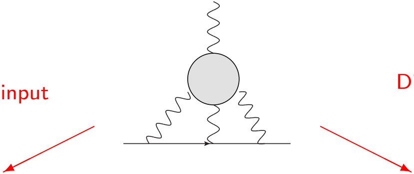
The HLbL contribution to the muon g-2



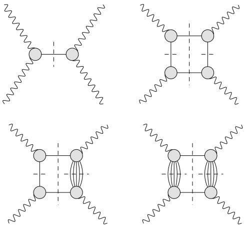
Talks by Antoine Gérardin (Tue/2:00), Luchang Jin (Tue/2:30)

Two new avenues for a model-independent value for the HLbL

Dispersive analysis +
Experimental/lattice input



Direct lattice calculation



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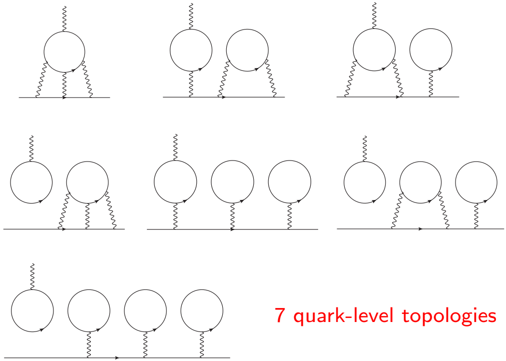
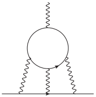
Truncation of cuts and states;
Talk by P.Stoffer



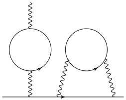
7 quark-level topologies

7 quark-level topologies of direct lattice calculation

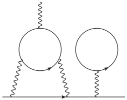
Hierarchy imposed by QED charges of dominant up- and down-quark contribution

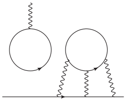


$Q_u^4 + Q_d^4 = 17/81$

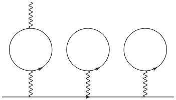


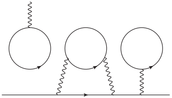
$(Q_u^2 + Q_d^2)^2 = 25/81$



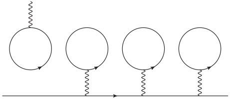


$(Q_u^3 + Q_d^3)(Q_u + Q_d) = 9/81$





$(Q_u^2 + Q_d^2)(Q_u + Q_d)^2 = 5/81$

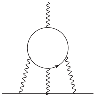


$(Q_u + Q_d)^4 = 1/81$

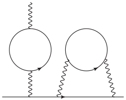
Further insight for magnitude of individual topologies can be gained by studying long-distance behavior of QCD correlation functions (Bijnens, RBC, ...)

7 quark-level topologies of direct lattice calculation

Hierarchy imposed by QED charges of dominant up- and down-quark contribution

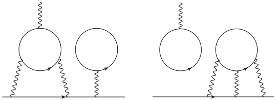


$Q_u^4 + Q_d^4 = 17/81$

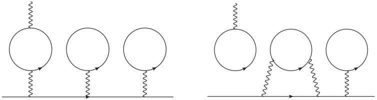


$(Q_u^2 + Q_d^2)^2 = 25/81$

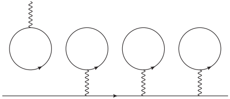
Dominant diagrams in top row: connected and leading disconnected diagram



$(Q_u^3 + Q_d^3)(Q_u + Q_d) = 9/81$



$(Q_u^2 + Q_d^2)(Q_u + Q_d)^2 = 5/81$



$(Q_u + Q_d)^4 = 1/81$

Further insight for magnitude of individual topologies can be gained by studying long-distance behavior of QCD correlation functions (Bijnens, RBC, ...)

