Spinning binary black holes: effective worldline actions, post-Newtonian vs. post-Minkowskian gravity, and input from Amplitudes

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Nonspinning black holes as effective point-masses

- Effective action for (monopole) point particles with worldlines $x = z(\tau)$ coupled to gravity, metric $g_{\mu\nu}(x)$:

$$\mathcal{S}[g,z] = \int d^4x \frac{\sqrt{-g}\,R}{16\pi G} - \sum_{\text{particles}} m \int d\tau\,\sqrt{-g_{\mu\nu}(z)\dot{z}^\mu \dot{z}^\nu}.$$

- Phase-space action (auxiliary field/Lagrangian multiplier λ):

$$\mathcal{S}[g,z,p] = \int d^4x \frac{\sqrt{-g}\,R}{16\pi G} + \sum \int d\tau \Big[p_\mu \dot{z}^\mu - \frac{\lambda}{2}(p^2 + m^2)\Big].$$

$$\left(\delta_p \mathcal{S} = 0 \quad \Rightarrow \quad p^\mu = m\frac{\dot{z}^\mu}{\sqrt{-\dot{z}^2}}\,; \qquad \delta_{p,z}\mathcal{S} = 0 \quad \Rightarrow \quad \lambda = \sqrt{-\dot{z}^2}\right)$$

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- Two particles: solve perturbatively (and integrate out the field)...
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Solving the relativistic two-body problem

- Numerical relativity (solve full Einstein field eqs. on supercomputers)
- Analytic approximation schemes:

Limit	Perturbation theory
Newtonian gravity $c \to \infty$	post-Newtonian $\frac{m_1}{m_2} \sim 1, \qquad \frac{Gm}{rc^2} \sim \frac{v^2}{c^2} \ll 1$
test-body motion in a stationary background $\frac{m_1}{m_2} \to 0$	post-test-body (“self-force”) $\frac{m_1}{m_2} \ll 1, \qquad \frac{Gm}{rc^2} \sim \frac{v^2}{c^2} \sim 1$
special relativity $G \to 0$	post-Minkowskian $\frac{m_1}{m_2} \sim 1, \qquad \frac{Gm}{rc^2} \ll \frac{v^2}{c^2} \sim 1$

post-Newtonian (PN) vs. post-Minkowskian (PM)

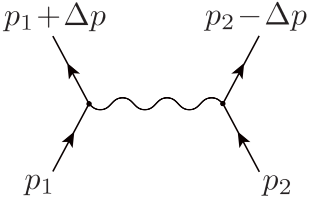
$$\mathcal{L} = \underbrace{-Mc^2 + \frac{\mu v^2}{2} + \frac{GM\mu}{r}}_{\downarrow} + \frac{1}{c^2}\Big(\dots\Big) + \frac{1}{c^4}\Big(\dots\Big) + \dots$$

		0PN	1PN	2PN	3PN	4PN	5PN	...
0PM:	1	v^2	v^4	v^6	v^8	v^{10}	v^{12}	
1PM:		$1/r$	v^2/r	v^4/r	v^6/r	v^8/r	v^{10}/r	
2PM:‡			$1/r^2$	v^2/r^2	v^4/r^2	v^6/r^2‡	v^8/r^2‡	
3PM:				$1/r^3$	v^2/r^3	v^4/r^3	v^6/r^3	
4PM:					$1/r^4$	v^2/r^4	v^4/r^4	
...						...	...	

$$1 \to Mc^2, \qquad v^2 \to \frac{v^2}{c^2}, \qquad \frac{1}{r} \to \frac{GM}{rc^2}.$$

current PN results; current PM results; overlap; unknown.

