State of the Art in PN Gravity Theory

Michèle Levi

Niels Bohr International Academy
Niels Bohr Institute
University of Copenhagen

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Complete state-of-the-art of PN theory for compact binary dynamics

l \ n	$(N^0)LO$	$N^{(1)}LO$	N^2LO	N^3LO	N^4LO
S^0	1	0	3	0	25
S^1	2	7	32		
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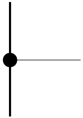
- Each entry at PN order $n + l + \text{Parity}(l)/2$
- A measure for loop computational scale: number of (highest) n-loop graphs that enter at N^nLO of up to the lth multipole moment S^l.
- All (but top right one) are derived in the public "EFTofPNG" code: `https://github.com/miche-levi/pncbc-eftofpng`

Kaluza-Klein decomposition of field

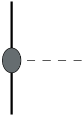
Reduction over time dimension à la Kaluza-Klein

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \equiv e^{2\phi}(dt - A_i dx^i)^2 - e^{-2\phi}\gamma_{ij} dx^i dx^j$$

- $\phi, A_i, \gamma_{ij} \equiv \delta_{ij} + \sigma_{ij}$, KK fields



- Newtonian potential scalar ϕ



- Gravitomagnetic vector A_i
- Hierarchy in coupling to mass and to spin
- Advantageous, preferable e.g. over Lorentz covariant, ADM decompositions...

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EFTs are Universal

There is a Hierarchy of Scales

1. r_s, scale of internal structure, $r_s \sim m$
2. r, orbital separation scale, $\frac{r_s}{r} \sim v^2$
3. λ, radiation wavelength scale, $\frac{r}{\lambda} \sim v$

$v \ll 1 \rightarrow nPN \equiv v^{2n}$ correction in Classical Gravity to Newtonian gravity

Multistage strategy for EFTs of inspiraling binaries

[Goldberger & Rothstein 2007]

1. One-Particle EFT
2. EFT of a Composite Particle
3. Effective Theory of Dynamical Multipoles

It's a multiscale!

One-Particle EFT

1st Stage Remove scale r_S of isolated compact object

In the full theory we only have a vacuum gravitational field:

$$S\left[g_{\mu\nu}\right] = -\frac{1}{16\pi G}\int d^4x\sqrt{g}R\left[g_{\mu\nu}\right]$$

Integrate out strong field modes $g^s_{\mu\nu}$, $g_{\mu\nu} \equiv g^s_{\mu\nu} + \bar{g}_{\mu\nu}$ via bottom-up approach:

$$S_{\text{eff}}\left[\bar{g}_{\mu\nu}, y^\mu(\sigma), e^\mu_A(\sigma)\right] = -\frac{1}{16\pi G}\int d^4x\sqrt{\bar{g}}R\left[\bar{g}_{\mu\nu}(x)\right] + \underbrace{\sum_{i=1}^{\infty} C_i(r_s)\int d\sigma \mathcal{O}_i(\sigma)}_{\equiv S_{pp}(\sigma)\ \text{with Wilson coefficients}}$$

The operators $\mathcal{O}_i(\sigma)$ must respect the symmetries that pertain at low energies.

$$S_{\text{eff}}[\bar{g}_{\mu\nu}, y^\mu] = -\frac{1}{16\pi G}\int d^4x\sqrt{\bar{g}}R\left[\bar{g}_{\mu\nu}(x)\right] - \int m d\sigma + \underbrace{c_{5\text{PN}}\int d\sigma\left(R_{\mu\alpha\nu\beta}\dot{y}^\alpha\dot{y}^\beta\right)^2 + \cdots}_{\text{finite size effects}}$$