Finite-volume and infinite-volume formulations

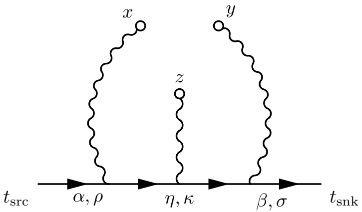
- a_μ^{HLbL} in finite-volume QCD and QED:
  - PRD93(2016)014503 (RBC/UKQCD): Connected diagram with $m_\pi = 171$ MeV; $a_\mu^{\mathrm{HLbL}} = 13.21(68) \times 10^{-10}$
  - PRL118(2017)022005 (RBC/UKQCD): Connected and leading disconnected diagram with $m_\pi = 139$ MeV; $a_\mu^{\mathrm{HLbL}} = 5.35(1.35) \times 10^{-10}$ (potentially large finite-volume systematics)

  Strategy: extrapolate away $1/L^n$ ($n \geq 2$) errors

- a_μ^{HLbL} in finite-volume QCD and infinite-volume QED:
  - Method proposed and successfully tested against the lepton-loop analytic result: arXiv:1510.08384 (Mainz), arXiv:1609.08454 (Mainz)
  - Similar method plus subtraction scheme to reduce systematic errors; successfully tested against lepton-loop analytic result: PRD96(2017)034515 (RBC/UKQCD)

  Strategy: FV errors exponentially suppressed but still may be significant, effect on noise?

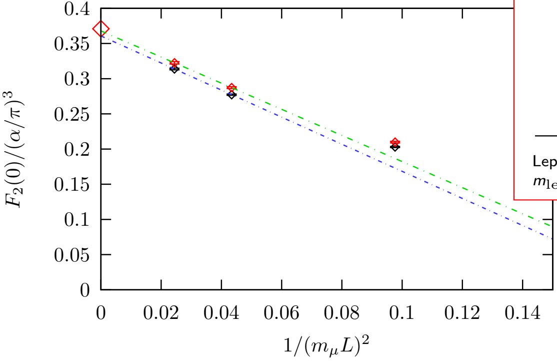
Finite-volume QED (PRD93(2016)014503 (RBC/UKQCD))



- The finite-volume QED_L prescription uses the photon propagator

$$G_{\rm L}^{\mu\nu}(x) = \frac{\delta^{\mu\nu}}{V} \sum_k{}' \frac{1}{\hat{k}^2} e^{ikx} , \tag{1}$$

  where $\hat{k}^2 = \sum_\mu 4\sin^2(k_\mu/2)$ and $V = \prod_\mu L_\mu$ with lattice dimensions L_μ. The sum is over all momenta with components $k_\mu = 2\pi n_\mu / L_\mu$ with $n_\mu \in [0, \ldots, L_\mu - 1]$ and the restriction that $k_0^2 + k_1^2 + k_2^2 \neq 0$.
- For fixed x and y can get result for all z in $\mathcal{O}(V \log V)$ time using convolutions starting at $t_{\rm src}$ and $t_{\rm snk}$; has statistical advantage for leading disconnected diagram (M^2 trick)



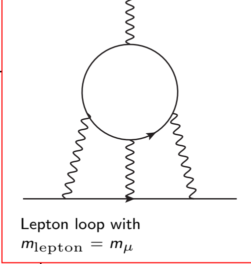
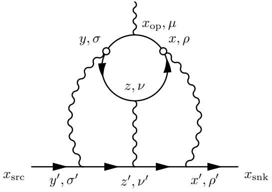
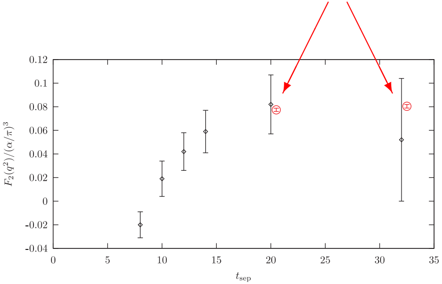


Figure 11. Results for $F_2(0)$ from QED connected light-by-light scattering. These results have been extrapolated to the $a^2 \to 0$ limit using two methods. The upper points use the quadratic fit to all three lattice spacings shown in Fig. 10 while the lower point uses a linear fit to the two left most points in that figure. Here we extrapolate to infinite volume using the linear fits shown to the two, left-most of the three points in each case.

