Generalized ***Prescriptive*** *Unitarity*

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QCD Meets Gravity 2017
University of California, Los Angeles

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VILLUM FONDEN
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Organization and Outline

- **Overview:** Generalized & *Prescriptive* Unitarity
- **Constructing Integrands *Constructively***
  - **Formalism**: tensor reduction & integrand bases
  - **Power counting**: notation (and subtleties); stratifications
  - **Illustration**: one-loop unitarity *redux*
- **Generalized Prescriptive Unitarity**
  - **The *Badness* of Box-Power Counting (in 4d)**
  - **The *Triviality* of Prescriptivity:** *the Triangularity of Cuts*
- **A New Strategy for Supergravity?**

Overview: Generalized Unitarity

[Bern, Dixon, Kosower; Dunbar; …]

- Integrands are rational functions—so may be expanded into an arbitrary (complete) basis:

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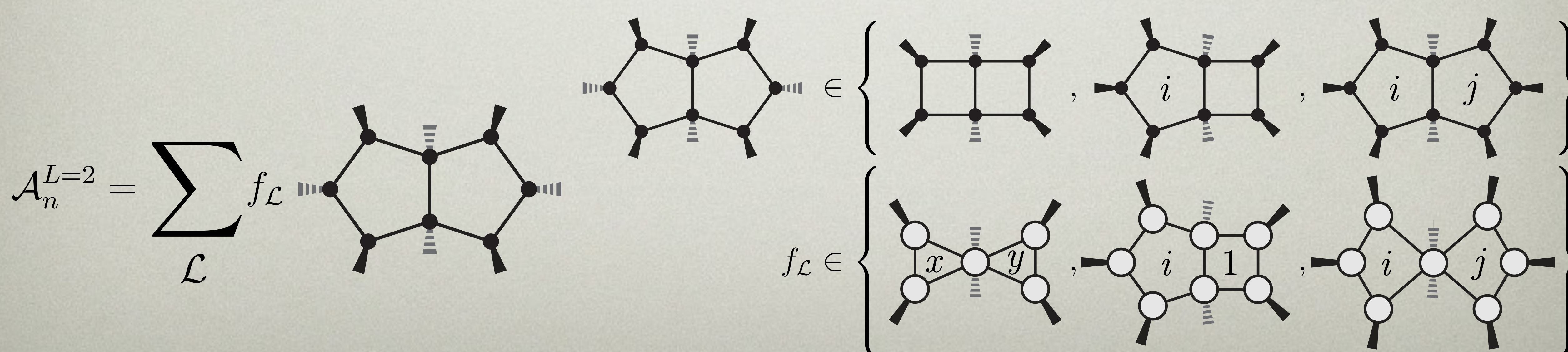

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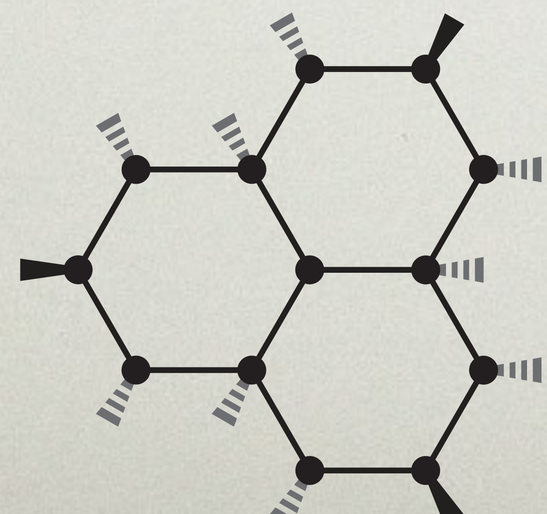
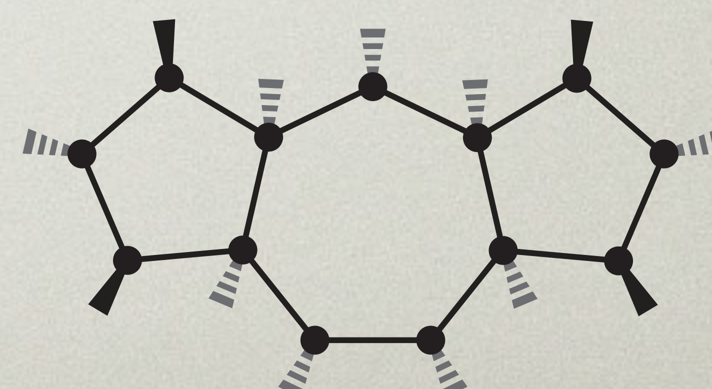


$$+ \sum_{\mathcal{L}} f_{\mathcal{L}}$$



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- Integrand bases are *stratified* by power-counting, so amplitudes can be constructed iteratively
- The maximal-(and next-to-maximal-)weight parts of any amplitude are always "cut-constructible"
- The maximal-weight part of SUGRA has better than BCJ power-counting for high enough loops…

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$$\textit{nota bene: } 1 \in [\ell] \;\Rightarrow [\ell]^a \subseteq [\ell]^{a+b} \quad \forall\, a, b \geq 0$$

Basics of Basic Basis Reduction

[Passarino, Veltman; van Neerven, Vermaseren]

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- Moreover, the only independent integrands with $(d{+}1)$ propagators can be chosen to be *parity-odd*

$$1 \in [\ell] = \text{span}\{\underbrace{(\ell, a_1), \ldots, (\ell, a_{d+1})}_{\text{“contact terms”}}, i\epsilon(\ell, a_1, \ldots, a_{d+1})\}$$

Power-Counting & Contructibility

- An integrand has 'p-gon power-counting' if:

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[Ossola, Papadopoulos, Pittau; Forde, Kosower]

