QCD meets Gravity

UCLA, December 11, 2019

Ultra-soft gravitational radiation from ultra-relativistic gravitational collisions

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An unsolved textbook exercise

The problem of computing the GWs emitted by a binary system is (almost) as old as GR.

Most of the time these processes are in the NR regime, with the exception of the merging itself when moderately relativistic speeds ($v/c \sim 0.3$-0.6) are reached.

Main tools: PN, PM, EOB, numerical relativity...

A tough but very relevant problem.

Much less attention has been devoted in the past to an easier(?), but apparently academic, problem.

Consider the collision of two massless (or highly relativistic, $\gamma = E/m \gg 1$) gravitationally interacting particles in the regime in which they deflect each other's trajectory by a small angle $\theta_s = \theta_E$:

$$\boxed{\theta_s \equiv \theta_E = \frac{8GE}{b} \equiv \frac{2R}{b} \;;\; c = 1}$$

"Exercise":compute the GW spectrum associated with this collision to lowest order in θ_E.

How can it possibly be an unsolved problem?

(A. Gruzinov, private conversation, early 2014)

What we do know

1. The zero frequency limit (Smarr, prl 1977)

A solid prediction for $dE^{GW}/d\omega\, d^2\theta$ as $\omega \to 0$. It goes to a constant obtained either by a classical or by a quantum argument.

The result (2->2 after integrating over angles) is classical (c=1 throughout):

$$\frac{dE^{GW}}{d\omega} \to \frac{Gs}{\pi}\theta_s^2 \log(4e\theta_s^{-2}) \quad ; \quad \omega \to 0 \quad ; \quad \theta_s \ll 1$$

2. Work in the seventies (P. D'Eath, K&T)

THE GENERATION OF GRAVITATIONAL WAVES.
IV. BREMSSTRAHLUNG*†‡

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ABSTRACT

This paper attempts a definitive treatment of "classical gravitational bremsstrahlung"—i.e., of the gravitational waves produced when two stars of arbitrary relative mass fly past each other with arbitrary relative velocity v, but with large enough impact parameter that

$$\text{(angle of gravitational deflection of stars' orbits)} \ll (1 - v^2/c^2)^{1/2} .$$

NB: $\theta_s < \gamma^{-1} \Rightarrow q = m\ v\ \gamma\ \theta_s < m$

Cf. extending recent PM calculations
of conservative process to UR regime

3. Numerical Relativity

(F. Pretorius, U. Sperhake, private comm. ~ 04.14)

The calculation in NR is challenging because the deflected particles carry with them two shock waves that travel (almost) as fast as the emitted GWs (and roughly in the same direction)

Disentangling the two becomes very tricky for γ's >~ 3 and θ_E a bit > γ^{-1}

Outline

- ~~I. Results & challenges on transplanckian gravitational scattering: a short summary (see also PdV's talk)~~

- II. Ultra-soft gravitational radiation from ultra-relativistic collisions via:

    - IIa. Classical GR

    - IIb. Quantum eikonal

    - IIc. Soft-theorems

Highlights

- Restoring elastic unitarity via eikonal resummation of s-channel ladders

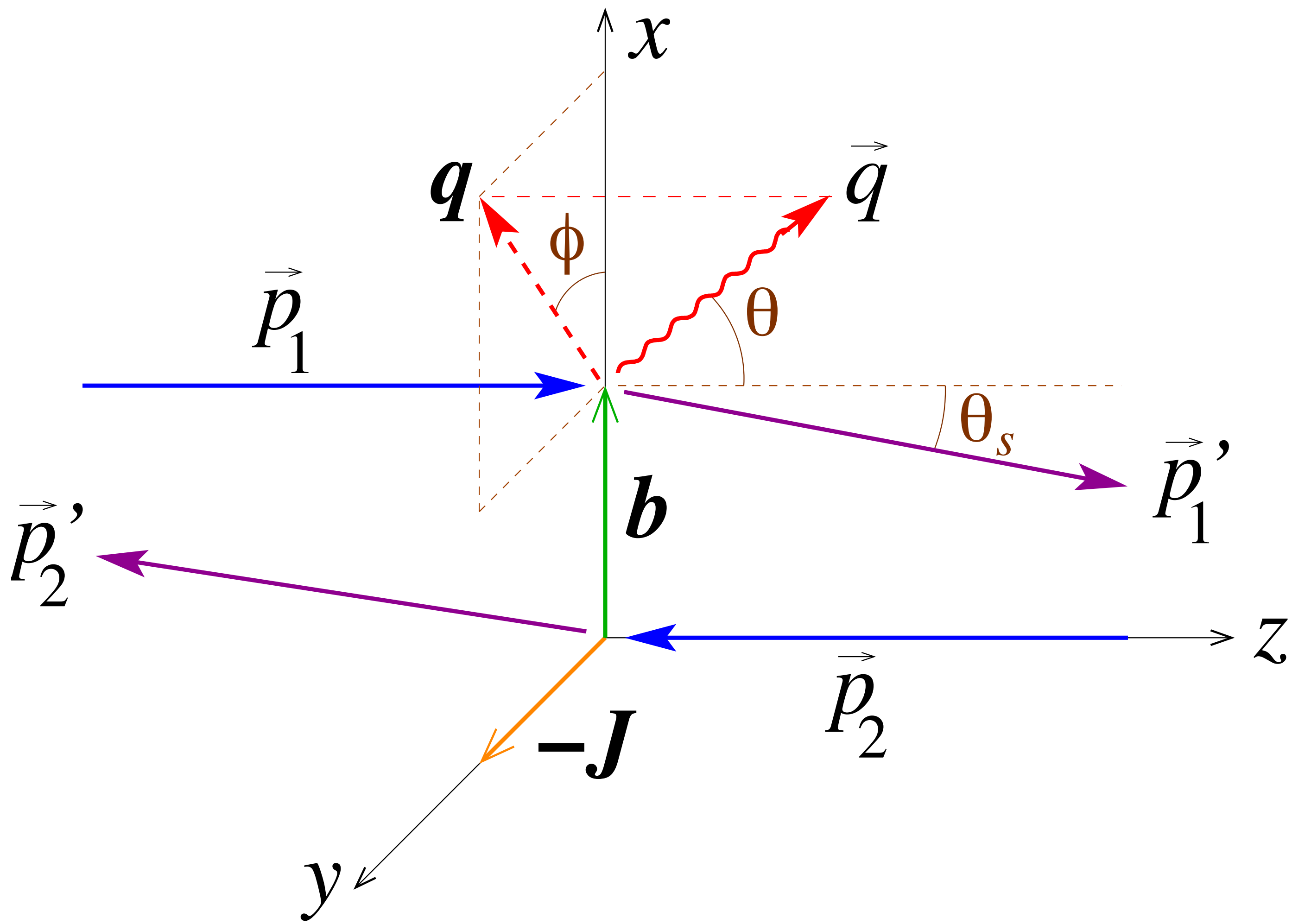
- Gravitational deflection up to 3PM (ACV90)

- Unitarity-preserving tidal excitation of colliding strings throuh quadrupole moment…

- "Pre-collapse", $\langle E_{final} \rangle \sim M_P^2 / \langle E_{initial} \rangle$, analog of pre-confinement in PQCD?

II: Ultra soft gravitational radiation from ultra-relativistic collisions

The process at hand



Three possible approaches

1. Classical GR
   (A. Gruzinov & GV, 1409.4555)
2. Quantum eikonal a la ACV
   (CC&Coradeschi & GV, 1512.00281, Ciafaloni, Colferai & GV, 1812.08137)
3. Soft-theorems
   (Laddha & Sen, 1804.09193; Sahoo & Sen 1808.03288, Addazi, Bianchi & GV, 1901.10986)

Anticipating:

a. 2. goes over to 1. in the classical limit;
b. They agree w/ 3. in the overlap of their respective domains of validity

Domains of validity

- The CGR and quantum eikonal approaches are limited to small-angle scattering but cover a wider range of GW frequencies.

- The soft-theorem approach is not limited to small deflection angles but is only valid in a smaller frequency range.