EFT of Composite Particle

2nd Stage Remove orbital scale r of binary, (first) via the top-down approach:

$$\bar{g}_{\mu\nu} \equiv \eta_{\mu\nu} + \underbrace{H_{\mu\nu}}_{\text{orbital}} + \underbrace{\widetilde{h}_{\mu\nu}}_{\text{radiation}}$$

$$\partial_t H_{\mu\nu} \sim \frac{v}{r} H_{\mu\nu}, \quad \partial_i H_{\mu\nu} \sim \frac{1}{r} H_{\mu\nu}, \quad \partial_\rho \widetilde{h}_{\mu\nu} \sim \frac{v}{r} \widetilde{h}_{\mu\nu}$$

$$S_{\text{eff}}\left[\bar{g}_{\mu\nu}, y_1^\mu, y_2^\mu, e_{(1)}{}^\mu_A, e_{(2)}{}^\mu_A\right] = -\frac{1}{16\pi G}\int d^4x \sqrt{\bar{g}} R\left[\bar{g}_{\mu\nu}\right] + S_{\text{pp}}(\sigma_1) + S_{\text{pp}}(\sigma_2)$$

Integrate out orbital field modes - in this classical context - only tree level

$$\Rightarrow e^{iS_{\text{eff(composite)}}\left[\widetilde{h}_{\mu\nu}, y^\mu, e^\mu_{(Comp)A}\right]} \equiv \int \mathcal{D}H_{\mu\nu} \; e^{iS_{\text{eff}}\left[\bar{g}_{\mu\nu}, y_1^\mu, y_2^\mu, e_{(1)}{}^\mu_A, e_{(2)}{}^\mu_A\right]}$$

Stop here for effective action strictly in conservative sector, that is **WITHOUT** any remaining (orbital scale) field modes

Spinning Particle: DOFs

Assume isolated object has no intrinsic permanent multipoles beyond mass (monopole) and spin (dipole)

1. *Gravitational field*
   - Metric $g_{\mu\nu}(x)$
   - Tetrad field $\eta^{ab}\tilde{e}_a{}^{\mu}(x)\tilde{e}_b{}^{\nu}(x) = g^{\mu\nu}(x)$

2. *Particle Coordinate*

   $y^{\mu}(\sigma)$ function of arbitrary affine parameter σ
   Particle worldline position does not in general coincide with object's 'center'

3. *Particle rotating DOFs*

   Worldline tetrad, $\eta^{AB}e_A{}^{\mu}(\sigma)e_B{}^{\nu}(\sigma) = g^{\mu\nu}$

   $\Rightarrow$ Angular velocity $\Omega^{\mu\nu}(\sigma) \equiv e^{\mu}_{A}\frac{De^{A\nu}}{D\sigma}$ + conjugate Spin $S_{\mu\nu}(\sigma)$

   $\Rightarrow$ Lorentz matrices $\eta^{AB}\Lambda_A{}^a(\sigma)\Lambda_B{}^b(\sigma) = \eta^{ab}$ + conjugate local spin $S_{ab}(\sigma)$

Spinning Particle: Symmetries

1. General coordinate invariance, and parity invariance
2. Worldline reparametrization invariance
3. Internal Lorentz invariance of local frame field
4. SO(3) invariance of 'body-fixed' spatial triad
5. Spin gauge invariance, that is invariance under choice of completion of 'body-fixed' spatial triad through timelike vector

Spin as extra particle DOF

Effective action of spinning particle

[Hanson & Regge 1974, Bailey & Israel 1975]

- $u^\mu \equiv dy^\mu/d\sigma$, $\Omega^{\mu\nu} \equiv e^\mu_A \frac{De^{A\nu}}{D\sigma} \Rightarrow L_{\text{pp}}\left[\bar{g}_{\mu\nu}, u_\mu, \Omega^{\mu\nu}\right]$
- $S_{\mu\nu} \equiv -2\frac{\partial L}{\partial \Omega^{\mu\nu}}$ spin as further particle DOF – classical source

$$\Rightarrow S_{\text{pp}}(\sigma) = \int d\sigma \left[-p_\mu u^\mu - \frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu} + L_{\text{SI}}\left[\bar{g}_{\mu\nu}\left(y^\mu\right), u^\mu, S_{\mu\nu}\right]\right]$$