PRD93(2015)014503 (RBC/UKQCD):

New sampling strategy with 10x reduced noise for same cost (red versus black):





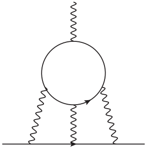
Stochastically evaluate the sum over vertices x and y:

- Pick random point x on lattice
- Sample all points y up to a specific distance $r = |x - y|$
- Pick y following a distribution $P(|x - y|)$ that is peaked at short distances

PRL118(2016)022005 (RBC/UKQCD):

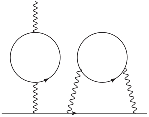
- Calculation at physical pion mass with finite-volume QED prescription ($\mathrm{QED_L}$) at single lattice cutoff of $a^{-1} = 1.73$ GeV and lattice size $L = 5.5$ fm.

- Connected diagram:



$$a_\mu^{\mathrm{cHLbL}} = 11.6(0.96) \times 10^{-10}$$

- Leading disconnected diagram:

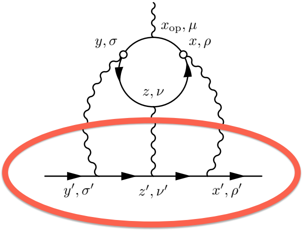


$$a_\mu^{\mathrm{dHLbL}} = -6.25(0.80) \times 10^{-10}$$

- Large cancellation expected from pion-pole-dominance considerations is realized: $a_\mu^{\mathrm{HLbL}} = a_\mu^{\mathrm{cHLbL}} + a_\mu^{\mathrm{dHLbL}} = 5.35(1.35) \times 10^{-10}$

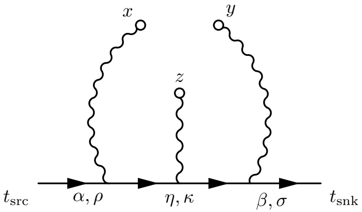
Potentially large systematics due to finite-volume QED!

Infinite-volume QED prescription (QED$_\infty$)



Remove power-law like finite-volume errors by computing the muon-photon part of the diagram in infinite volume (C.L. talk at lattice 2015 and Green et al. 2015, PRL115(2015)222003; Asmussen et al. 2016, PoS,LATTICE2016 164; PRD96(2017)034515)

Details:



We define

$$i^3 \mathcal{G}_{\rho,\sigma,\kappa}(x,y,z) \quad = \quad \mathfrak{G}_{\rho,\sigma,\kappa}(x,y,z) + \mathfrak{G}_{\sigma,\kappa,\rho}(y,z,x) + \text{other 4 permutations}\,.$$

and add the Hermitian conjugate with permuted indices (does not alter F_2 but makes this kernel infrared finite)

$$\mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(x,y,z) \quad = \quad \frac{1}{2}\mathfrak{G}_{\rho,\sigma,\kappa}(x,y,z) + \frac{1}{2}[\mathfrak{G}_{\kappa,\sigma,\rho}(z,y,x)]^\dagger$$

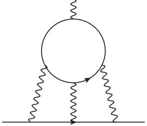
For $m_{\rm line} = 1$ this yields the kernel

$$\begin{aligned}\mathfrak{G}^{(1)}_{\sigma,\kappa,\rho}(y,z,x) \quad &= \quad \frac{\gamma_0+1}{2} i\gamma_\sigma \left(- \not{\partial}_y + \gamma_0 + 1\right) i\gamma_\kappa \left(\not{\partial}_x + \gamma_0 + 1\right) i\gamma_\rho \frac{\gamma_0+1}{2} \\ &\times \frac{1}{4\pi^2} \int d^4\eta \frac{1}{(\eta - z)^2} f(\eta - y) f(x - \eta).\end{aligned}$$