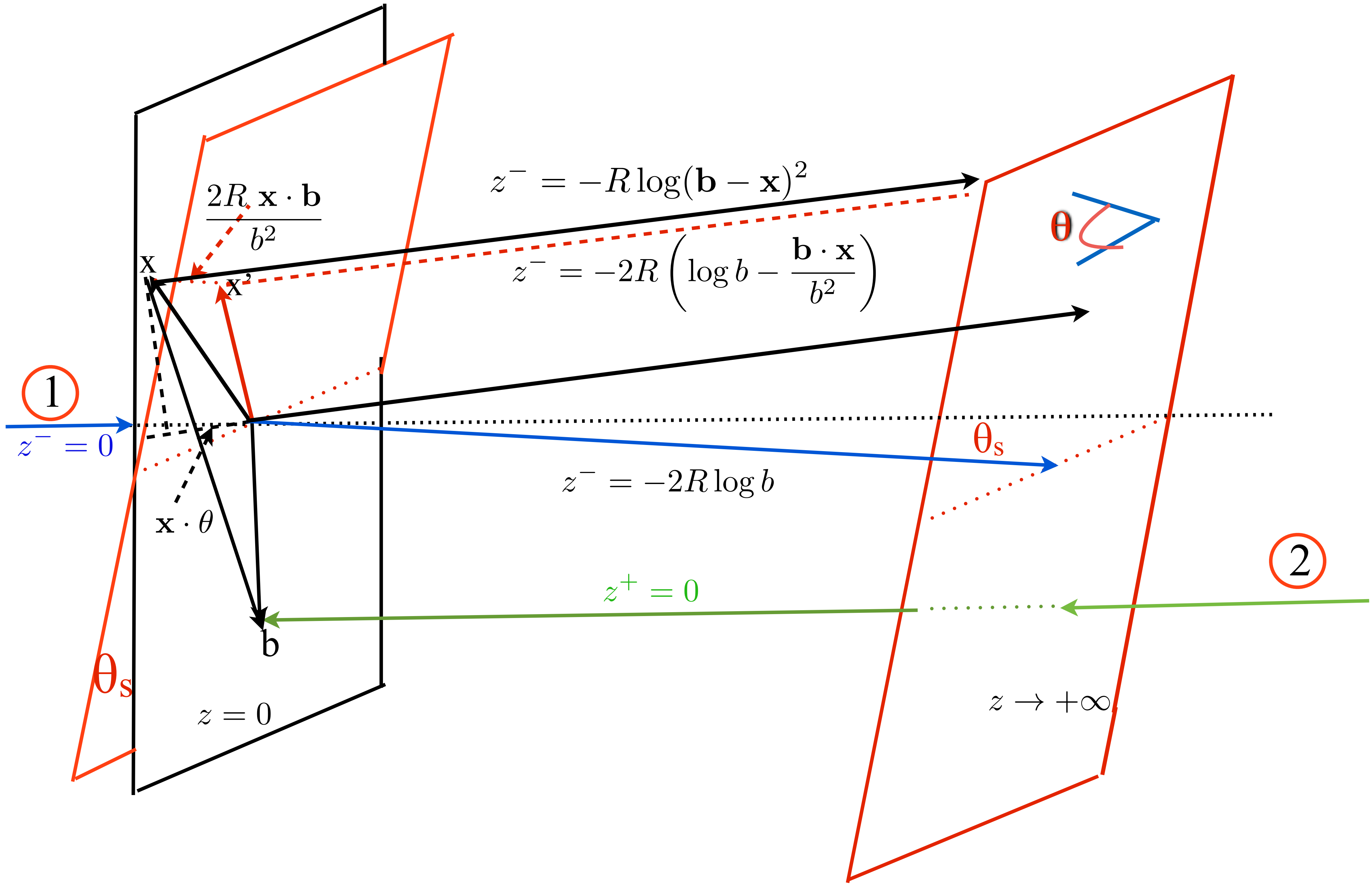
A classical GR approach

(A. Gruzinov & GV, 1409.4555)

Based on Huygens superposition principle.

For gravity this includes in an essential way the gravitational (Shapiro) time delay in AS metric.

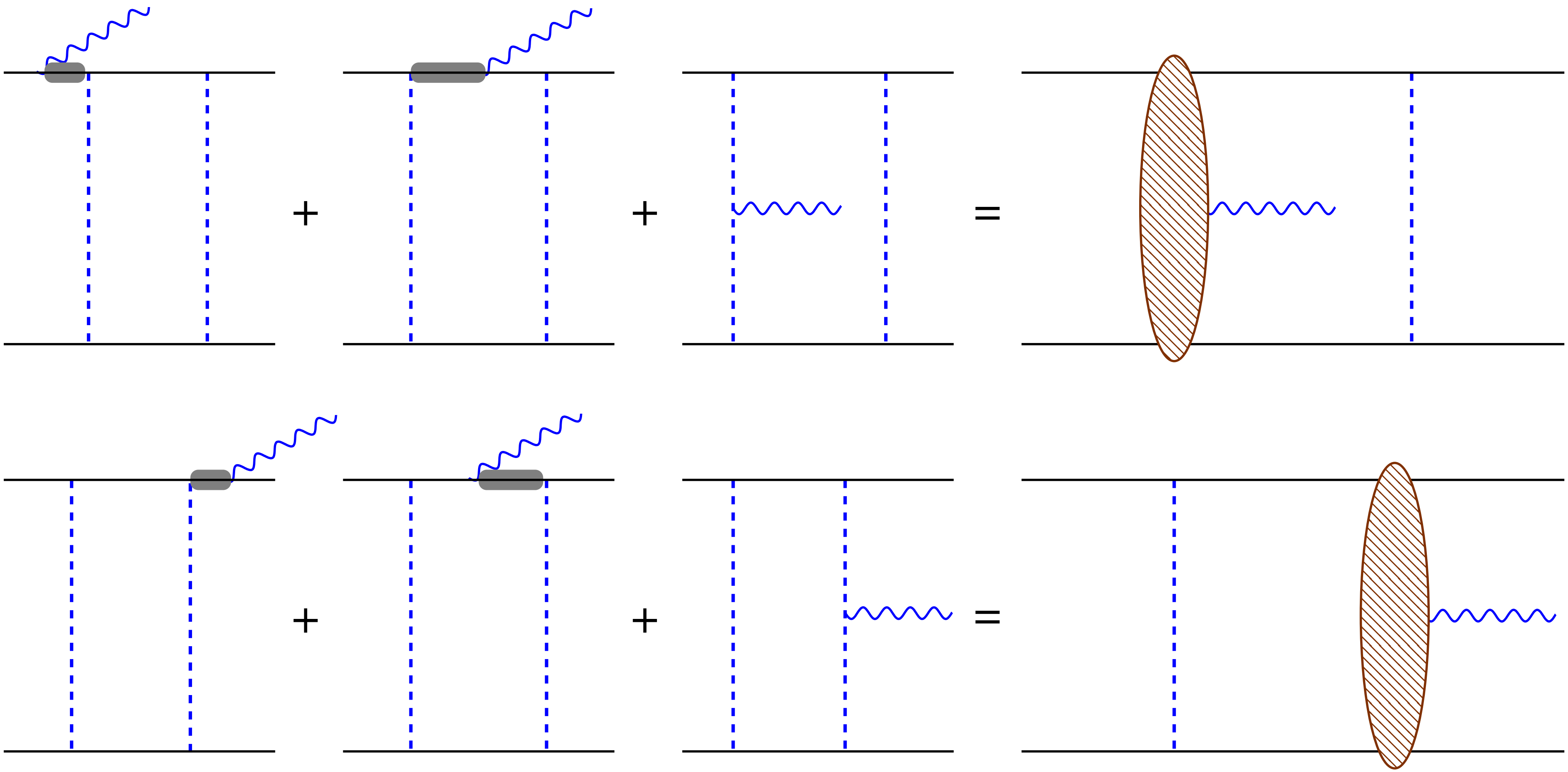
In pictures (formulae to be given later)



A quantum eikonal approach

(Ciafaloni, Colferai& GV, 1505.06619, CC&Coradeschi & GV, 1512.00281)

Emission from external and internal legs throughout the whole ladder (with its suitable phase) has to be taken into account for not-so-soft gravitons.



One should also take into account the (finite) difference between the (infinite) Coulomb phase of the final 3-particle state and that of an elastic 2-particle state.

When this is done, the classical result of G+V is exactly recovered for $h\omega/E \to 0$!

Here it comes!

The classical result/limit

Frequency + angular spectrum ($s = 4E^2$, $R = 4GE$)

$$\frac{dE^{GW}}{d\omega\ d^2\tilde{\theta}} = \frac{GE^2}{\pi^4}|c|^2\ ;\ \tilde{\boldsymbol{\theta}} = \boldsymbol{\theta} - \boldsymbol{\theta}_s\ ;\ \boldsymbol{\theta}_s = 2R\frac{\boldsymbol{b}}{b^2}$$

$$c(\omega, \tilde{\boldsymbol{\theta}}) = \int \frac{d^2x\ \zeta^2}{|\zeta|^4}\ e^{-i\omega\mathbf{x}\cdot\tilde{\boldsymbol{\theta}}}\left[e^{-2iR\omega\Phi(\mathbf{x})} - 1\right]$$

$$\zeta = x + iy \qquad \Phi(\mathbf{x}) = \frac{1}{2}\ln\frac{(\mathbf{x}-\mathbf{b})^2}{b^2} + \frac{\mathbf{b}\cdot\mathbf{x}}{b^2}$$

$$c(\omega, \boldsymbol{\theta}) = \int \frac{d^2x\ \zeta^2}{|\zeta|^4} e^{-i\omega\mathbf{x}\cdot\boldsymbol{\theta}}\left[e^{-iR\omega\ln\frac{(\mathbf{x}-\mathbf{b})^2}{b^2}} - e^{+2iR\omega\frac{\mathbf{b}\cdot\mathbf{x}}{b^2}}\right]$$

Re ζ^2 and Im ζ^2 correspond to usual (+,x) GW polarizations, ζ^2, ζ^{*2} to the two circular ones (not each other's cc!).

Subtracting the deflected shock wave is crucial!

Analytic results:

A Hawking knee

(CC&Coradeschi & GV, 1512.00281)

& an unexpected bump

(Ciafaloni, Colferai & GV, 1812.08137)

Below $\omega = b^{-1}$ the GW-spectrum "freezes" => ZFL

$$\boxed{\frac{dE^{GW}}{d\omega} \to \frac{4G}{\pi}\,\theta_s^2 E^2 \,\log(\theta_s^{-2})}$$

For $b^{-1} < \omega < R^{-1}$ it is almost flat in ω

$$\boxed{\frac{dE^{GW}}{d\omega} \sim \frac{4G}{\pi}\theta_s^2 E^2 \log(\omega R)^{-2}}$$

Above $\omega = R^{-1}$ drops, takes a "scale-invariant" form:

Hawking knee!

$$\boxed{\frac{dE^{GW}}{d\omega} \sim \theta_s^2 \frac{E}{\omega}}$$

This gives a log ω^* in the "efficiency" for a cutoff at ω^*

At $\omega \sim R^{-1} \theta_s^{-2}$ the above spectrum becomes $O(Gs\, \theta_s^4)$ i.e. of the same order as terms we neglected.

