Isospin breaking in tau input for $(g - 2)$ from lattice QCD

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Motivations

Final states $I = 1$ charged

$\tau$ data can improve $a_\mu[^{\pi\pi}] 
\rightarrow 72\%$ of total Hadronic LO

or $a^{ee}_\mu \neq a^\tau \rightarrow$ NP [Cirigliano et al '18]

Final states $I = 0, 1$ neutral
**Isospin Corrections**

Restriction to $e^+e^- \rightarrow \pi^+\pi^-$ and $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$

$$v_0(s) = \frac{s}{4\pi\alpha^2} \sigma_{\pi^+\pi^-}(s)$$

$$v_-(s) = \frac{m^2_\tau}{6|V_{ud}|^2} \frac{B_{\pi\pi^0}}{B_e} \frac{1}{N_{\pi\pi^0}} \frac{dN_{\pi\pi^0}}{ds} \left(1 - \frac{s}{m^2_\tau}\right)^{-1} \left(1 + \frac{2s}{m^2_\tau}\right)^{-1} \frac{1}{S_{EW}}$$

Isospin correction $v_0 = R_{IB}v_-$  

$R_{IB} = \frac{FSR}{G_{EM}} \frac{\beta_0^3|F^0_\pi|^2}{\beta_-^3|F^-_\pi|^2}$  

[Alemani et al. '98]

0. $S_{EW}$ electro-weak radiative correct.  
[Marciano, Sirlin '88][Braaten, Li '90]

1. Final State Radiation of $\pi^+\pi^-$ system  
[Schwinger '89][Drees, Hikasa '90]

2. $G_{EM}$ (long distance) radiative corrections in $\tau$ decays  
Chiral Resonance Theory  
[Cirigliano et al. '01, '02]
Meson Dominance  
[Flores-Talpa et al. '06, '07]

3. Phase Space $(\beta_0, -\beta_0)$ due to $(m_{\pi^\pm} - m_{\pi^0})$
LONG DISTANCE QED - I

At low energies relevant degrees of freedom are mesons

Chiral Perturbation Theory

Meson dominance model

Corrections casted in one function $\nu_-(s) \rightarrow \nu_-(s)G_{EM}(s)$

Real photon corrections

Virtual photon corrections

Real + virtual

$\rightarrow$ IR divergences cancel
Pion form factors

\[ F_\pi^0(s) \propto \frac{m_\rho^2}{D_\rho(s)} \]
\[ \times \left[ 1 + \delta_{\rho \omega} \frac{s}{D_\omega(s)} \right] \]
\[ + \frac{m_X^2}{D_X(s)} \quad X = \rho', \rho'' \]
\[ F_{\pi^-}(s) \propto \frac{m_{\rho^-}^2}{D_{\rho^-}(s)} + (\rho', \rho'') \]

Sources of IB breaking in phenomenological models

\[ m_{\rho^0} \neq m_{\rho^\pm}, \quad \Gamma_{\rho^0} \neq \Gamma_{\rho^\pm}, \quad m_{\pi^0} \neq m_{\pi^\pm} \]
\[ \rho - \omega \text{ mixing } \delta_{\rho \omega} \simeq O(m_u - m_d) + O(e^2) \]
Lattice field theories

Lattice spacing $a \rightarrow$ regulate UV divergences
finite size $L \rightarrow$ finite dimensional integral
Euclidean metric $\rightarrow$ Boltzman interpretation
of path integral
Continuum theory $a \rightarrow 0$, $L \rightarrow \infty$

$$\langle O \rangle = \mathcal{Z}^{-1} \int [D U] e^{-S[U]} O(U) \approx \frac{1}{N} \sum_{i=1}^{N} O[U_i]$$

Very high dimensional integral (but finite) $\rightarrow$ Monte-Carlo methods
Markov Chain of gauge field configs $U_0 \rightarrow U_1 \rightarrow \cdots \rightarrow U_N$
Contribution to $a_\mu$

Time-momentum representation

$$G^\gamma(t) = \frac{1}{3} \sum_k \int d\vec{x} \langle \jmath^\gamma_k(x) \jmath^\gamma_k(0) \rangle \rightarrow a_\mu = 4\alpha^2 \sum_t w_t G^\gamma(t)$$

