The double-copy structure of one-loop open-string amplitudes

based on 1711.09104
with Carlos Mafra

& work in progress

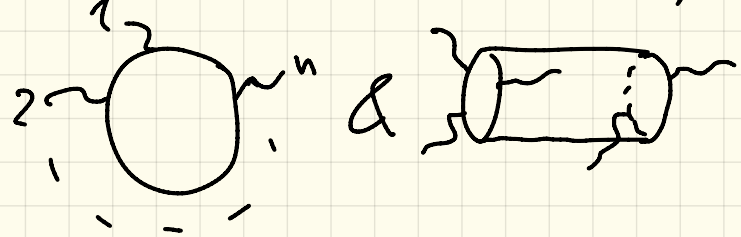
key result: open-string n-pt amplitudes

A_n^{tree} = [gauge-thy trees] $\otimes_{KLT}$ [disk integrals]

on symmetric footing

$A_n^{\text{1-loop}}$ = [gauge-thy kinematics] $\otimes$ [cylinder / Möbius-strip integrals]

see $M^{R^4}_{\text{tree},n}$ tree-lv. matrix element of R^4



origin : integrand $K_n^{g\text{-loop}}$ in Koba Nielsen

from $V_j \supset e^{ik_j \cdot x(z_j)}$

$$A_n^{g\text{-loop}} = \int_{\mathcal{M}_{g,n}} \frac{|I_n^{g\text{-loop}}|^2 \; K_n^{g\text{-loop}}}{\langle V_1 V_2 \cdots V_n \rangle_{\text{genus-}g}}$$

$\Rightarrow$ KLT-like formula

$$K_n^{\text{tree}} = \sum_{\rho,\tau\in S_{n-3}} PT(1,\rho,n,n-1)\; S_{\alpha'=0}[\rho|\tau]\; A_{\text{gauge}}^{\text{tree}}(1,\tau,n-1,n)$$

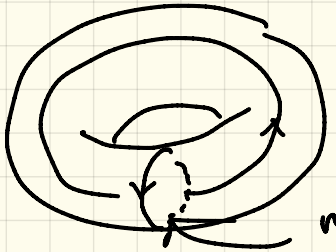
$p(2,3,\ldots,n-2)$

KLT matrix/ momentum kernel

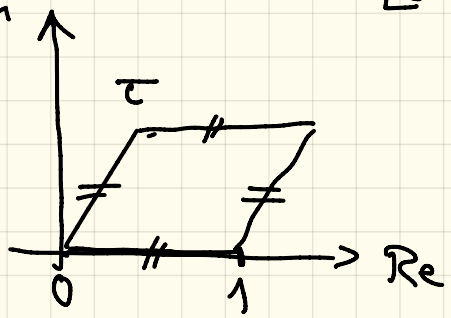
[Mafra, OS, Stieberger 1106.2645
Brödel, OS, Stieberger 1304.7267]

What is 1-loop analogue of PT?

one-loop setup: open-string worldsheets descend from



$\longleftrightarrow$ measure l here



L^2

$z \cong z+1$

$z \cong z+\tau$

$$I_n^{1\text{-loop}}(l) = \exp\left(\sum_{i<j} s_{ij} \log \theta_1(z_{ij}|\tau) + \sum_{j=1}^{n} z_j (l\cdot k_j) + \frac{\tau l^2}{4\pi i} \right)$$

only double-periodic after $\int d^D l$

$$I_n^{1\text{-loop}}(l) \Big|_{z_j \to z_j+\tau} = I_n^{1\text{-loop}}(l - 2\pi i k_j)$$

Generalized elliptic function (GEF)

defined by periodicity $z_j \to z_j + 1$ and

$$E(\{z_j, k_j\}, \tau, \ell)\Big|_{z_j \to z_j + \tau} = E(\dots, \ell - 2\pi i k_j)$$

$\longrightarrow$ $K_n^{1\text{-loop}}$ must be expressible in terms of GEF's, e.g.

$$E_{1|23.4,5} = \partial\log\theta_1(z_{12};\tau) + \partial\log\theta_1(z_{23};\tau) + \partial\log\theta_1(z_{31};\tau)$$

additive change @ $z_2 \to z_2 + \tau$: $\quad \hookrightarrow +2\pi i \qquad \hookrightarrow -2\pi i \qquad \hookrightarrow 0$

$$E^{\nu}_{1|2,3,4,5} = \ell^{\nu} + \Big[k_2^{\nu}\,\partial\log\theta_1(z_{12};\tau) + (2 \to 3,4,5)\Big]$$

(k_2^{ν} $\hookrightarrow 2\pi i$; $(2 \to 3,4,5)$ $\hookrightarrow 0$)

ℓ^{ν}: compensating shift by $-2\pi i\, k_2^{\nu}$

Examples: $K_4^{1\text{-loop}} = t_8(1,2,3,4) = \overbrace{E_{1|2,3,4}}^{=1} C_{1|2,3,4}$

$K_5^{1\text{-loop}} = E^{\nu}_{1|2,3,4,5} C^{\nu}_{1|2,3,4,5}$ ← gauge-inv. $e_1^{\nu} t_8(2,3,4,5) + \dots$ appears in N_{pent} @ $O(\ell^1)$

$+ \left[s_{23} E_{1|23,4,5} C_{1|23,4,5} + (23 \to 24, 25, 34, 35, 45) \right]$

$s_{45} \left(s_{34} A^{\text{tree}}_{\text{gauge}}(1\,2\,3\,4\,5) - (2 \leftrightarrow 3) \right)$

identical "BCJ-like" relations for E & C

$0 = \underbrace{k_2^{\nu} E^{\nu}_{1|2,3,4,5} + \left[s_{23} E_{1|23,4,5} + (3 \leftrightarrow 4,5) \right]}_{\int \frac{d}{dz_2}(\dots) \Rightarrow 0}$ & $E \leftrightarrow C$ "BRST exact"

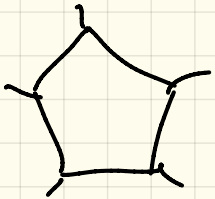
$\Rightarrow$ double copy $K_n^{1\text{-loop}} = E_1^{\mu_1\mu_2\dots} \otimes C^{\mu_1\mu_2\dots}$ -

Duality to $R+R^4$ supergravity

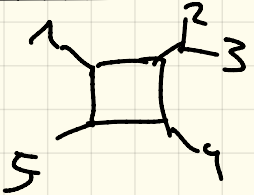
$$M_n^{R^4} = C^{\mu_1\mu_2\cdots} \otimes \tilde{C}^{\mu_1\mu_2\cdots} = K_n^{1\text{-loop}}\Big|_{E\to C}$$

e.g. $M_4^{R^4} = C_{1|2,3,4}\,\tilde{C}_{1|2,3,4}$

$$M_5^{R^4} = C^{\mu}_{1|2,3,4,5}\,\tilde{C}^{\mu}_{1|2,3,4,5} + \underbrace{\left[s_{23}\,C_{1|23,4,5}\,\tilde{C}_{1|23,4,5} + \text{perm's}\right]}$$



$|\ell_\mu C^{\mu}_{1|2,3,4,5}|^2$



UV-divergence of max SUGRA in $D = 8 - 2\varepsilon$