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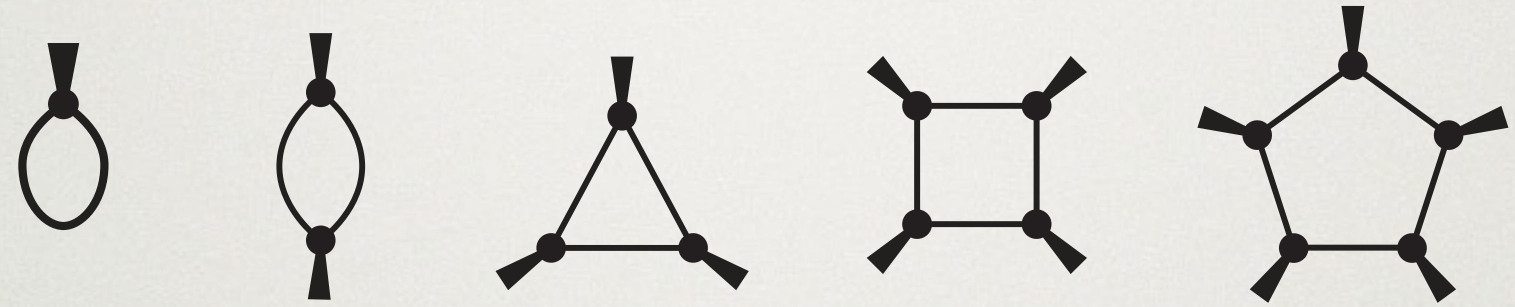
1 

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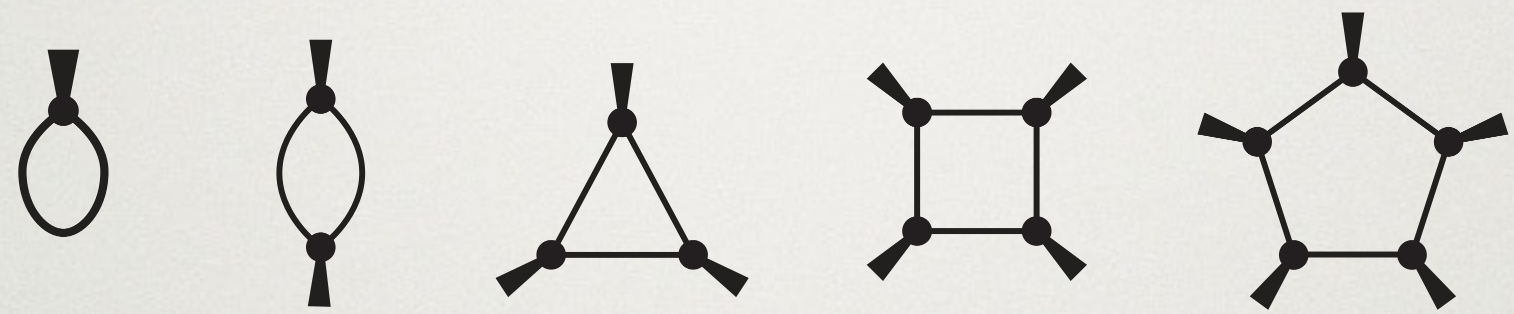
1 

$\mathcal{B}_4$					1	$[\ell]^1$
$\mathcal{B}_3$				1	$[\ell]^1$	$[\ell]^2$
$\mathcal{B}_2$			1	$[\ell]^1$	$[\ell]^2$	$[\ell]^3$
$\mathcal{B}_1$		1	$[\ell]^1$	$[\ell]^2$	$[\ell]^3$	$[\ell]^4$
$\mathcal{B}_0$	1	$[\ell]^1$	$[\ell]^2$	$[\ell]^3$	$[\ell]^4$	$[\ell]^5$

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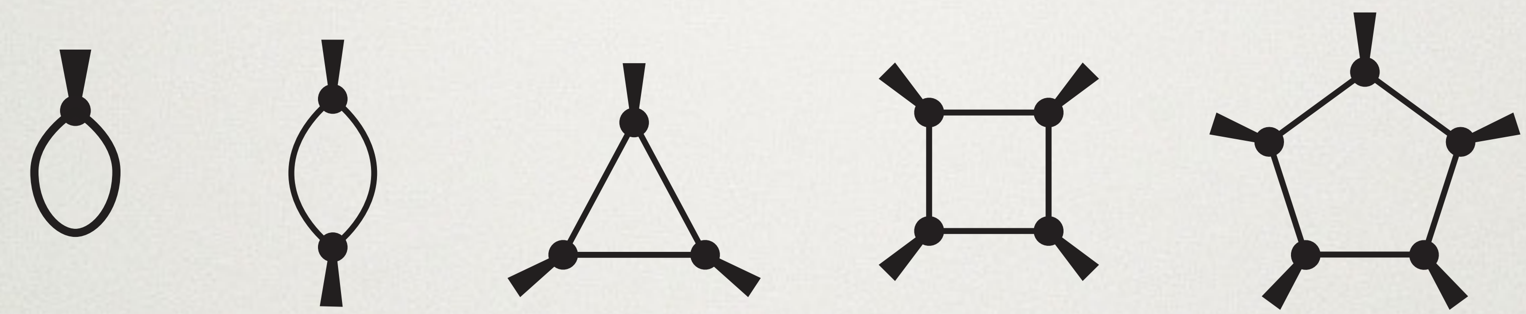
1 

$\mathcal{B}_4$					1	6
$\mathcal{B}_3$				1	6	20
$\mathcal{B}_2$			1	6	20	50
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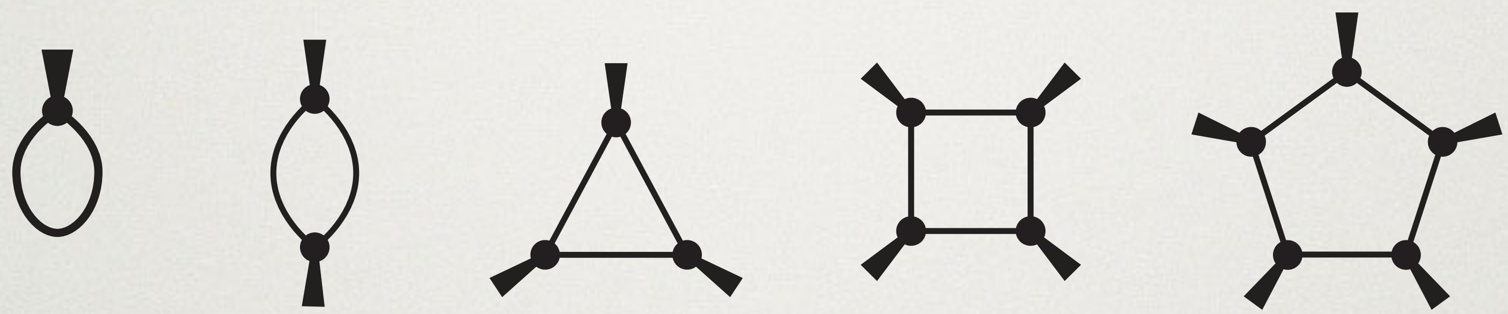