Isospin decomposition of $u, d$ current

$$j^\gamma_\mu = \frac{i}{6} (\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) + \frac{i}{2} (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) = j^{(0)}_\mu + j^{(1)}_\mu$$

$$G^\gamma_{00} \leftarrow \langle \jmath^{(0)}_k(x) \jmath^{(0)}_k(0) \rangle = \begin{array}{c}
\circ \quad \circ \\
\circ \quad \circ \\
\circ \quad \circ \\
\circ \quad \circ \\
\circ \quad \circ
\end{array} + \begin{array}{c}
\circ \quad \circ \\
\circ \quad \circ \\
\circ \quad \circ \\
\circ \quad \circ \\
\circ \quad \circ
\end{array} + \begin{array}{c}
\circ \quad \circ \\
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\circ \quad \circ
\end{array} + \begin{array}{c}
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\circ \quad \circ \\
\circ \quad \circ
\end{array} + \begin{array}{c}
\circ \quad \circ \\
\circ \quad \circ \\
\circ \quad \circ \\
\circ \quad \circ \\
\circ \quad \circ
\end{array} + \cdots$$

$$G^\gamma_{01} \leftarrow \langle \jmath^{(0)}_k(x) \jmath^{(1)}_k(0) \rangle = \begin{array}{c}
\circ \quad \circ \\
\circ \quad \circ \\
\circ \quad \circ \\
\circ \quad \circ \\
\circ \quad \circ
\end{array} + \begin{array}{c}
\circ \quad \circ \\
\circ \quad \circ \\
\circ \quad \circ \\
\circ \quad \circ \\
\circ \quad \circ
\end{array} + \cdots$$

$$G^\gamma_{11} \leftarrow \langle \jmath^{(1)}_k(x) \jmath^{(1)}_k(0) \rangle = \begin{array}{c}
\circ \quad \circ \\
\circ \quad \circ \\
\circ \quad \circ \\
\circ \quad \circ \\
\circ \quad \circ
\end{array} + \begin{array}{c}
\circ \quad \circ \\
\circ \quad \circ \\
\circ \quad \circ \\
\circ \quad \circ \\
\circ \quad \circ
\end{array} + \cdots$$

Decompose $a_\mu = a^{(0,0)}_\mu + a^{(0,1)}_\mu + a^{(1,1)}_\mu$
Neutral vs Charged

\[ \frac{i}{2}(\bar{u}\gamma_{\mu} u - \bar{d}\gamma_{\mu} d), \left[ \begin{array}{c} I = 1 \\ I_3 = 0 \end{array} \right] \rightarrow \frac{i}{\sqrt{2}}(\bar{u}\gamma_{\mu} d), \left[ \begin{array}{c} I = 1 \\ I_3 = -1 \end{array} \right] \]

Isospin 1 charged correlator

\[ G_W^{11} = \frac{1}{3} \sum_k \int d\vec{x} \langle j_k^{(1,+)}(x) j_k^{(1,-)}(0) \rangle \]

\[ \delta G^{(1,1)} \equiv G_1^{\gamma} - G_1^{W} = Z_4^4 \left( Q_u - Q_d \right)^4 \left( 4\pi \alpha \right) \left[ \right. \]

\[ G_0^{\gamma} = Z_4^4 \left( Q_u^2 - Q_d^2 \right)^2 \left( 4\pi \alpha \right) \left[ \right. \]

\[ + Z_4^2 \left( Q_u^2 - Q_d^2 \right) (m_u - m_d) \left[ 2 \times \right. \]

\[ \left. + \ldots \right] \]

\[ \ldots = \text{subleading diagrams currently not included} \]
**Lattice: Preliminary results - I**

\[ \Delta a_\mu \text{ from } G_{01}^\gamma \text{ (QED and SIB):} \]

![Graph 1: \( \Delta a_\mu(t) \) vs. \( t \text{ [fm]} \)]

- \( \text{conn, SIB} \)
- \( \text{discon, QED} \)
- \( \text{conn, QED} \)

\[ V = \begin{array}{c}
\end{array} \quad F = \begin{array}{c}
\end{array} \quad S = \begin{array}{c}
\end{array} \]

\[ M = \begin{array}{c}
\end{array} \quad O = \begin{array}{c}
\end{array} \quad \text{relevant, negative, neglected} \]
LATTICE: PRELIMINARY RESULTS - II

Study integrand in euclidean time → as important as integral

direct comparison
1. validate previous estimates of $R_{IB}$

Lattice vs. EFT+Pheno
2. study neutral/charged $\rho$ and $\omega$ properties

Preliminary lattice (full) calculation: $G_{01}^\gamma + \delta G$

Not included:
1. relevant
2. sub-leading $1/N_c, 1/N_f$
3. finite-volume errors
4. discretization errors
Towards a comparison - I