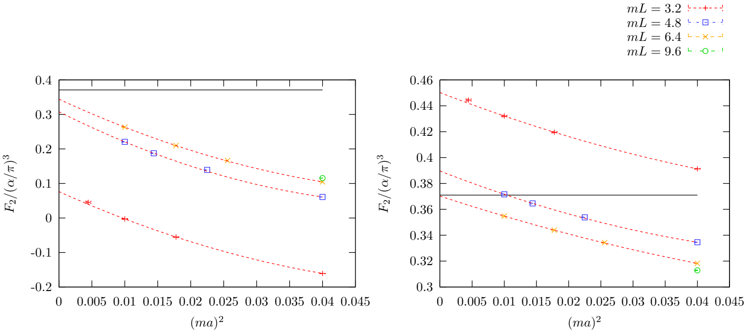
Due to current conservation, we can also devise a subtraction scheme that we found suppresses significantly finite-volume and discretization errors (demonstrated in the lepton loop case)

$$\mathfrak{G}^{(2)}_{\rho,\sigma,\kappa}(x,y,z) \quad = \quad \mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(x,y,z) - \mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(y,y,z) - \mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(x,y,y) + \mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(y,y,y)$$

Test of lepton-loop for infinite-volume method



Lepton loop with
$m_{\text{lepton}} = m_\mu$

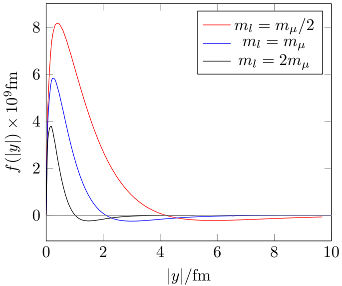


Without subtraction (left), with subtraction (right)

Same test for Mainz program (slide thanks to Andreas Nyffeler)

Lepton loop contribution a_μ^{LbL} in QED

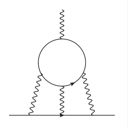
Integrand of lepton loop contribution a_μ^{LbL}:



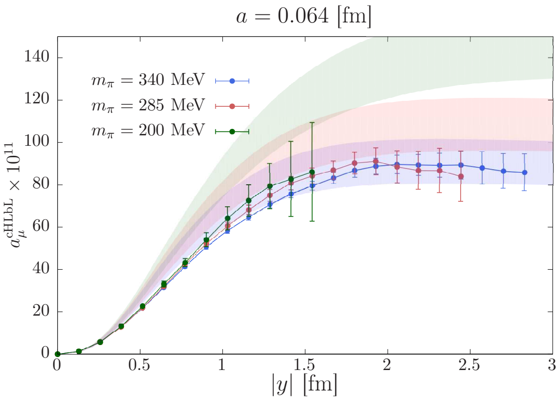
m_l/m_μ	$a_\mu^{\text{LbL}} \times 10^{11}$ (exact)	$a_\mu^{\text{LbL}} \times 10^{11}$	Precision	Deviation
1/2	1229.07	1257.5(6.2)(2.4)	0.5%	2.3%
1	464.97	470.6(2.3)(2.1)	0.7%	1.2%
2	150.31	150.4(0.7)(1.7)	1.2%	0.06%

1st uncertainty from 3D integration, 2nd uncertainty from extrapolation to small $|y|$.
Behavior for small $|y|$ compatible with $f(|y|) \propto m_\mu |y| \log^2(m_\mu |y|)$.
Analytical results for a_μ^{LbL} with $m_l = m_\mu, 2m_\mu$ reproduced at the percent level.
(Laporta + Remiddi '93, numbers courtesy of Massimo Passera)

Preliminary results from Mainz program earlier this year (Nils Asmussen at Mainz g-2 workshop)

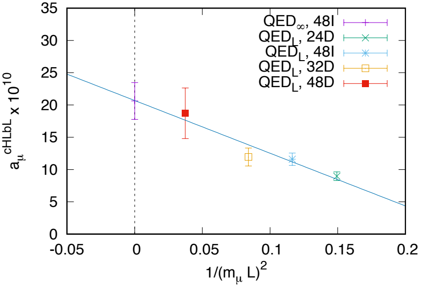


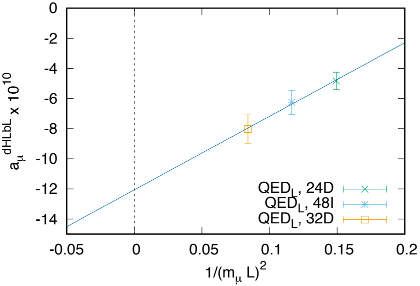
Pion Mass Dependence of a_μ^{cHLbL}

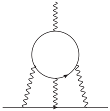


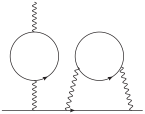
- the results show an upward trend for decreasing pion mass
- currently collecting more statistics in long distance regime