	1					
$\mathcal{B}_4$					**1**+0	**1**+5
$\mathcal{B}_3$				**1**+0	**2**+4	**0**+20
$\mathcal{B}_2$			**1**+0	**3**+3	**2**+18	**0**+50
$\mathcal{B}_1$		**1**+0	**4**+2	**5**+15	**2**+48	**0**+105
$\mathcal{B}_0$	**1**+0	**5**+1	**9**+11	**7**+43	**2**+103	**0**+196

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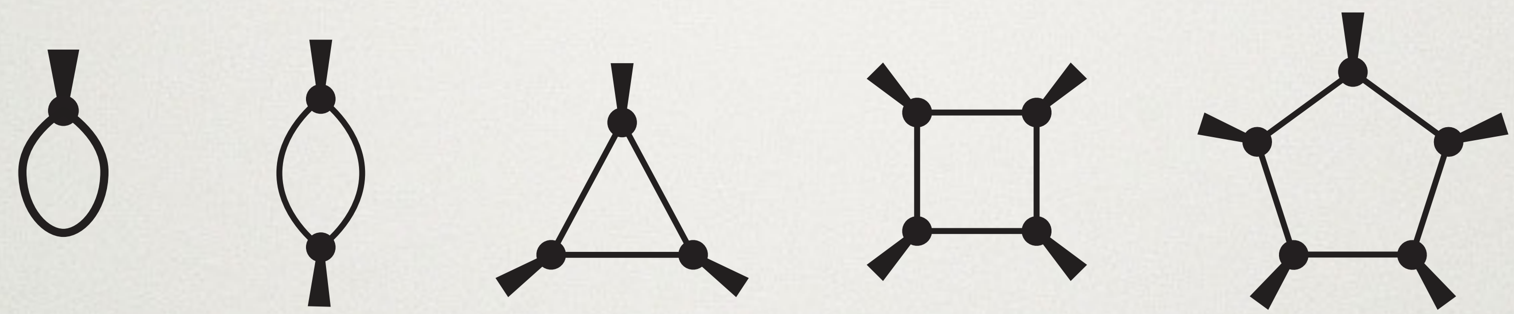

	1					
$\widehat{\mathcal{B}}_4$					**1**+0	**0**+5
$\widehat{\mathcal{B}}_3$				**1**+0	**1**+4	**0**+14
$\widehat{\mathcal{B}}_2$			**1**+0	**2**+3	**0**+14	**0**+30
$\widehat{\mathcal{B}}_1$		**1**+0	**3**+2	**2**+12	**0**+30	**0**+55
$\widehat{\mathcal{B}}_0$	**1**+0	**4**+1	**5**+9	**2**+28	**0**+55	**0**+91

One Loop Unitarity Redux (4d)

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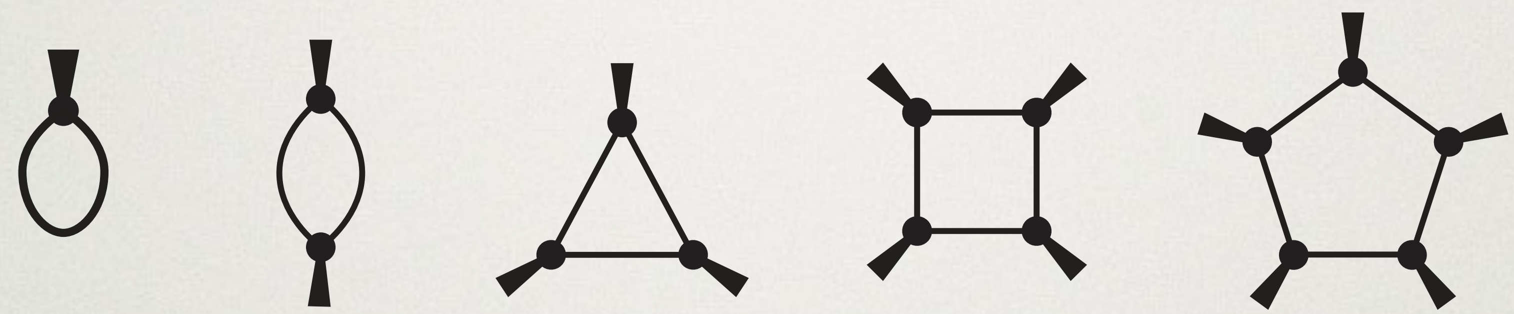
1 

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$\widehat{\mathcal{B}}_4$					1+0	0+5	weight = 2
$\widehat{\mathcal{B}}_3$				1+0	1+4	0+14	
$\widehat{\mathcal{B}}_2$			1+0	2+3	0+14	0+30	weight = 1
$\widehat{\mathcal{B}}_1$		1+0	3+2	2+12	0+30	0+55	weight = 0
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- Consider a theory that is bubble constructible (such as $\mathcal{N} \geq 1$ SYM)

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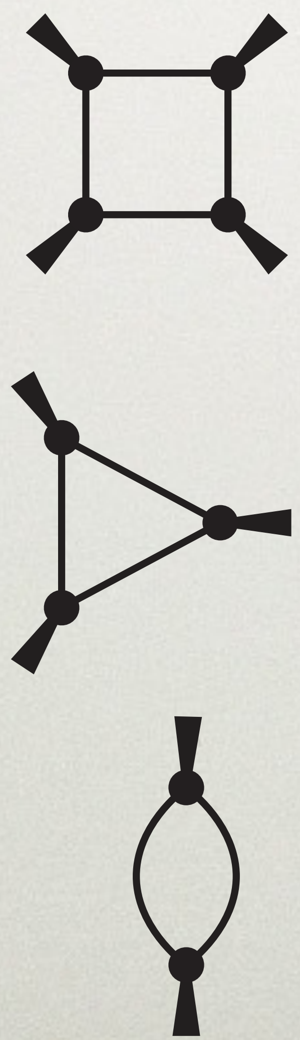