Restriction to $2\pi \to$ neglect pure $I = 0$ part $a^{(0,0)}_\mu [\pi^0 \gamma, 3\pi, \ldots ]$

Lattice: $\Delta a_\mu[\pi\pi, \tau] = 4\alpha^2 \sum_t w_t \times \left[ G^\gamma_{01}(t) + G^\gamma_{11}(t) - G^W_{11}(t) \right]$

Pheno: $\Delta a_\mu[\pi\pi, \tau] = \int_{4m^2_{\pi}}^{m^2_{\tau}} ds K(s) \left[ v_0(s) - v_-(s) \right]$

Conversion to Euclidean time for direct comparison

$\Delta a_\mu[\pi\pi, \tau] = 4\alpha^2 \sum_t w_t \times \left\{ \frac{1}{12\pi^2} \int d\omega \omega e^{-\omega t} \left[ R_{1B}(\omega^2) - 1 \right] v_-(\omega^2) \right\}$
Towards a comparison - II

Lattice contains $\pi^0\pi^-\gamma$ states →

Re-evaluation of $G_{EM} \rightarrow G_{EM}^{\pi}$ [in collab. with Cirigliano]

Real photon corrections

Virtual photon corrections

$G_{EM}^{\pi}$ w/o $\pi^0\pi^-\gamma$ FSR

$\frac{v_-}{G_{EM}^{\pi}}$ w $\pi^0\pi^-\gamma$ FSR
Towards a comparison - III

[1] = [Jegelehner, Szafron '17]

modified $\rho \gamma$ coupling
large negative $\Delta a_\mu$

(standard) $G_{EM}$

$G^{\pi}_{EM}$ required for:
experim vs lattice
phenom vs lattice

Modified $\rho \gamma$ not required to describe lattice data

prelim. estimate $G_{EM}^{\pi} \rightarrow$ qualitatively similar to $G_{EM}$
affects $F_{\pi}^0$ in HVP for $(g - 2)_\mu \rightarrow$ risk for double-counting?
potential pole enhancement in $O(\alpha^k)$ $k > 1$?
Conclusions

For precise prediction:

study systematic errors → ongoing finite volume study
improvement of errs → high stat. data set from HLbL

Outlook:

1. full lattice calculation of $\Delta a_\mu[\tau]$ almost complete
2. tests/checks previous calculations
   comparing $\nu_-$ with experiment requires $G_{EM}^\pi$
   study $G_{01}^\gamma$ alone → $\rho\omega$ mixing; $\delta G^{(1,1)}$ alone → $\rho^0$ vs $\rho^-$
3. possibly sensitive to new physics

Thanks for your attention
Presently only leading diagrams are computed $V, F, S, M$ [Blum et al. ’18] working on improving precision between 2 and 4 times working on $SU(3)$ and $1/N_c$ diagrams presently not computed
Peeking at the data - I

Lattice fully inclusive → comparison with $v_-$ problematic

manipulate correlator to implement energy cut

fit lowest energy state $(c_0 + c_1 t)e^{-Et}$

lattice correlator more precise at short distances

fit with fixed energy $E_{\pi\pi I=1}, E_{\pi\gamma}$

temporary solution: not required with better precision
$$\Delta a_\mu = 4\alpha^2 \sum_t w_t \delta G(t) \rightarrow \text{weights suppress short distance}$$

lattice correlator more precise at short distances

fit \((c_0 + c_1 t)e^{-Et}\)

\(E \rightarrow \pi\pi \text{ or } \pi\gamma\)

reduction of stat. noise

temporary solution: not required with better precision
LATTICE IMPROVEMENTS


contribution of diagram $F$ to pure $I = 1$ part of $\Delta a_\mu$

$\Delta a_\mu^{(I=1)}[F \ only]$

$O(1000)$ point-src per conf.
$5 \cdot 10^5$ combinations
80 configurations
$\times 4$ reduction in error
finite volume errs relevant
→ dedicated study

data from [Blum et al. ’18]: $O(500)$ point-src per conf.
76 configurations
Radiative corrections

Some QED corrections computed in Chiral PT

\[ \text{e.g. photon exchange between } \tau \text{ and hadrons} \]

relevant to compare lattice data vs \( v_- \)

is current precision enough?

alternative calculation from lattice possible

\[ \text{[Giusti et al. '17]} \]