Preliminary QCD results for infinite-volume extrapolation (RBC/UKQCD 2018); see talk by L.Jin for complete analysis



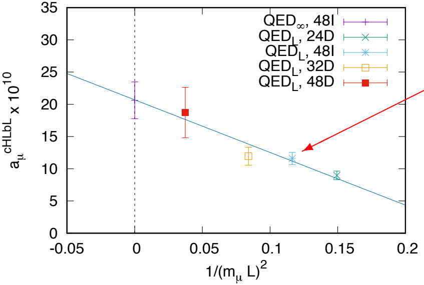


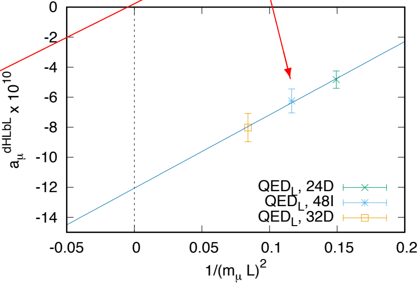


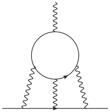


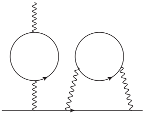
Preliminary QCD results for infinite-volume extrapolation (RBC/UKQCD 2018); see talk by L.Jin for complete analysis

Data used for finite-volume result in PRL118(2016)022005









Roadmap to complete first-principles light-by-light calculation with all errors controlled

- Calculation of connected plus leading disconnected diagram at physical pion mass completed
- Infinite-volume extrapolation done (to be published)
- Discretization errors are now controlled for (four different lattice spacings over two different actions, to be published)
- Calculation of sub-leading disconnected diagrams, starting with 3-1 topology first results
- Crosscheck of dispersive versus lattice (see, e.g., arXiv:1712.00421) desirable
- Progress by two groups (Mainz & RBC/UKQCD), cross checks will be very valuable!

Summary

Summary

- Lattice QCD has indeed come of age
- Significant improvements in methodology and growing computing power go hand-in-hand
- There is a vibrant community with many groups working on the HVP and currently two groups working on the HLbL
- HVP purely from lattice QCD+QED with competitive errors within reach over the next few years
- HLbL first lattice result with all errors controlled soon to be published

Backup

We perform the calculation as a perturbation around an isospin-symmetric lattice QCD computation with two degenerate light quarks with mass m_{light} and a heavy quark with mass m_{heavy} tuned to produce a pion mass of 135.0 MeV and a kaon mass of 495.7 MeV.

The correlator is expanded in the fine-structure constant α as well as $\Delta m_{\mathrm{up,\ down}} = m_{\mathrm{up,\ down}} - m_{\mathrm{light}}$, and $\Delta m_{\mathrm{strange}} = m_{\mathrm{strange}} - m_{\mathrm{heavy}}$. We write

$$\begin{aligned} C(t) = C^{(0)}(t) + \alpha C^{(1)}_{\mathrm{QED}}(t) + \sum_f \Delta m_f C^{(1)}_{\Delta \mathrm{m_f}}(t) \\ + \mathcal{O}(\alpha^2, \alpha \Delta m, \Delta m^2) \,. \end{aligned}$$