Also, if continued above $R^{-1} \theta_s^{-2}$, the so-called "Dyson bound" ($dE/dt < 1/G$) would be violated. Using $\omega^* \sim R^{-1} \theta_s^{-2}$ we find (to leading-log accuracy) a GW "efficiency"

$$\boxed{\frac{E^{GW}}{\sqrt{s}} = \frac{1}{2\pi}\, \theta_s^2\, \log(\theta_s^{-2})}$$

The fine spectrum below 1/b

The above results were very suggestive of a monotonically decreasing spectrum

This appears not to be the case…

A careful study of the region $\omega R \ll 1$, but with ωb generic, shows that:

- At $\omega b < (\ll) 1$ there are corrections of order $(\omega b)\log(\omega b)$, $(\omega b)^2\log^2(\omega b)$.

- First noticed by Sen et al. in the context of soft theorems in $D=4$.

- These logarithmically enhanced sub and sub-sub leading corrections disappear at $\omega b > 1$ so that the previously found $\log(1/\omega R)$ behavior (for $\omega b > 1 > \omega R$), as well as the Hawking knee, remain valid.

- The ωb (both w/ and w/out $\log(\omega b)$) correction only appears for circularly polarized (definite helicity) GWs but disappear either for the linear + and x polarizations, or after summing over them, or, finally, after integration over the azimuthal angle.

- The $(\omega b)\log(\omega b)$ terms are in complete agreement with what had been previously found by A. Sen and collaborators using soft-graviton theorems to sub-leading order (see below).

- The leading $(\omega b)^2 \log^2(\omega b)$ correction to the total flux is positive and produces a bump at $\omega b \sim 0.5$.

- Could not be compared to Sen et al. who only considered $\omega b \log(\omega b)$ corrections.

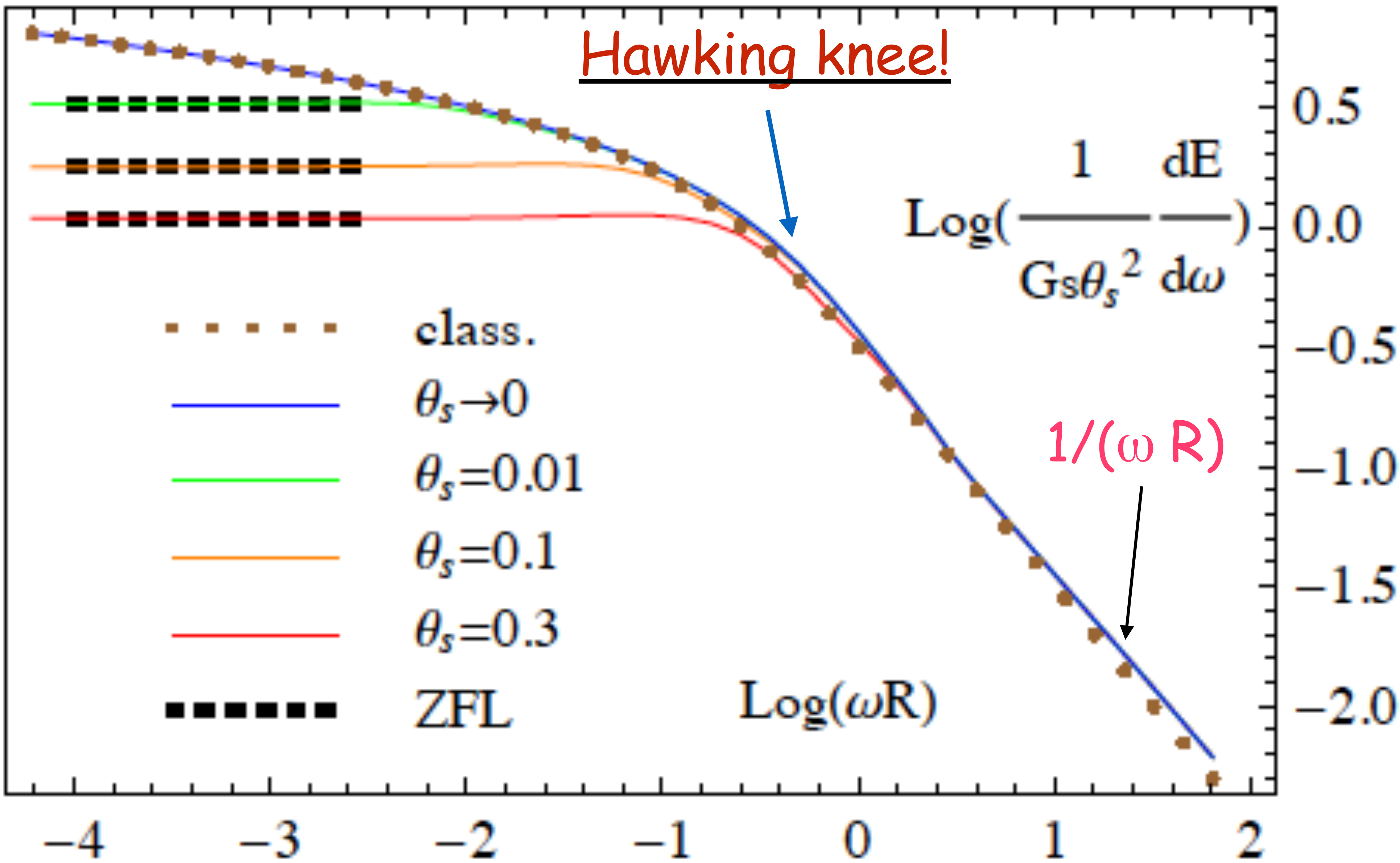
- Confirmed by Sahoo (private comm. by Sen).

- Can be compared successfully with soft-graviton approach if Sen et al.'s recipe is adopted at $O(\omega^2)$, see below.

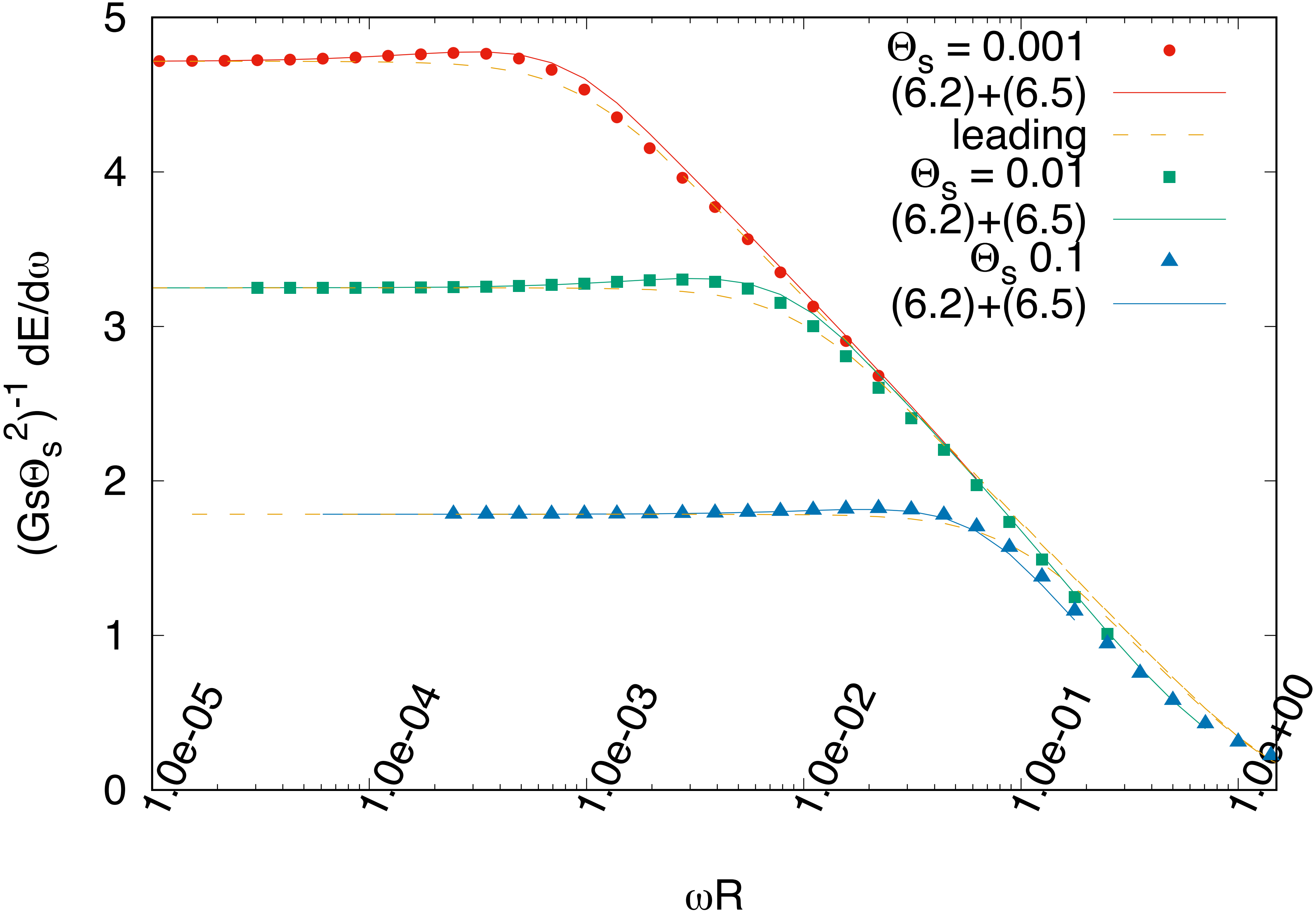
Numerical results

Ciafaloni, Colferai, Coradeschi & GV-1512.00281
Ciafaloni, Colferai & GV-1812.08137