- This form assumes covariant gauge, e.g. $e^\mu_{[0]} = \frac{p^\mu}{\sqrt{p^2}}$, $\quad S_{\mu\nu}p^\nu = 0$
- Linear momentum $p_\mu \equiv -\frac{\partial L}{\partial u^\mu} = m\frac{u^\mu}{\sqrt{u^2}} + \mathcal{O}(RS^2)$

For EFT of spin – gauge of both rotational DOFs
should be fixed at level of one-particle action

Extra term in minimal coupling

Introduce gauge freedom into tetrad by boosting its timelike component
$\rightarrow$ entails transformed gauge of spin $\hat{S}_{\mu\nu}$, traditionally called "SSC"

$\Rightarrow$ Extra term in action appears!

- From minimal coupling

$$\frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} = \frac{1}{2}\hat{S}_{\mu\nu}\hat{\Omega}^{\mu\nu} + \frac{\hat{S}^{\mu\rho}p_{\rho}}{p^2}\frac{Dp_{\mu}}{D\sigma}$$

- Extra term with covariant derivative of momentum, contributes to finite size effects, yet carries no Wilson coefficient
- As of LO with spin, to all orders in spin!
- Essentially Thomas precession

- Beyond minimal coupling we use the relation

$$S_{\mu\nu} = \hat{S}_{\mu\nu} - \frac{\hat{S}_{\mu\rho}p^{\rho}p_{\nu}}{p^2} + \frac{\hat{S}_{\nu\rho}p^{\rho}p_{\mu}}{p^2}$$

LO non-minimal couplings to all orders in spin

$$S^{\mu} \equiv *S^{\mu\nu}\frac{p_{\nu}}{\sqrt{p^2}}, *S_{\alpha\beta} \equiv \frac{1}{2}\epsilon_{\alpha\beta\mu\nu}S^{\mu\nu};\ E_{\mu\nu} \equiv R_{\mu\alpha\nu\beta}u^{\alpha}u^{\beta},\ B_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\alpha\beta\gamma\mu}R^{\alpha\beta}{}_{\delta\nu}u^{\gamma}u^{\delta}$$

New spin-induced Wilson coefficients:

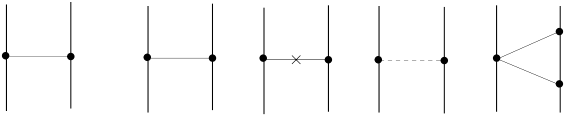
$$L_{\rm SI} = \sum_{n=1}^{\infty}\frac{(-1)^n}{(2n)!}\frac{C_{ES^{2n}}}{m^{2n-1}}D_{\mu_{2n}}\cdots D_{\mu_3}\frac{E_{\mu_1\mu_2}}{\sqrt{u^2}}S^{\mu_1}S^{\mu_2}\cdots S^{\mu_{2n-1}}S^{\mu_{2n}}$$
$$+\sum_{n=1}^{\infty}\frac{(-1)^n}{(2n+1)!}\frac{C_{BS^{2n+1}}}{m^{2n}}D_{\mu_{2n+1}}\cdots D_{\mu_3}\frac{B_{\mu_1\mu_2}}{\sqrt{u^2}}S^{\mu_1}S^{\mu_2}\cdots S^{\mu_{2n-1}}S^{\mu_{2n}}S^{\mu_{2n+1}}$$

LO spin couplings up to 5PN order

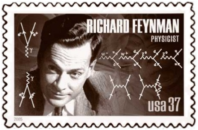
- $L_{ES^2} = -\frac{C_{ES^2}}{2m}\frac{E_{\mu\nu}}{\sqrt{u^2}}S^{\mu}S^{\nu}$, Quadrupole @2PN
- $L_{BS^3} = -\frac{C_{BS^3}}{6m^2}\frac{D_{\lambda}B_{\mu\nu}}{\sqrt{u^2}}S^{\mu}S^{\nu}S^{\lambda}$, Octupole @3.5PN
- $L_{ES^4} = \frac{C_{ES^4}}{24m^3}\frac{D_{\lambda}D_{\kappa}E_{\mu\nu}}{\sqrt{u^2}}S^{\mu}S^{\nu}S^{\lambda}S^{\kappa}$, Hexadecapole @4PN