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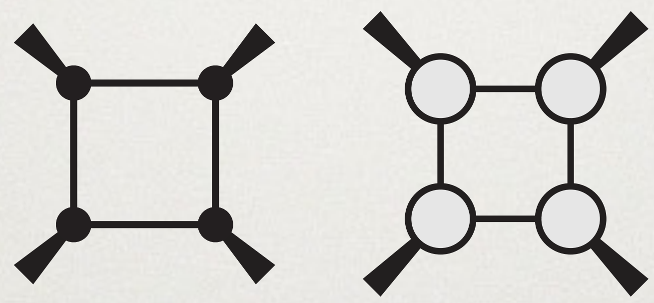

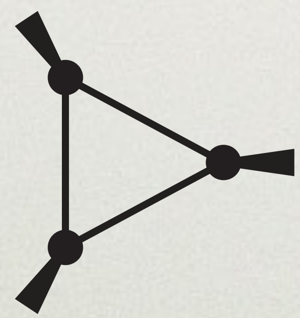

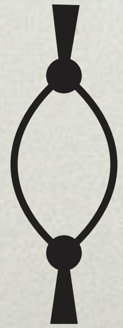

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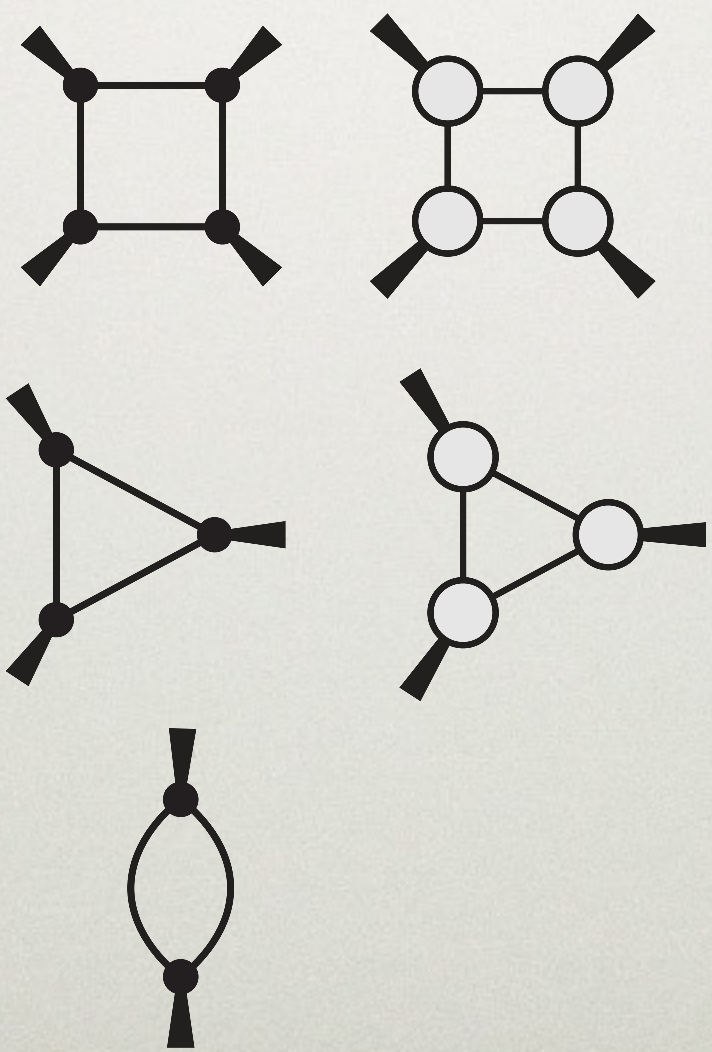

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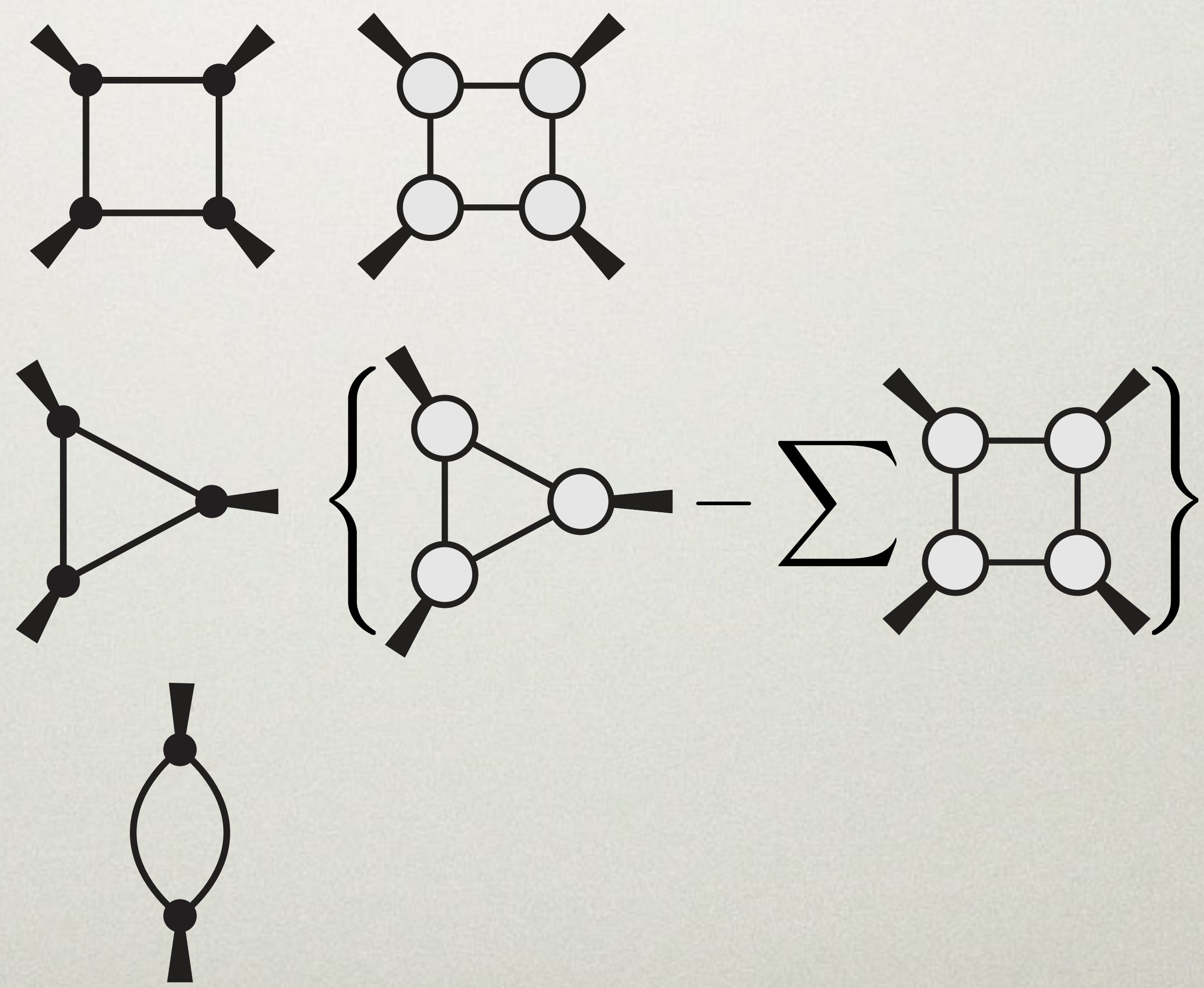