(CCCV 1512.00281)



(CCV 1812.08137)



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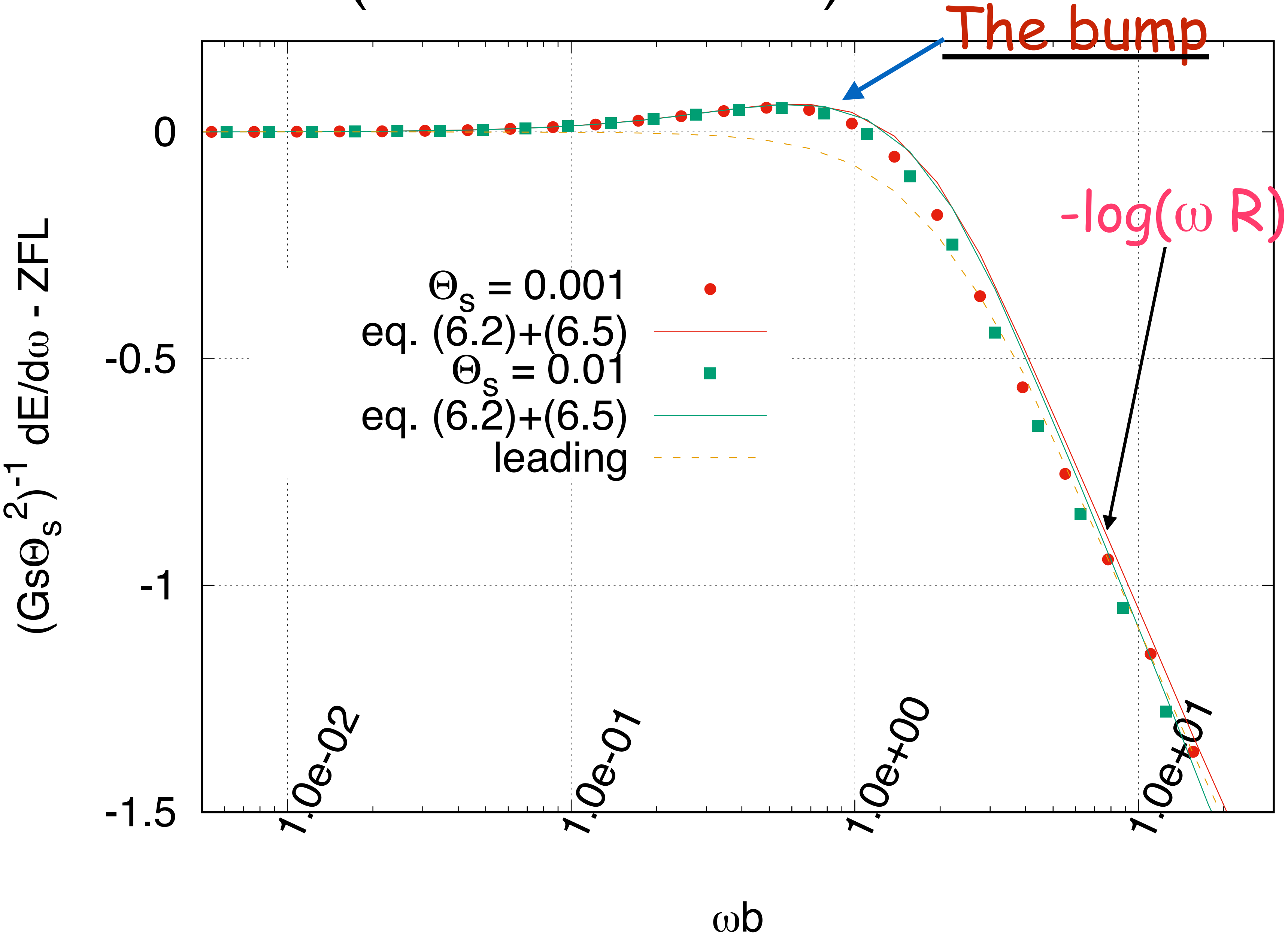
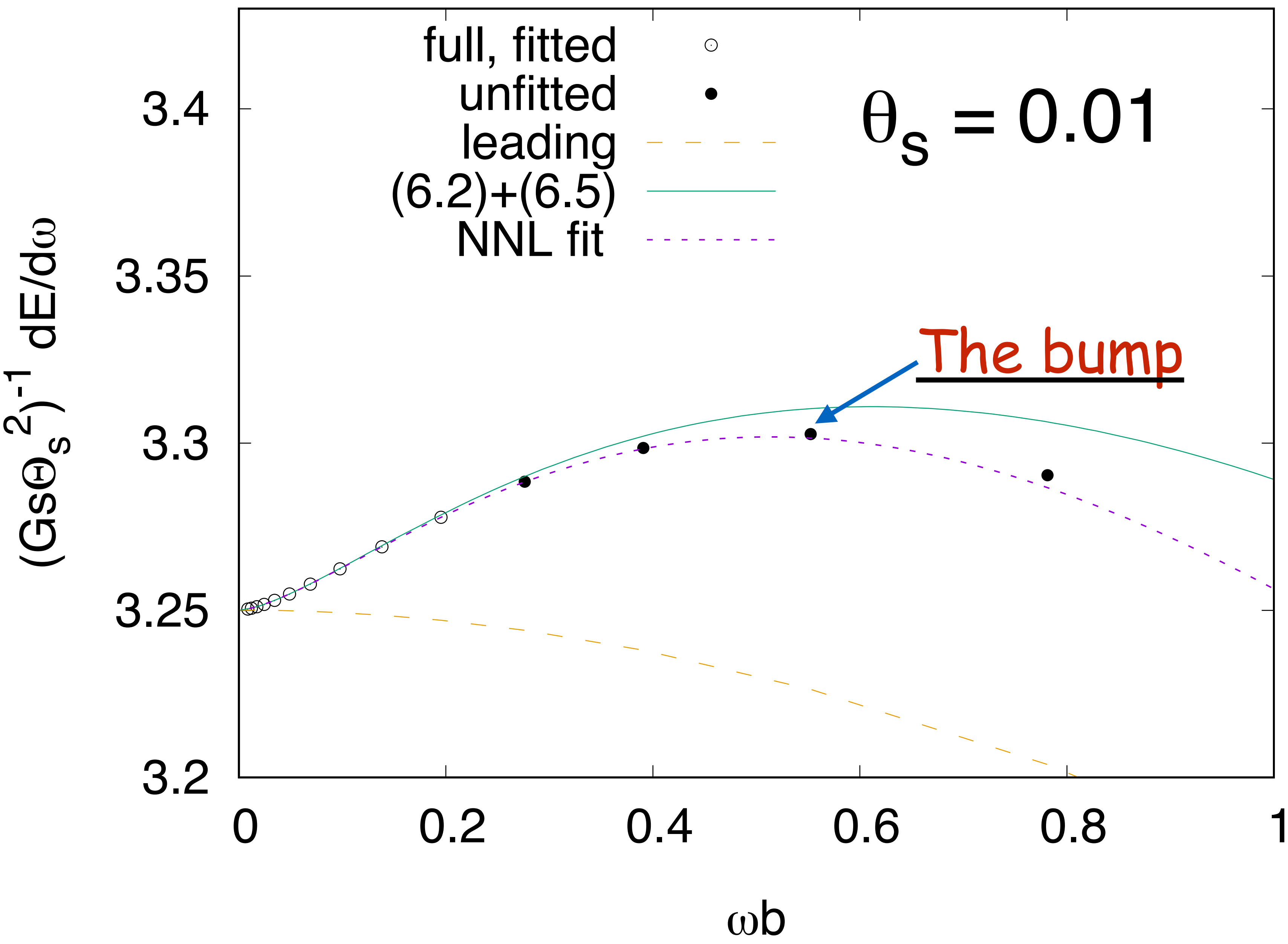
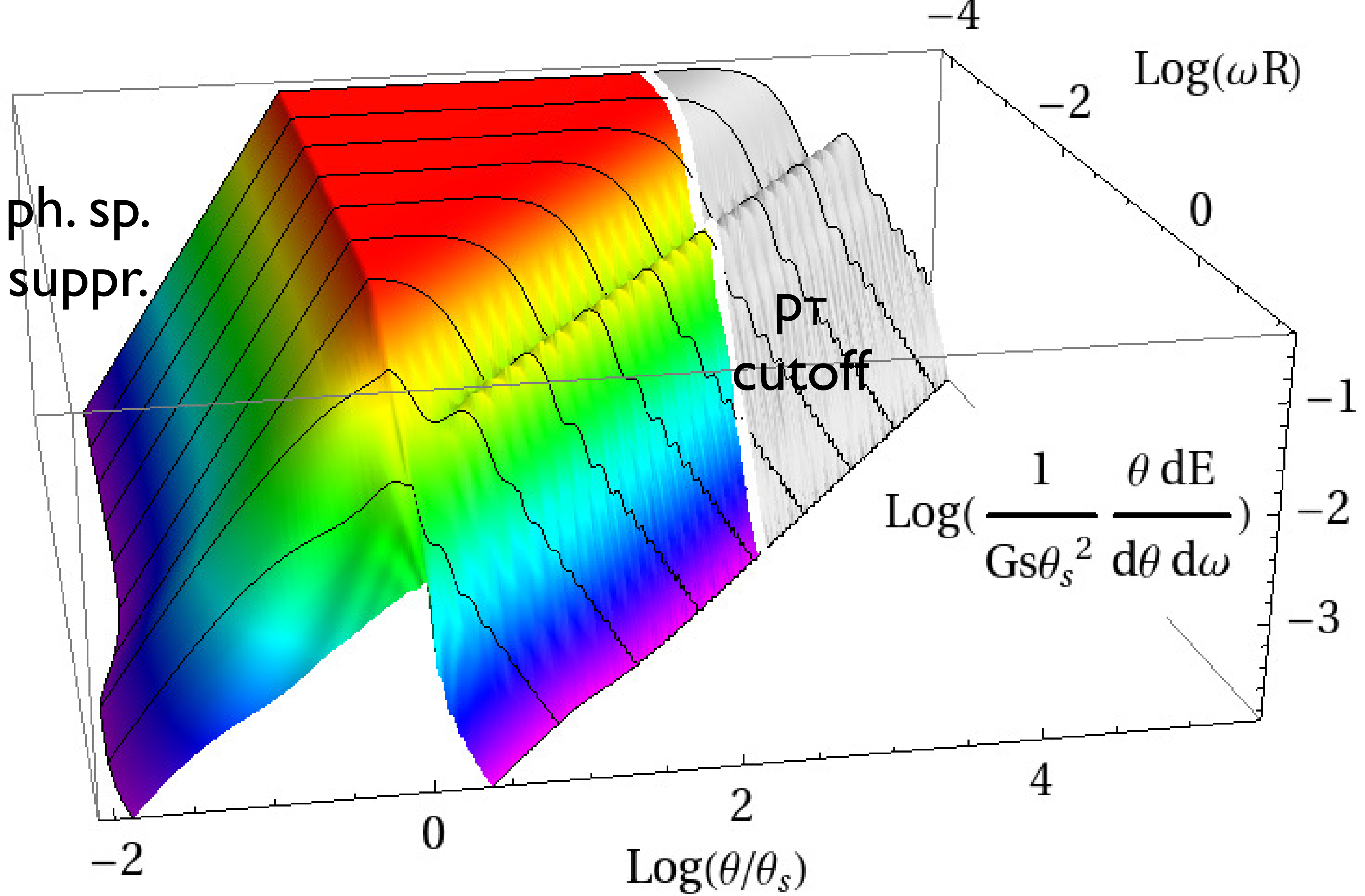