1PM (monopole/scalar) gravitational scattering



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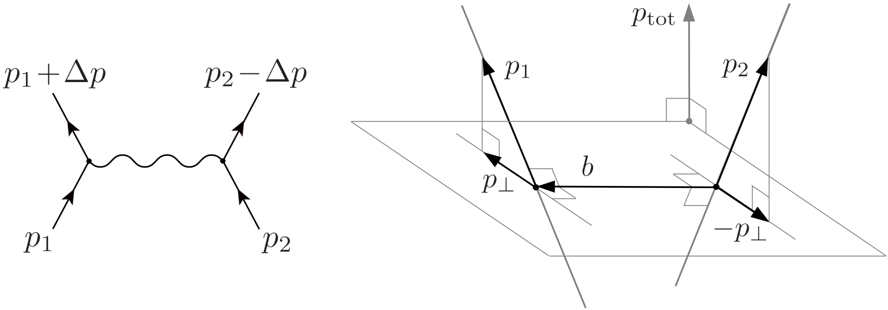

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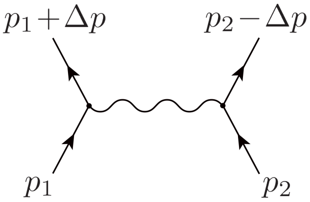

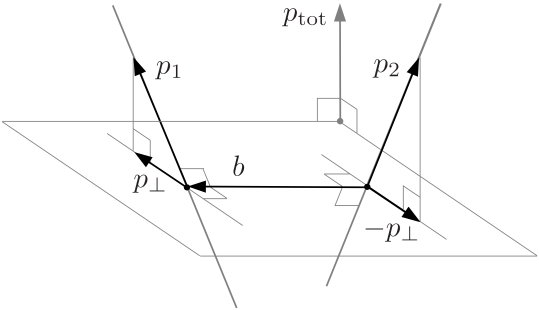

- Impact parameter: $b^\mu = (x_1^\mu - x_2^\mu)_{\rm min}, \qquad b \cdot p_1 = b \cdot p_2 = 0,$

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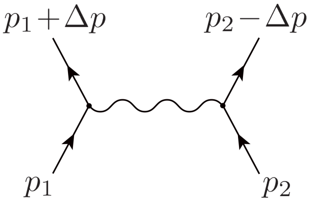

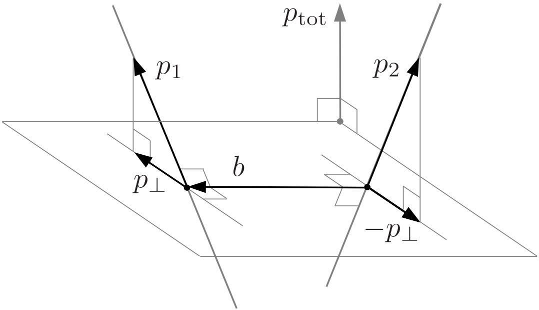

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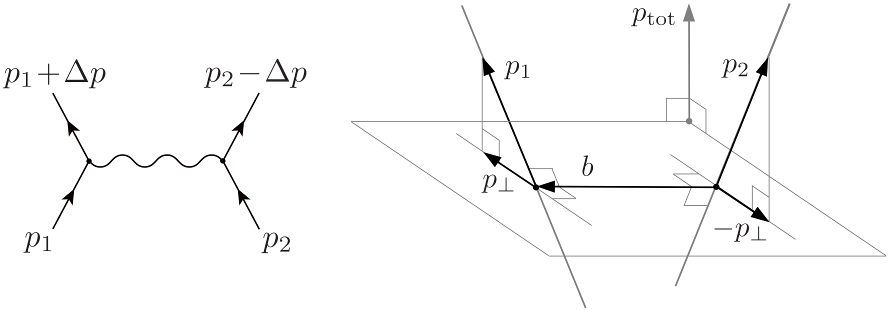

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- Deflection: $$\boxed{\Delta p^\mu = -2Gm_1m_2 \frac{2\gamma^2 - 1}{\sqrt{\gamma^2 - 1}} \frac{b^\mu}{b^2} + O(G^2).}$$

[Portilla ‘79, ‘80; Westpfahl+ ‘79, ‘85; Bel+ ‘81; Damour+ ‘81; ...]