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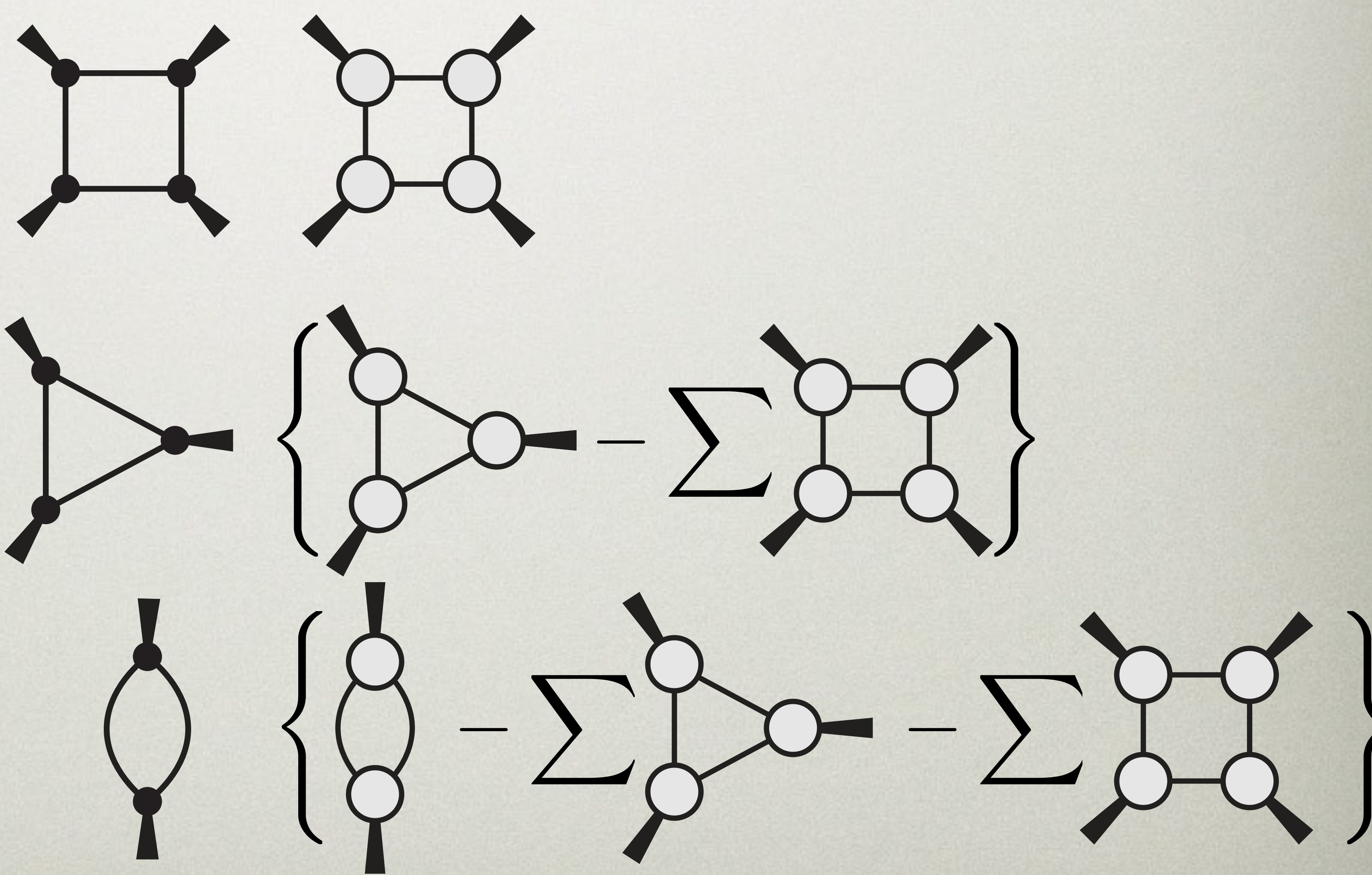