(CCV 1812.08137)



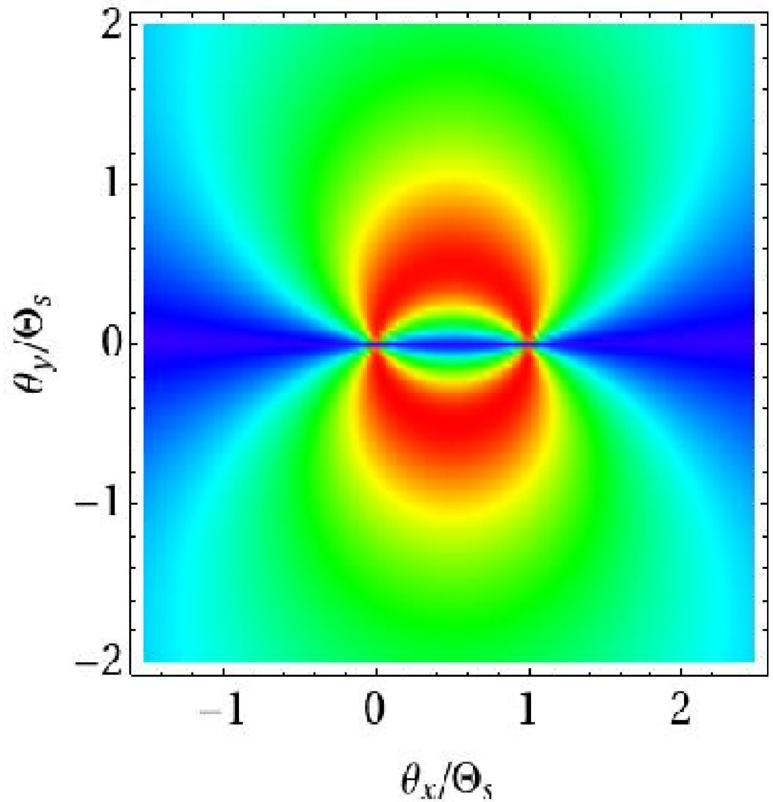
$\theta_s = 10^{-3}$



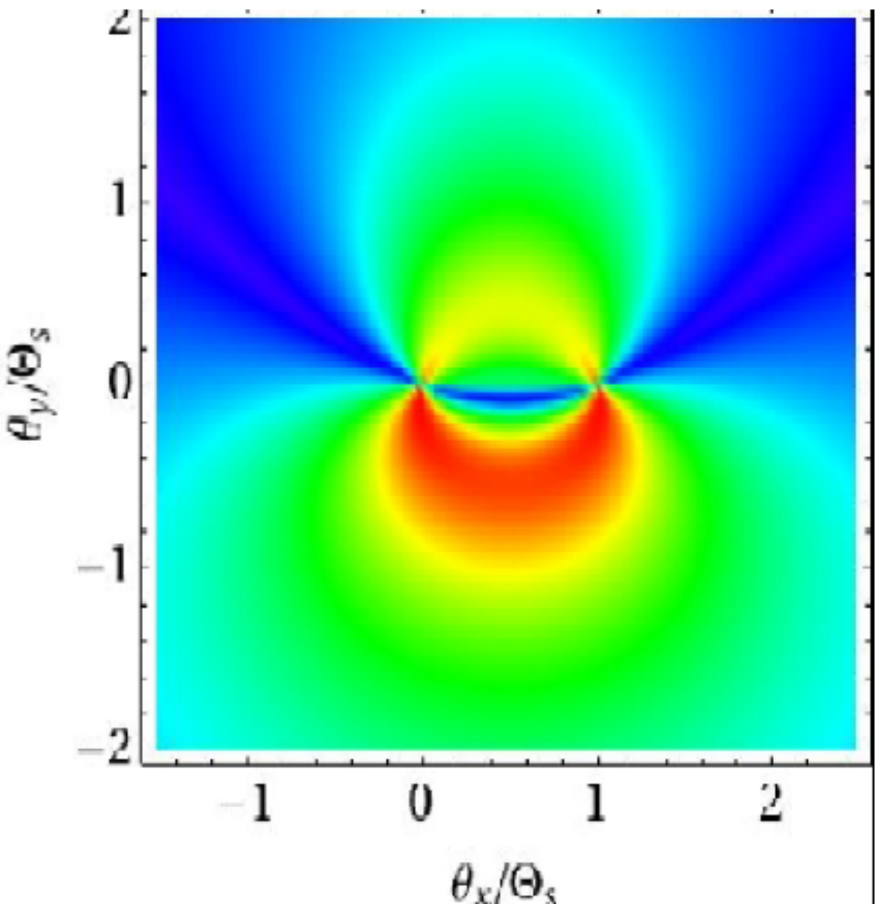
M. Ciafaloni, D. Colferai & GV, 1505.06619