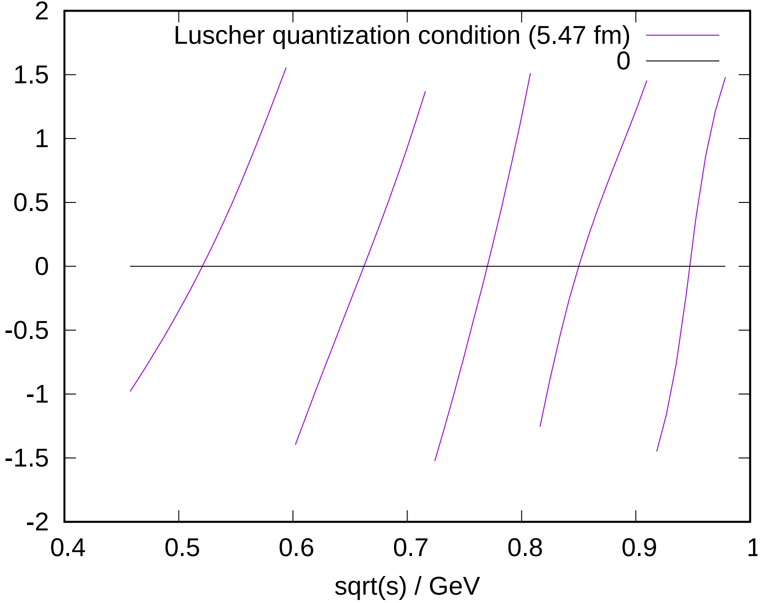
The correlators of this expansion are computed in lattice QCD with dynamical up, down, and strange quarks. We compute the missing contributions to a_μ from charm sea quarks in perturbative QCD (RHAD) by integrating the time-like region above 2 GeV and find them to be smaller than 0.3×10^{-10}.

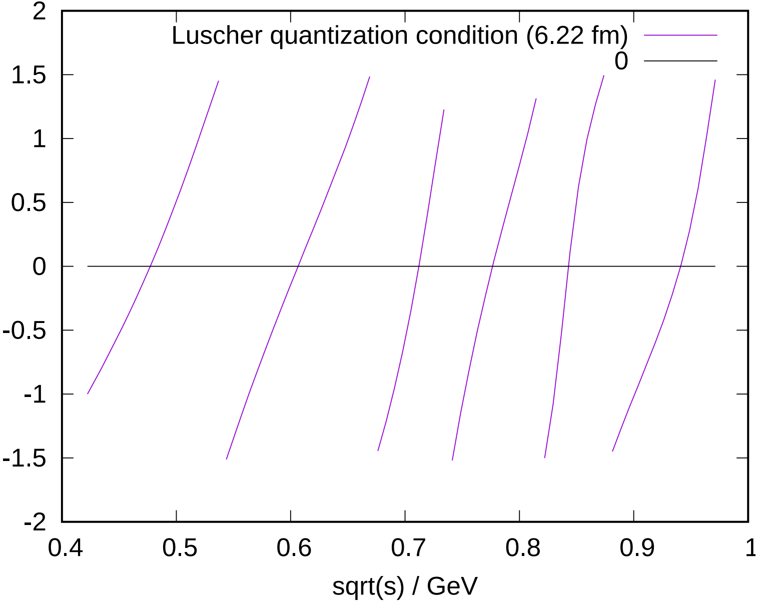
We tune the bare up, down, and strange quark masses m_{up}, m_{down}, and m_{strange} such that the π^0, π^+, K^0, and K^+ meson masses computed in our calculation agree with the respective experimental measurements. The lattice spacing is determined by setting the Ω^- mass to its experimental value.

We perform the lattice calculations for the light quark contributions using RBC/UKQCD's 48I and 64I lattice configurations with lattice cutoffs $a^{-1} = 1.730(4)$ GeV and $a^{-1} = 2.359(7)$ GeV and a larger set of ensembles with up to $a^{-1} = 2.774(10)$ GeV for the charm contribution.

From the parameter tuning procedure on the 48I we find $\Delta m_{\mathrm{up}} = -0.00050(1)$, $\Delta m_{\mathrm{down}} = 0.00050(1)$, and $\Delta m_{\mathrm{strange}} = -0.0002(2)$.

The shift of the Ω^- mass due to the QED correction is significantly smaller than the lattice spacing uncertainty and its effect on $C(t)$ is therefore not included separately.