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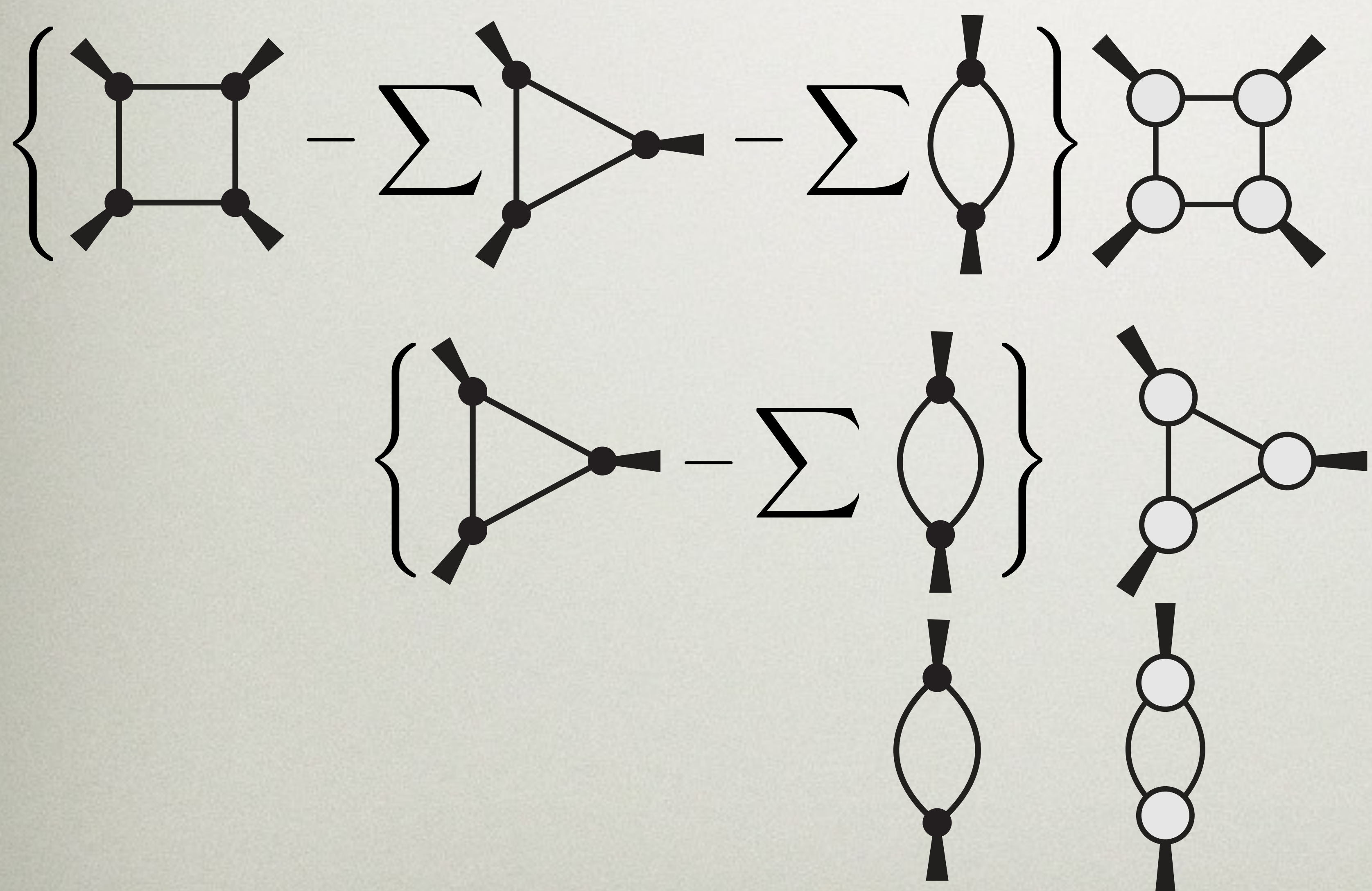

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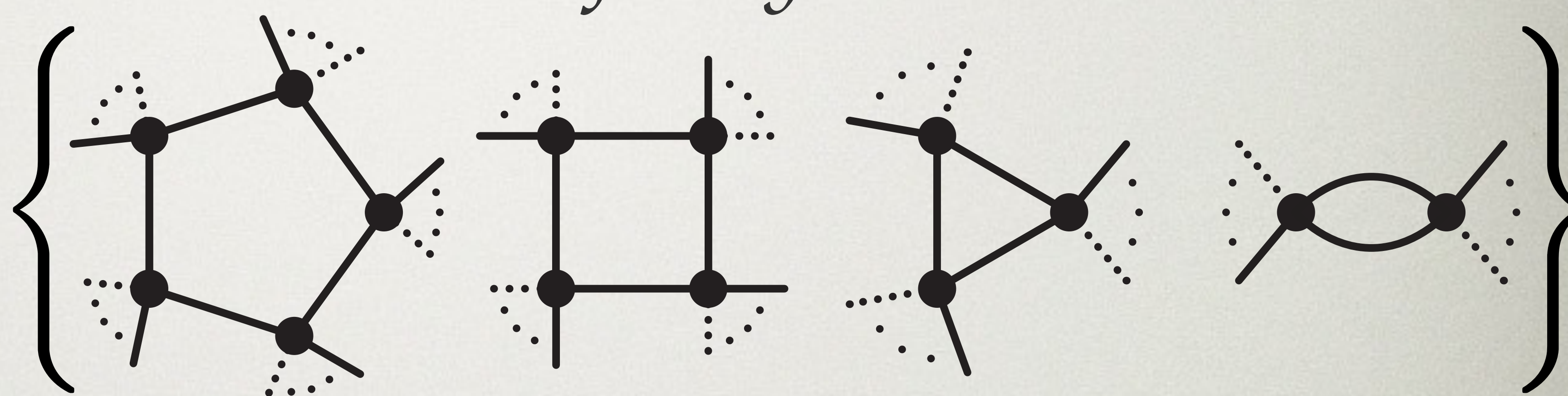

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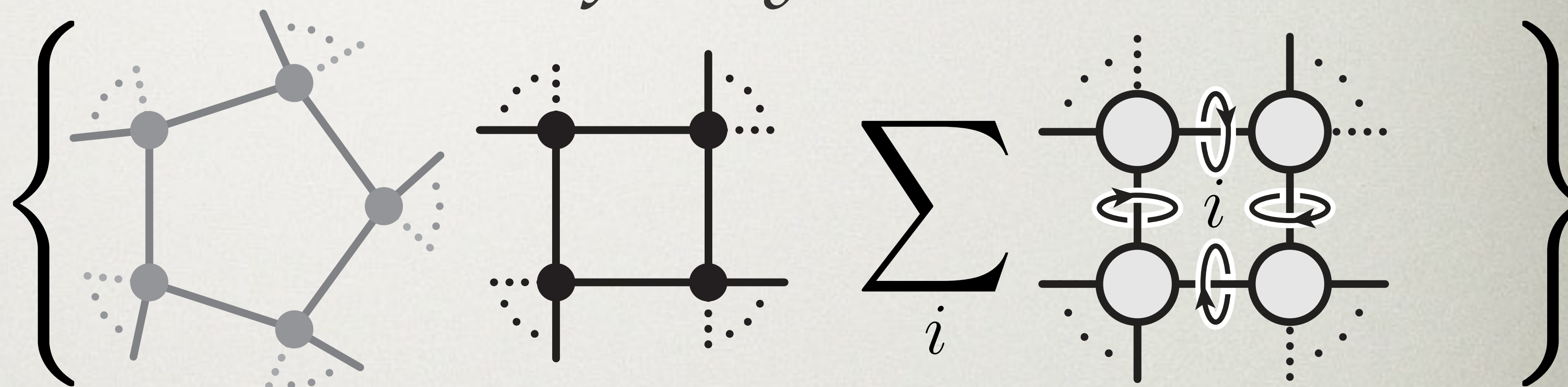