Angular (polar and azimuthal) distribution

$\omega R = 10^{-3}$



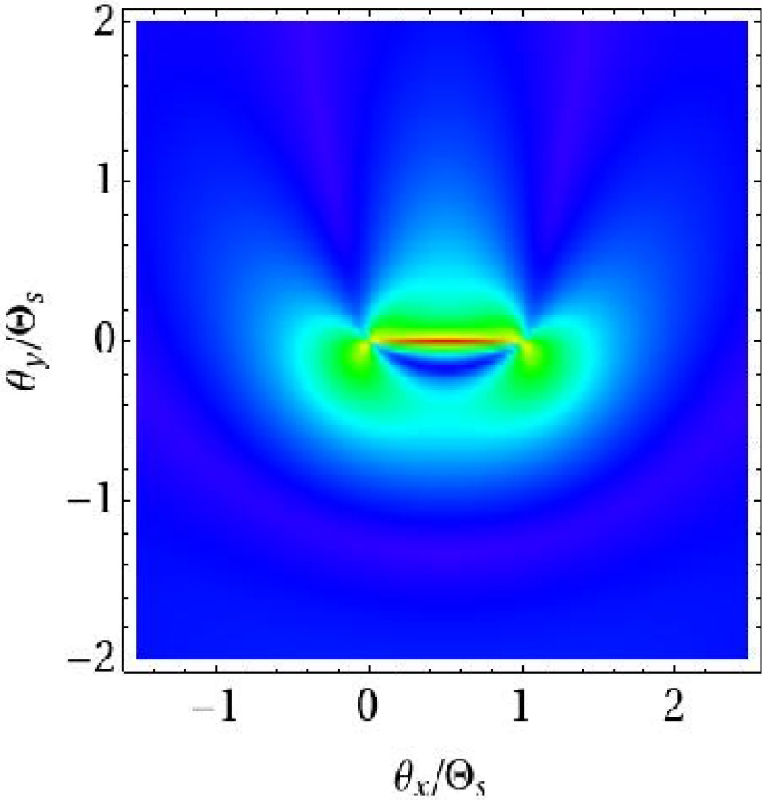
$\omega R = 0.125$



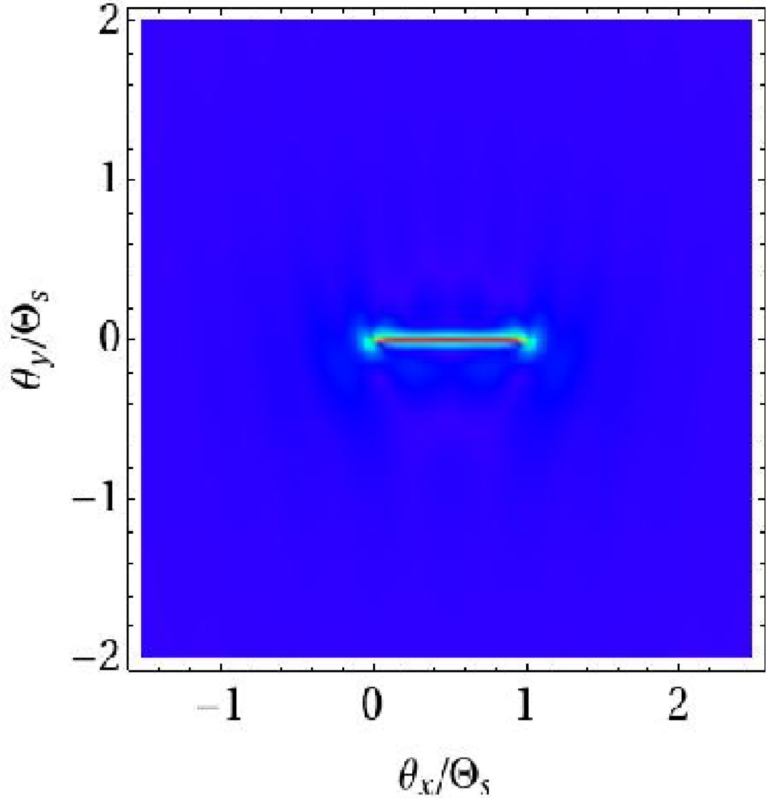
M. Ciafaloni, D. Colferai, F. Coraldeschi & GV, 1512.00281

Angular (polar and azimuthal) distribution

$\omega R = 1.0$



$\omega R = 8.0$



Selected for PRD's picture gallery…

A soft-theorem approach

Beyond the ZFL via soft theorems
(Laddha & Sen, 1804.09193;
Sahoo & Sen, 1808.03288,
Addazi, Bianchi & GV, 1901.10986)

Low-energy (soft) theorems for photons and gravitons (Low, Weinberg, … sixties) had a revival recently (Strominger, Cachazo, Bern, Di Vecchia, Bianchi…). In the case of a soft graviton of momentum q we have (for spinless hard particles)

$$\mathcal{M}_{N+1}(p_i;q) \approx \kappa \sum_{i=1}^{N} \left[\frac{p_i h p_i}{q p_i} + \frac{p_i h J_i q}{q p_i} - \frac{q J_i h J_i q}{2 q p_i} \right] \mathcal{M}_N(p_i)$$

$$\equiv S(q)\mathcal{M}_N(p_i) \; ; \; S(q) = S_0(q) + S_1(q) + S_2(q)$$

$$J_i^{\mu\nu} = p_i^{\mu} \partial / \partial p_{\nu}^{i} - p_i^{\nu} \partial / \partial p_{\mu}^{i}$$

The amplitude for emitting many soft gravitons should factorize and the same should be true for virtual soft-graviton corrections. As a result the "bare" S-matrix element: $\mathcal{S}^{(0)}_{fi} = \langle f|\mathcal{S}^{(0)}|i\rangle$ gets dressed by a unitary coherent-state operator:

$$\mathcal{S}^{(0)} \to \mathcal{S} = \exp\left(\int \frac{d^3q}{\sqrt{2\omega}}(\lambda^*_q a^\dagger_q - \lambda_q a_q)\right)\mathcal{S}^{(0)}$$

The expectation value of the energy carried by the soft-gravitons in the process at hand will be given by

$$\langle 0|\langle i|\mathcal{S}^\dagger|f\rangle \int d^3q\hbar\omega a^\dagger_q a_q \langle f|\mathcal{S}|i\rangle|0\rangle = \int_\lambda^\Lambda \frac{d^3q}{2\omega}\hbar\omega|\lambda_q|^2 \sum_{\mathrm{sgr}(\Lambda)} |\langle f;\mathrm{sgr}|\mathcal{S}|i\rangle|^2$$

where we have used properties of coherent states.

Finally, we have:

$$\frac{dE^{GW}(i \to f)}{d^3q} = \frac{\hbar}{2} |\lambda_q^{(i \to f)}|^2$$

At subleading order λ_q includes differential operators that act of the amplitude itself. Thus a better way to write the above equation is

$$\frac{dE^{GW}(i \to f)}{d^3q} = \frac{\langle i|\mathcal{S}^\dagger|f\rangle \frac{\hbar}{2} |\lambda_q^{(i \to f)}|^2 \langle f|\mathcal{S}|i\rangle}{|\langle f|\mathcal{S}|i\rangle|^2}$$

where λ_q is the soft operator S_q we defined earlier that can act on either side.

We want to find a general expression for $|\lambda_q|^2$ (after integrating over angles) without reference to the particular amplitude it is acting on.

Recovering the ZFL (m=0 case)

Keeping just S_0, summing over polarizations, and integrating over the angles while keeping $\omega = |q|$ (in c.o.m.) fixed, we find (all p_i incoming)

$$\frac{dE_0^{GW}}{d\omega} = \hbar\omega \frac{dN_0}{d\omega} = -\frac{2G}{\pi} \sum_{i,j} (p_i p_j) \log \frac{|p_i p_j|}{\mu^2}$$

NB:result does not depend on μ, free of mass (collinear) divergences. For 2->2 scattering:

$$\boxed{\frac{dE^{GW}}{d\omega}(\omega = 0) = \frac{4G}{\pi}\left(s \log s + t \log(-t) + u \log(-u)\right)}$$

At small deflection angle ($|t| \ll s$):

$$\frac{dE^{GW}}{d\omega} \to \frac{Gs}{\pi} \theta_E^2 \log(4e\theta_E^{-2}) \quad ; \quad \omega \to 0$$

NL ($O(\omega)$) correction to the spectrum

Comes from interference between S_0 and S_1 soft operators. Basic integral is

$$B_1 = 8\pi G \int \frac{d^3q}{2|q|(2\pi)^3} \sum_{i,j} \sum_{s=\pm 2} \left[\frac{(p_i h^s p_i)(p_j h^{(-s)} J_j q)}{q p_i q p_j} + (i \leftrightarrow j) \right]$$

to which we add a $\delta(qP + 2E\omega_0)$ (w/ P the c.o.m. momentum) to fix the c.o.m. $\omega = \omega_0$ in a covariant way. Summing over polarizations and integrating over angles we get:

$$\boxed{\frac{dE_1}{d\omega} = -2\frac{G\sqrt{s}\hbar\omega}{\pi}\sum_{ij}\frac{\log\left[\frac{-s(p_i p_j)}{2(Pp_i)(Pp_j)}\right]}{\tilde{s}_{ij}}\left[(p_i p_j)P - (Pp_j)p_i - (Pp_i)p_j\right]^\mu\left(\frac{\overleftarrow{\partial}}{\partial p_i} + \frac{\overrightarrow{\partial}}{\partial p_j}\right)_\mu}$$

$$\tilde{s}_{ij} = s + \frac{2(Pp_i)(Pp_j)}{p_i p_j}$$

To be sandwiched (divided) between (by) $S_{if}^{+}S_{fi}$.

Surprisingly, when applied to a 2->2 elastic process, it gives a vanishing result.

This agrees with what was obtained in the eikonal (and CGR) approach. It also agrees with Sen et al. for the log-enhanced term (recall that we summed over pol.^{s}!).

The sub-sub leading ($O(\omega^2)$) correction

The calculation ($|S_1|^2$ & $Re[S_0 S_{2^*}]$) is more involved, but final result takes a (relatively) simple, elegant form

$$\frac{dE_2^{GW}}{d(\hbar\omega)}|\mathcal{S}_{if}|^2 = \mathcal{S}_{if}^{\dagger}\frac{G\hbar\omega^2}{\pi}\left(C_1 + C_2 + C_3\right)\mathcal{S}_{fi}$$

$$C_1 = -3\sum_i \overleftarrow{D}_i \sum_j \overrightarrow{D}_j + 4\sum_i (\overleftarrow{D}_i + \overrightarrow{D}_i)^2$$

$$C_2 = \sum_{i\neq j}\frac{P^2}{\tilde{s}_{ij}}\log\frac{P^2 p_i p_j}{2Pp_i Pp_j}[p_i p_j(\overleftrightarrow{\partial}_{ij})^2 - 2p_i(\overleftrightarrow{\partial}_{ij})p_j(\overleftrightarrow{\partial}_{ij})]$$

$$C_3 = \sum_{i\neq j}\frac{2}{p_i p_j \tilde{s}_{ij}}\left[1 + \frac{P^2}{\tilde{s}_{ij}}\log\frac{P^2 p_i p_j}{2Pp_i Pp_j}\right](p_i p_j)^2\left(Q_{ij}^{\mu}(\overleftrightarrow{\partial}_{ij})_{\mu}\right)^2$$

$$D_i = p_i \partial_i \quad \text{(no sum)}$$

$$\overleftrightarrow{\partial}_{ij\,\nu} \equiv \overleftarrow{\partial}_{i\nu} + \overrightarrow{\partial}_{j\nu} \qquad Q_{ij}^{\mu} \equiv \left(P^{\mu} - \frac{Pp_j}{p_i p_j}p_i^{\mu} - \frac{Pp_i}{p_i p_j}p_j^{\mu}\right)$$