Luscher quantization condition (5.47 fm)
0
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0.4
0.5
0.6
0.7
0.8
0.9
1
sqrt(s) / GeV



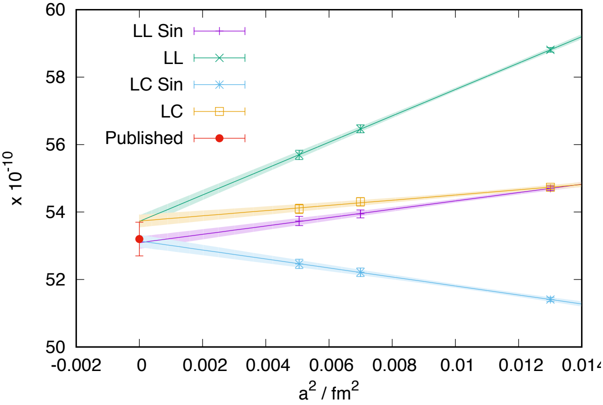
Luscher quantization condition (6.22 fm)
0
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0.4
0.5
0.6
0.7
0.8
0.9
1
sqrt(s) / GeV

Consolidate continuum limit

Adding a finer lattice

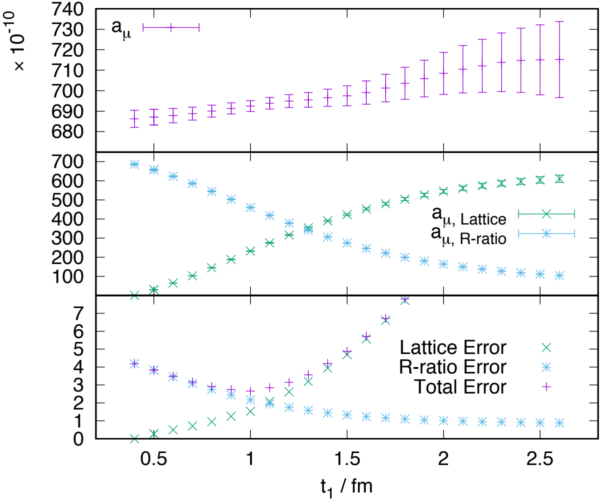
Add $a^{-1} = 2.77$ GeV lattice spacing

- Third lattice spacing for strange data ($a^{-1} = 2.77$ GeV with $m_\pi = 234$ MeV with sea light-quark mass corrected from global fit):



- For light quark need new ensemble at physical pion mass. Proposed for early science time at Summit Machine at Oak Ridge later this year ($a^{-1} = 2.77$ GeV with $m_\pi = 139$ MeV).

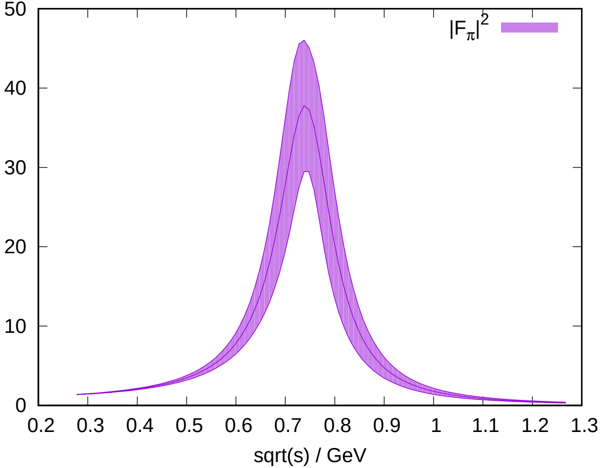
Window method with fixed $t_0 = 0.4$ fm



For $t = 1$ fm approximately 50% of uncertainty comes from lattice and 50% of uncertainty comes from the R-ratio. Is there a small slope? More in a few slides!

Can use this to check experimental data sets; see my KEK talk for more details

Predicts $|F_\pi(s)|^2$:



We can then also predict matrix elements and energies for our other lattices; successfully checked!