LO sectors beyond Newtonian

Feynman graphs of non-spinning sector to 1PN order



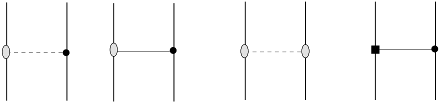
(a0) (a) (b) (c) (d)



Newton
.

One-loop diagram – absent from 1PN
with KK parametrization of field

LO Feynman diagrams with spin – to quadratic-in-spin

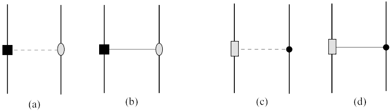


(a) (b) (a1) (a2)

Spin-Orbit Spin-Spin

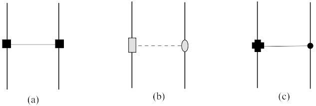
LO cubic & quartic in spin

Feynman diagrams of LO **cubic** in spin sector



- On left pair – quadrupole-dipole, on right – octupole-monopole
- Note analogy of each pair with LO spin-orbit

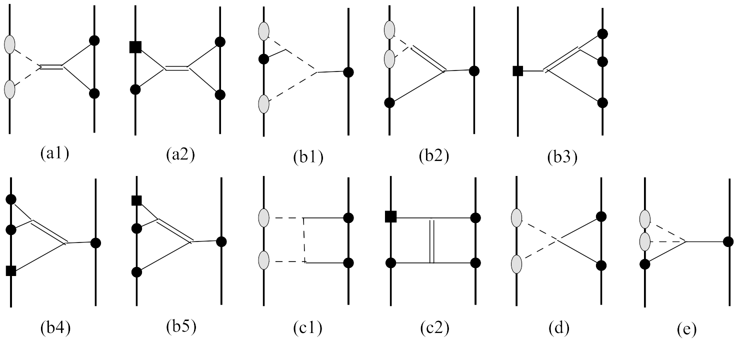
Feynman diagrams of LO **quartic** in spin sector



- On left and right – quadrupole-quadrupole and hexadecapole-monopole
  Each is analogous to LO spin-squared
- In middle – octupole-dipole analogous to LO spin1-spin2

NNLO spin-squared sector

Feynman diagrams of order G^3 with 2 loops



- In general with spin at N^{n}LO – n-loop graphs are realized
- Five 2-loop topologies actually fall into 3 kinds
- One of which – topology (c1,c2) – is the leading nasty one!

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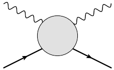
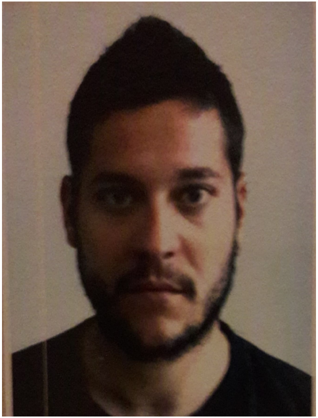
- Gray area corresponds to where we can no longer take $p^\mu \simeq \frac{m}{u} u^\mu$, but have to take into account corrections from non-minimal coupling part of spinning particle action. What happens then?
- Also corresponds to gravitational Compton scattering with $s > 1$



- Can we get insight on the non-uniqueness of fixing the graviton Compton amplitude with $s > 2$ from PN gravity?