Specializing to a 2->2 process

$$\frac{dE_2^{GW}}{d\omega}|\mathcal{S}_{if}|^2 = 2\frac{G\hbar^2\omega^2}{\pi} \times \mathcal{S}_{if}^{\dagger}\left\{\overleftarrow{D}^2 + \overrightarrow{D}^2 + \left[st + us\log\left(-\frac{u}{s}\right)\right]\overleftrightarrow{\Delta}_{st}^2 + \left[su + ts\log\left(-\frac{t}{s}\right)\right]\overleftrightarrow{\Delta}_{su}^2\right\}\mathcal{S}_{fi}$$

$$D \equiv s\partial_s + t\partial_t + u\partial_u \ : \ \overleftrightarrow{\Delta}_{st} \equiv \left(\overleftarrow{\partial}_s - \overleftarrow{\partial}_t - \overrightarrow{\partial}_s + \overrightarrow{\partial}_t\right) \ldots$$

The above combinations of derivatives are unambiguous. They act on either A(s,t) or on A'(s,u) or on A''(t,u) yielding the same result for the same physical amplitude. Checked at tree level in N=8 SUGRA.

Applying this after eikonal resummation

Because of phase O(action/h) derivatives act, to leading order, on the exponent (Cf. WKB). The powers of h cancel and we get a classical contribution. Unfortunately, the infinite Coulomb phase does NOT drop out.

The reason is quite clear: the derivative operators in J_i acting on the IR-div Coulomb phase give IR div. results (Cf. time delay as opposed to deflection angle). However, also the final soft graviton contributes an IR div. Coulomb phase which is exactly as needed for the cancellation (Cf. CCCV15).

The standard soft-graviton recipe misses this piece and should be amended.

If we follow Sen et al's recipe for dealing with the Coulomb IR logs we can match the result with the one obtained in CCV-18 (for the unpolarized, angle-integrated flux).

We get, like in CCV18, a positive correction of order $(\omega b)^2 \log^2(\omega b)$ confirming the already mentioned bump in the spectrum around $\omega b = 0.5$.

Summarizing

- GW's from ultra-relativistic collisions is an interesting (though probably academic) theoretical problem.
- It is challenging both analytically and numerically, both classically and quantum mechanically.
- The ZFL (for $dE^{GW}/d\omega$) is classical & well understood

To go beyond two approaches have been followed:

- The first follows the eikonal ACV approach, is limited (so far) to small deflection angles, but extends to frequencies beyond $1/R \gg 1/b$
- It is free from IR problems which, interestingly, lead to finite logarithmic enhancements at $\omega < 1/b$ which are responsible for a peak in the flux around $\omega b = 1$.

- There is a break/knee in the spectrum at "Hawking's" frequency $\omega = 1/R$
- The second approach goes via soft-graviton theorems. It is not limited to small-angle scattering but is restricted to the $\omega b < 1$ regime.
- The sub and sub-sub leading corrections to the ZFL start to be understood.
- Because of IR sensitivity in 4D, they produce interesting new effects in $dE/d\omega$ in the region $\omega b < 1$.

- A recipe due to Sen and collaborators looks to be confirmed by the eikonal-based results.
- At sub-sub leading level that same recipe confirms the CCV-18 prediction of a bump in the flux @ $\omega b \sim 1$
- Eventually one would like to extend these results about gravitation radiation to arbitrary masses and kinematics and to combine them with the results that start to come in on the conservative gravitational potential at 3PM level for a full understanding of gravitational scattering at the 2-loop level.

Thank you…

and a reminder

Workshop on

Gravitational scattering, inspiral, and radiation

(GGI, May 18-July 5, 2020)

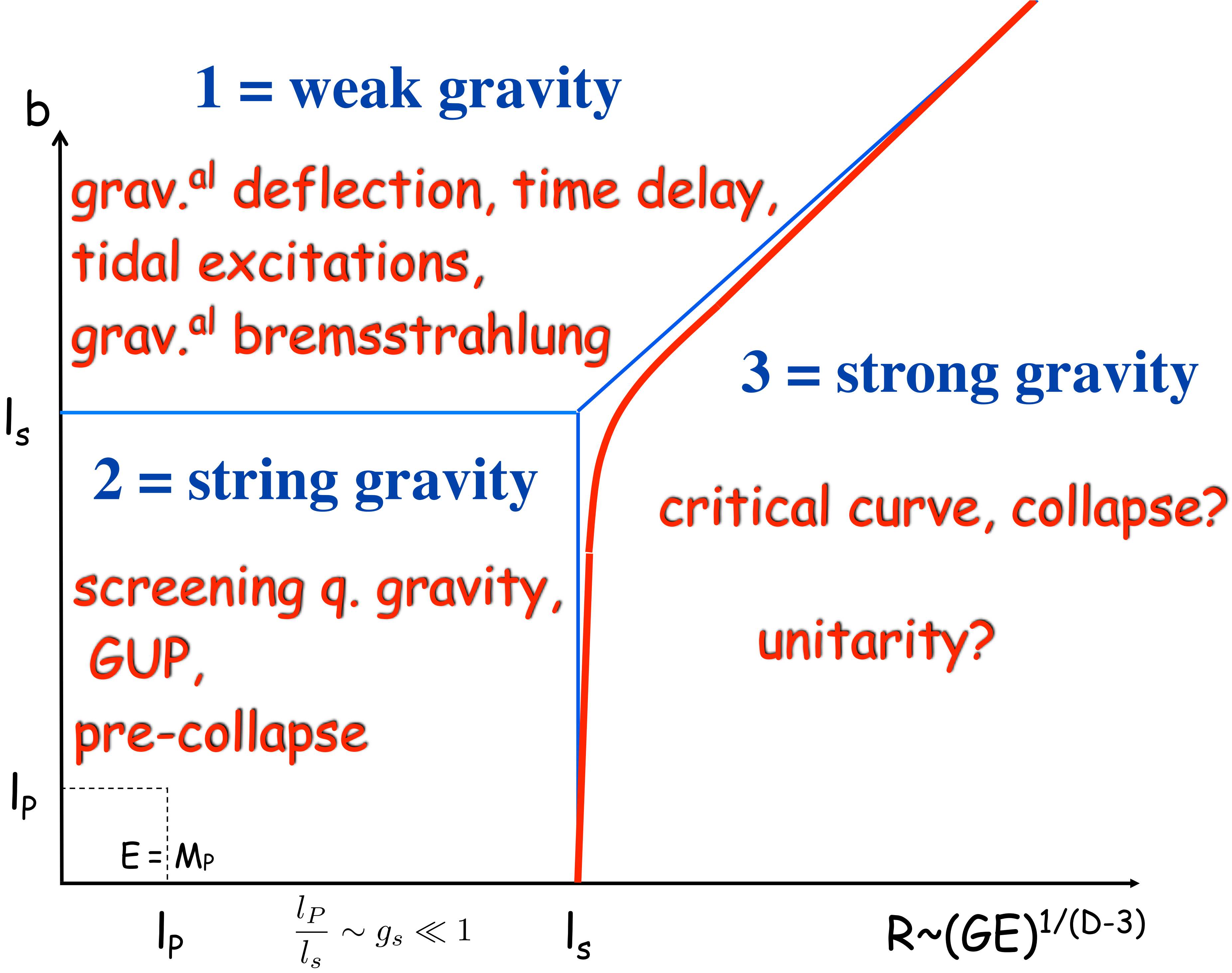
I. Results & challenges on the transplanckian gravitational scattering problem: a short summary

For a longer summary see my slides at the focus week of this year's GGI workshop: "string theory from a world-sheet perspective" or at my 2015 Les Houches lecture notes

Parameter-space for string-string collisions @ $s \gg M_P^2$

$$b \sim \frac{2J}{\sqrt{s}} \quad ; \quad R_D \sim \left(G\sqrt{s}\right)^{\frac{1}{D-3}} \quad ; \quad l_s \sim \sqrt{\alpha'\hbar} \quad ; \quad G\hbar = l_P^{D-2} \sim g_s^2 l_s^{D-2}$$

- 3 relevant length scales (neglecting l_P @ $g_s \ll 1$)
- Playing w/s and g_s we can make R_D/l_s arbitrary
- Several regimes emerge. Roughly just three:


1 = weak gravity
b
grav.al deflection, time delay,
tidal excitations,
grav.al bremsstrahlung
3 = strong gravity
l_s
2 = string gravity
critical curve, collapse?
screening q. gravity,
GUP,
unitarity?
pre-collapse
l_P
E = M_P
l_P
$\frac{l_P}{l_s} \sim g_s \ll 1$
l_s
$R \sim (GE)^{1/(D-3)}$