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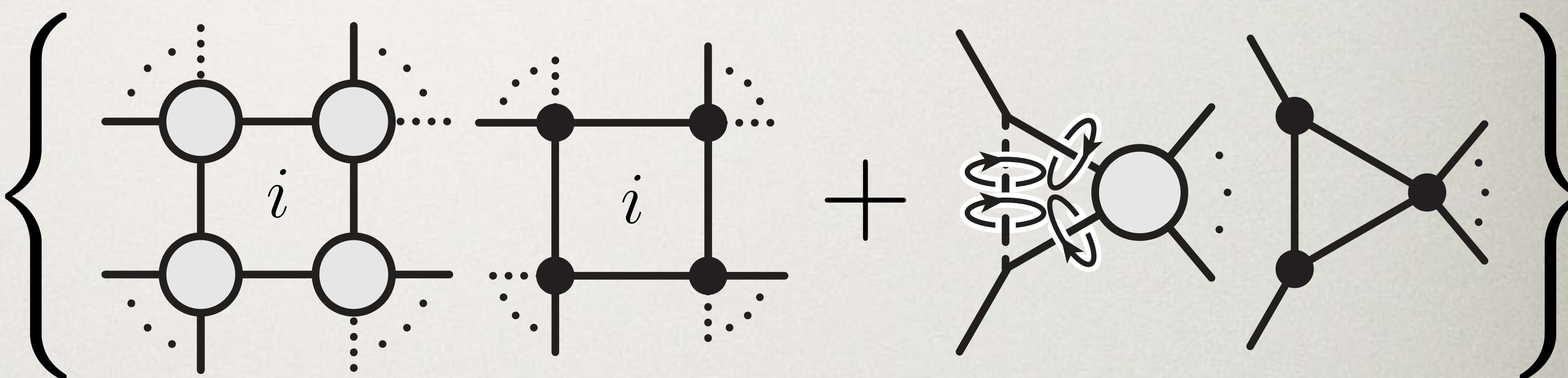

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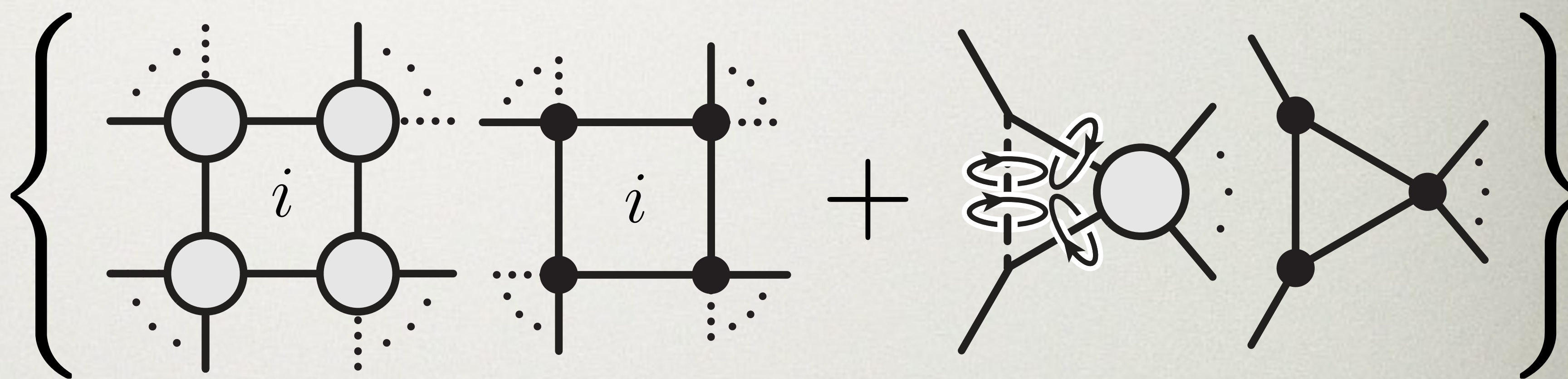

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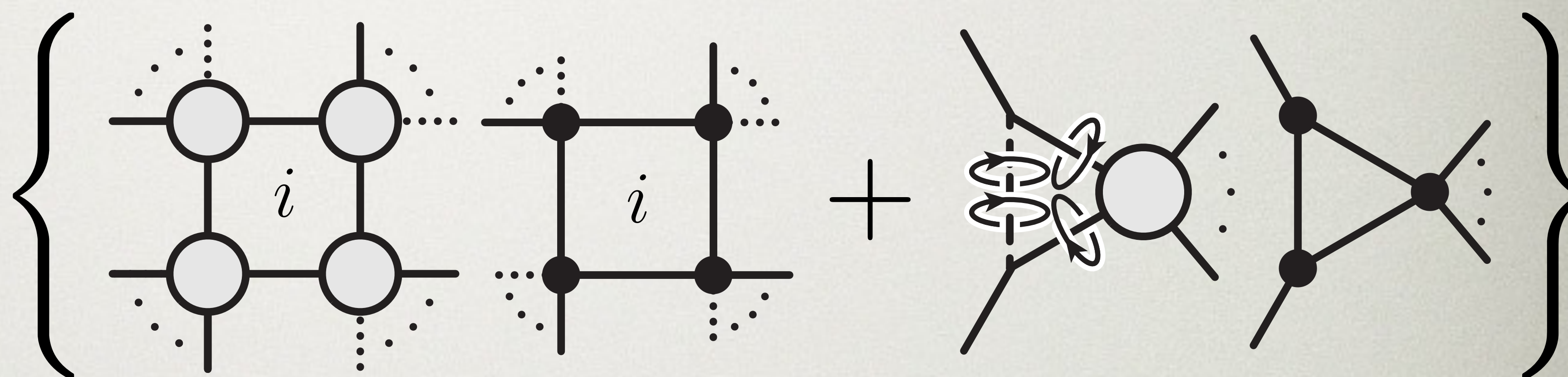