NLO Cubic-in-Spin – Task Force

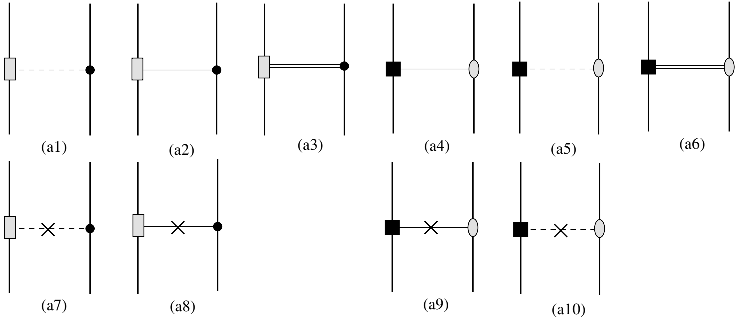


Stavros Mougiakakos
IPhT Saclay

Mariana Vieira
NBI Copenhagen

NLO cubic-in spin sector

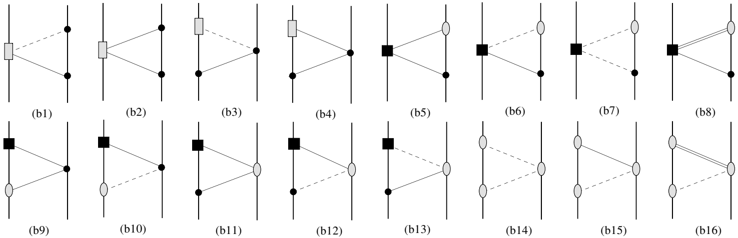
One-Graviton Exchange



- At 1-graviton level we only have 2 kinds of interaction, similar to LO
- 4 graphs appeared at LO come in with insertions on the propagators
- New octupole coupling to tensor component of the KK fields

NLO cubic-in spin sector

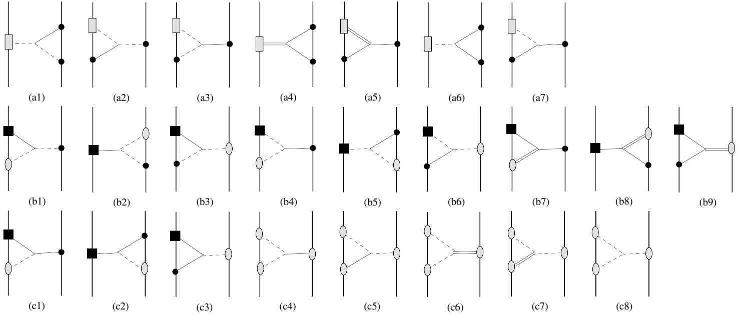
Two-Graviton Exchange



- Graphs include all relevant interactions among the spin-induced quadrupole, octupole, and the mass and spin
- There are nonlinearities originating strictly from minimal coupling
- New octupole–two-graviton couplings

NLO cubic-in spin sector

Cubic self-interaction



- Graphs include all relevant interactions among the spin-induced quadrupole, octupole, and the mass and spin
- There are nonlinearities originating strictly from minimal coupling
- Cubic vertices with time derivatives, similar to NLO (odd P) spin-orbit sector

NLO cubic-in spin sector

New Feature: Extra one- and two-graviton exchange

- $p_\mu = \frac{m}{u} u_\mu + \Delta p_\mu(RS^2) \Rightarrow L_{S^3}$
- New type of worldline-graviton couplings to "composite" octupole with similar graphs as with "elementary" spin-induced octupole

Conclusions

ML, Rept. Prog. Phys. 2019
ML, Stavros Mougiakakos, Mariana Vieira, arXiv:1912.xxxxx

- EFT of gravitating spinning objects
  has pushed state of the art in PN Gravity
- "Even is easier than odd"
- NLO cubic-in-spin - 1st complete sector
  beyond current state of the art at 4PN order
- New features from gauge of rotational DOFs
  where the difference between p_μ and u_μ matters
- Going beyond this sector into the "gray area"
  – may become impossibly intricate
- Possible insight for the grav. Compton amplitude for higher spins.
  Can amplitudes computations capture these classical effects too?