Spinning black holes

- Effective phase-space action for a spinning black hole:

  worldline $z(\tau)$, momentum $p_\mu(\tau)$, $\left(\hat{p}^\mu = p^\mu/\sqrt{-p^2}\right)$

  tetrad ${\Lambda_a}^\mu(\tau)$, spin $S^{\mu\nu}(\tau)$,

$$\mathcal{S}_{\rm BH}[z,p,\Lambda,S,g] = \int d\tau \left\{ p_\mu \dot{z}^\mu + \frac{1}{2} S_{\mu\nu} \Lambda^{a\,\mu} \frac{D{\Lambda_a}^\nu}{d\tau} \right.$$
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- $\delta S_{\rm BH} = 0 \Rightarrow$ Mathisson-Papapetrou-Dixon (MPD) eqs.:

$$\frac{Dp_\mu}{d\tau} = -\frac{1}{2} R_{\mu\nu\rho\sigma} \dot{z}^\nu S^{\rho\sigma} + \frac{p\cdot\dot{z}}{2} \frac{\mathcal{D}}{\partial z^\mu} \log \mathcal{M}^2,$$
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- "Dynamical mass function" for a BH:

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\mathcal{M}^2 = m^2 + 2m^2 \hat{p}_\mu \hat{p}_\nu \Big(& -\frac{1}{2!} R^\mu{}_\alpha{}^\nu{}_\beta \, a^\alpha a^\beta && : \text{quadrupole} \propto S^2 \\
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Eikonal limits of minimally coupled massive fields

- Scalar field —— geodesic motion (monopole only)
- Dirac field —— pole - dipole
- Spin-1 —— pole - dipole - (black-hole-)quadrupole (PN) [Holstein, Ross '08]

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- Spin-2 —— pole - ... - (black-hole-)hexadecapole (PN) [Vaidya '15]

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- Spin-s —— pole - ... - (black-hole-)2^{2s}pole [Guevara '17]
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- All the linear-in-Riemann terms [the only ones that matter at $O(G)$] are fixed by symmetries and matching to the (linearized) Kerr solution...

Spinning black holes

- Effective phase-space action for a spinning BH:

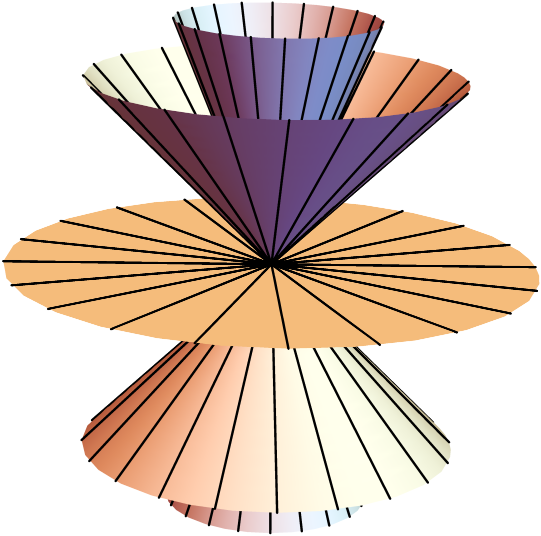
$$\mathcal{S}_{\rm BH} = \int d\tau \left\{ p_\mu \dot{z}^\mu + \frac{1}{2} S_{\mu\nu} \Lambda^{a\mu} \frac{D\Lambda_a{}^\nu}{d\tau} - \chi^\mu S_{\mu\nu} p^\nu - \frac{\lambda}{2} \left[p^2 + \mathcal{M}^2 \right] \right\}.$$

- "Dynamical mass function" for a BH:

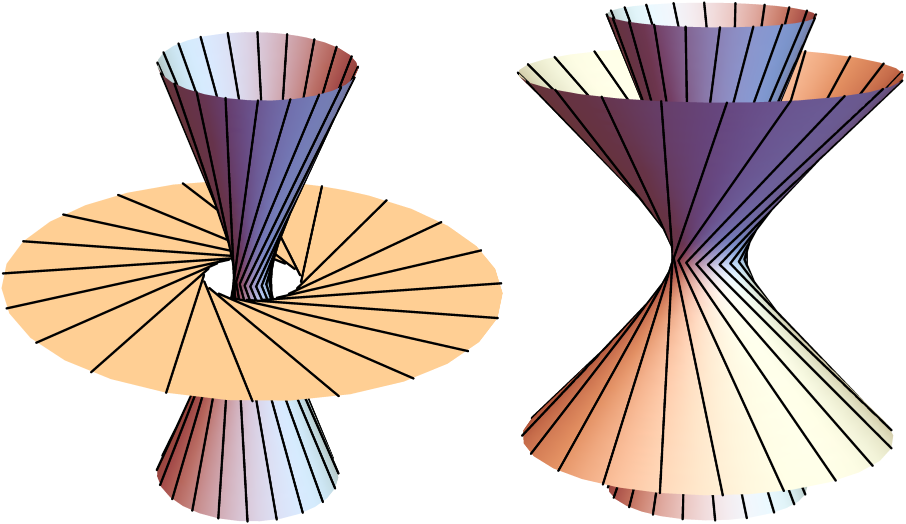
$$\begin{aligned}
\mathcal{M}^2 = m^2 + 2m^2 \hat{p}_\mu \hat{p}_\nu \Big(& - \frac{1}{2!} R^\mu{}_\alpha{}^\nu{}_\beta \, a^\alpha a^\beta && : \text{quadrupole} \propto S^2 \\
& + \frac{1}{3!} {}^*R^\mu{}_\alpha{}^\nu{}_{\beta;\gamma} \, a^\alpha a^\beta a^\gamma && : \text{octupole} \propto S^3 \\
& + \frac{1}{4!} R^\mu{}_\alpha{}^\nu{}_{\beta;\gamma\delta} \, a^\alpha a^\beta a^\gamma a^\delta && : \text{hexadecapole} \propto S^4 \\
& + \dots \Big)
\end{aligned}$$

- All the linear-in-Riemann terms [the only ones that matter at $O(G)$] are fixed by symmetries and matching to the (linearized) Kerr solution...

Kerr-Schild structure of Schwarzschild...



... vs. Kerr...



Spin-multipole structure of Kerr

- Exact Kerr: $$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + 2\partial_{(\mu}\xi_{\nu)}.$$
- Linearized harmonic-gauge part $h_{\mu\nu}$: $$\bar{h}_{\mu\nu} = \mathcal{P}_{\mu\nu}{}^{\alpha\beta} h_{\alpha\beta}$$

$$\mathcal{P}_{\mu\nu}{}^{\alpha\beta} = \delta_{(\mu}{}^{(\alpha}\delta_{\nu)}{}^{\beta)} - \tfrac{1}{2}\eta_{\mu\nu}\eta^{\alpha\beta},$$

$$\bar{h}_{\mu\nu} = \left(1 - \frac{1}{2!}a^\alpha a^\beta \partial_{\alpha\beta} + \frac{1}{4!}a^\alpha a^\beta a^\gamma a^\delta \partial_{\alpha\beta\gamma\delta} - \dots\right)\frac{4m}{r}u_\mu u_\nu$$

$$+ \left(a^\sigma \partial_\alpha - \frac{1}{3!}a^\sigma a^\beta a^\gamma \partial_{\alpha\beta\gamma} + \dots\right)\frac{4m}{r}u_{(\mu}\epsilon_{\nu)\rho\sigma}{}^\alpha u^\rho$$

$$= 4m \; \exp(a * \partial)^\alpha{}_\gamma \; \frac{u^\gamma u^\beta}{r},$$

$$(a * \partial)^\mu{}_\nu = \epsilon^\mu{}_{\nu\alpha\beta} a^\alpha \partial^\beta.$$

[Steinhoff, JV '16; Harte, JV '16; JV '17]

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Linear interaction of two spinning black holes

(see [JV '17])

$$\mathcal{L}_{\text{int}} = \frac{1}{2}\,\hat{\mathcal{T}}^{\mu\nu}(p_1, a_1, -\partial)\,h_{2\mu\nu}(x)\,\Big|_{x=z_1}$$

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- This depends on the spins only through $a_1 + a_2$!

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- The double sum over BH1-multipole–BH2-multipole couplings factorizes into a single sum over the multipoles of one effective spinning BH coupled to an effective point-mass —at linear order in G.

Post-Minkowskian scattering angles {amplitudes}

- Scattering angle χ (encodes the full dynamics...)
- Impact parameter b ; $\gamma = -p_1 \cdot p_2 / m_1 m_2$; $M = m_1 + m_2, \quad \nu = m_1 m_2 / M^2$

- Schw.-Schw. 1PM/tree:

$$\chi = 2\frac{GM}{b}\,\frac{\sqrt{1+2\nu(\gamma-1)}}{\gamma^2-1}\,(2\gamma^2-1) + O(G^2).$$

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