Results in the weak-gravity regime

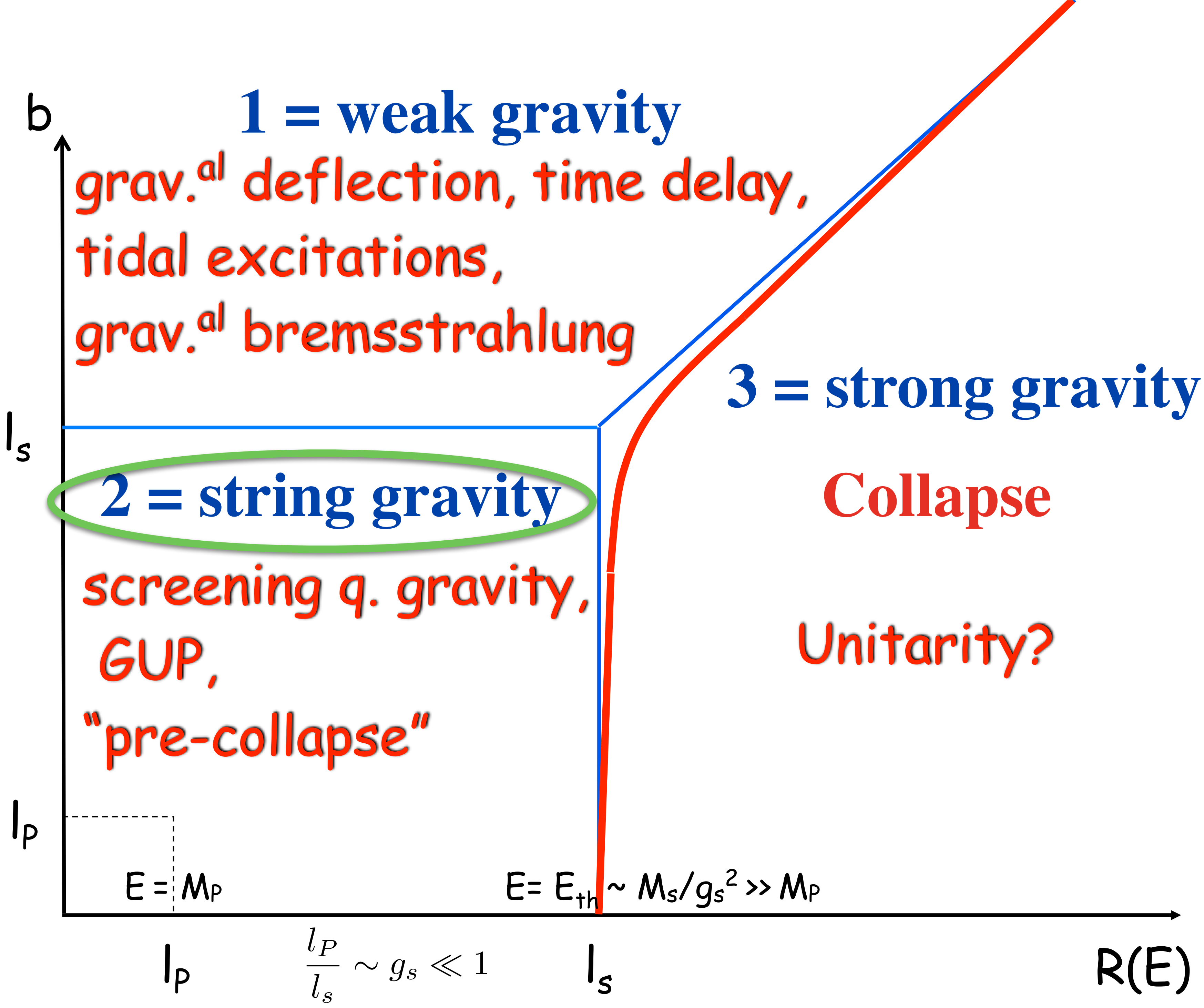
- Restoring elastic unitarity via eikonal resummation of s-channel ladders (incl. xed ones)

- Gravitational deflection & time delay:an emerging Aichelburg-Sexl (AS) metric

- t-channel "fractionation" and hard scattering (large Q) from large-distance (b >> h/Q) physics

- Tidal excitation of colliding strings when $Gs(l_s/b)^2>1$, inelastic unitarity OK, comparison with string in AS metric.

- Gravitational bremsstrahlung (see Part II)


b
1 = weak gravity
grav.al deflection, time delay,
tidal excitations,
grav.al bremsstrahlung
3 = strong gravity
l_s
2 = string gravity
Collapse
screening q. gravity,
GUP,
"pre-collapse"
Unitarity?
l_P
$E = M_P$
$E = E_{th} \sim M_s/g_s^2 >> M_P$
l_P
$\frac{l_P}{l_s} \sim g_s \ll 1$
l_s
R(E)

Results in the string-gravity regime

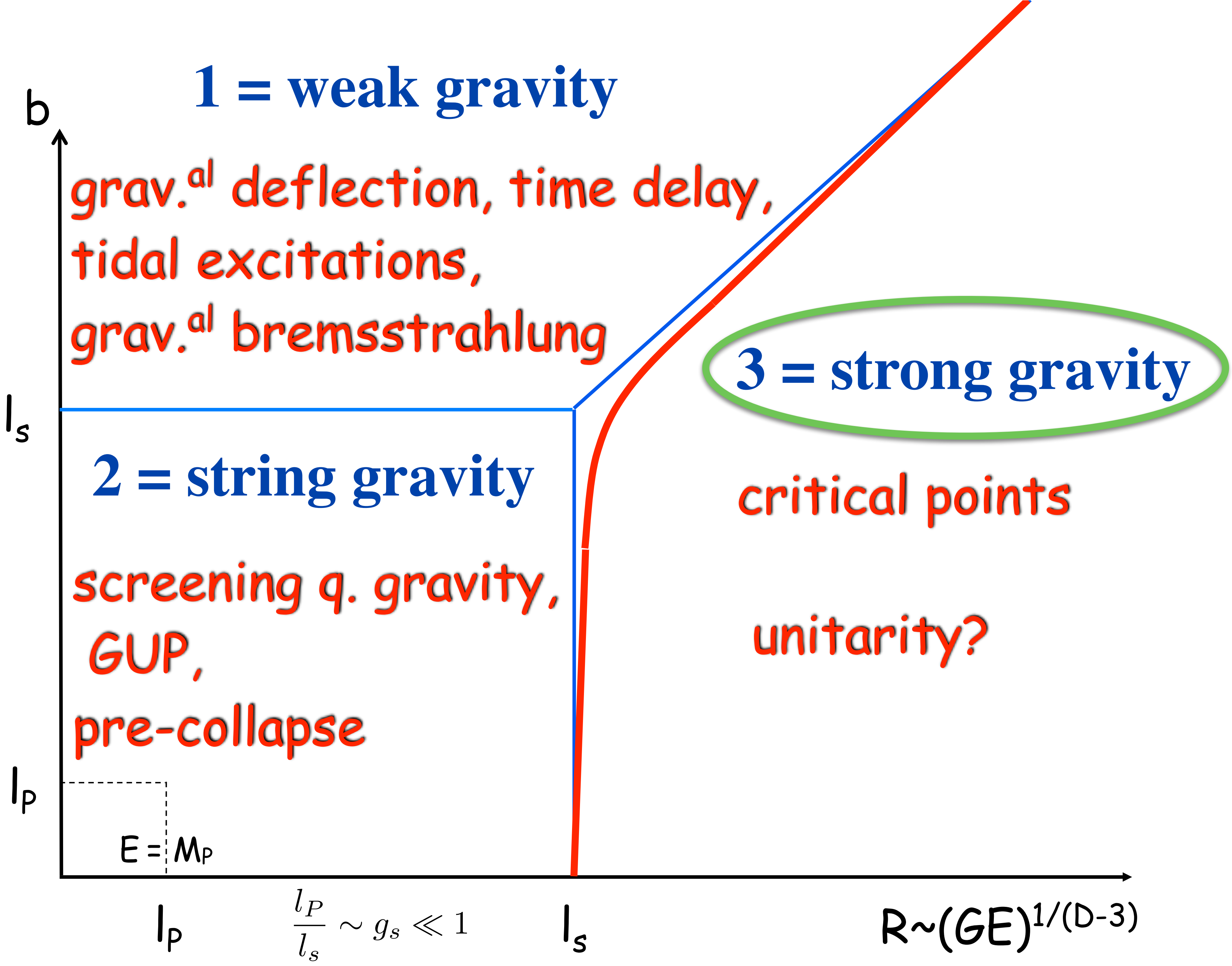
- String softening of quantum gravity @ small b: solving a causality problem (Edelstein et al)

- Maximal classical deflection and comparison/ agreement w/ Gross-Mende-Ooguri

- Generalized Uncertainty Principle

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' \Delta p \geq l_s$$

- s-channel "fractionation" and precocious black-hole-like behavior ($\langle E_{final} \rangle \sim M_P^2 / \langle E_{initial} \rangle$)


1 = weak gravity
b
grav.al deflection, time delay,
tidal excitations,
grav.al bremsstrahlung
3 = strong gravity
l_s
2 = string gravity
critical points
screening q. gravity,
GUP,
unitarity?
pre-collapse
l_P
E = M_P
l_P
$\frac{l_P}{l_s} \sim g_s \ll 1$
l_s
R~(GE)^{1/(D-3)}

Results in the strong gravity regime

(D=4, in point-particle limit. D > 4 easier?)

- Identifying (semi) classical contributions as effective trees. No classical correction to deflection at $O(R^2/b^2)$; correction estimated (correctly?) at $O(R^3/b^3)$. See also PdV's talk.
- An effective 2D field theory (~ Lipatov) to resum trees.
- Emergence of critical parameters in agreement w/ collapse criteria (via CTS constructions).
- Unitarity beyond cr. surf?