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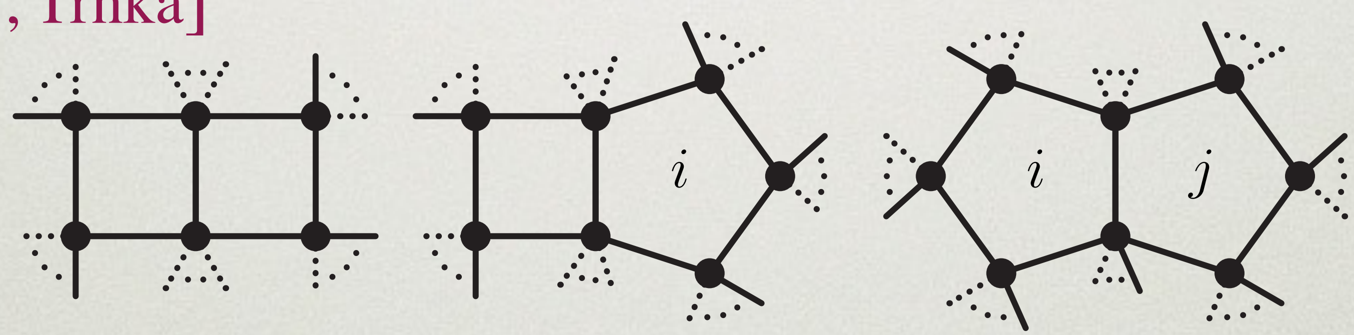

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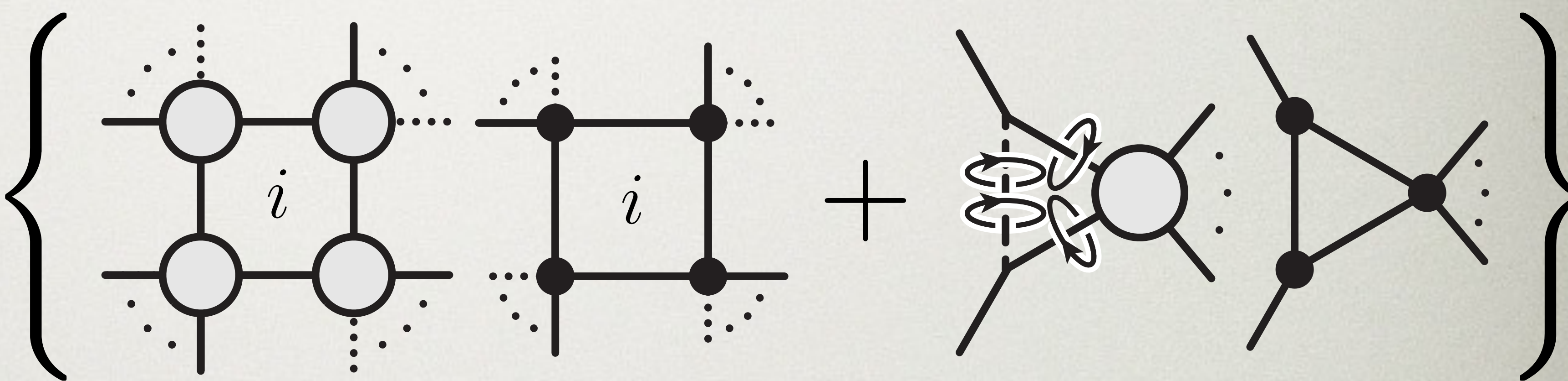

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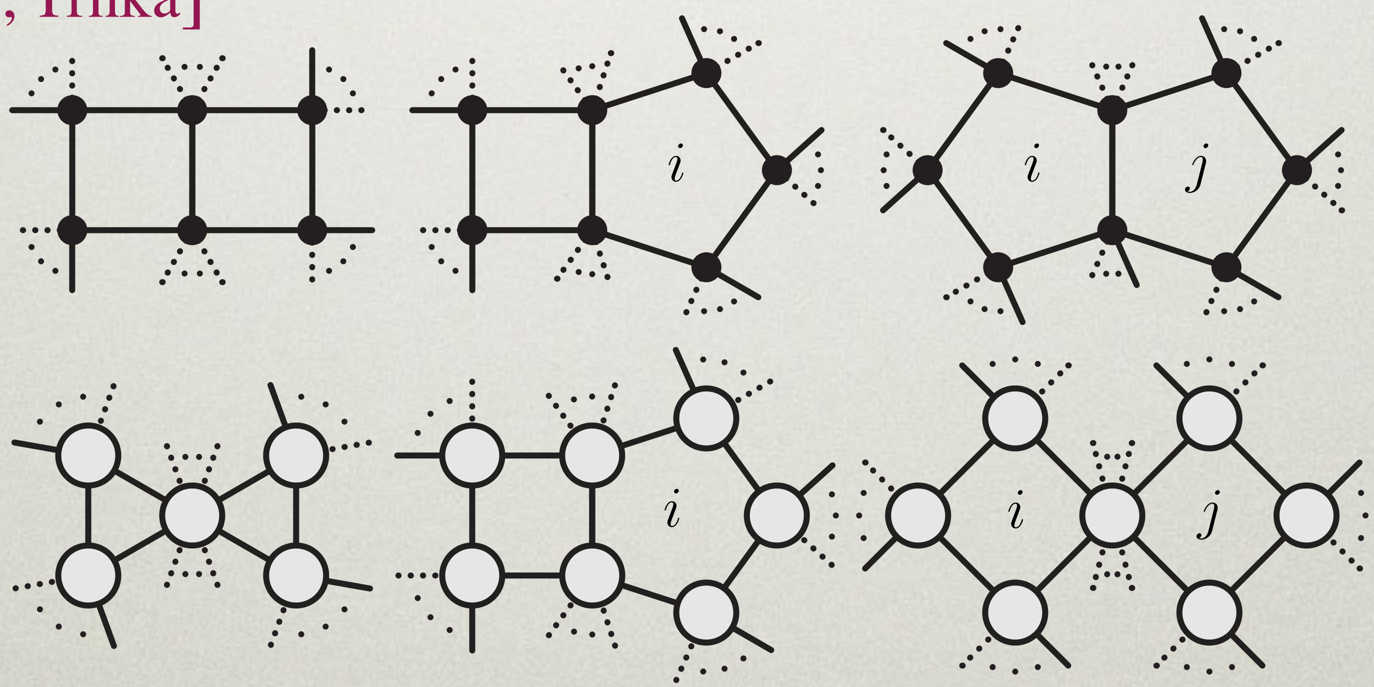

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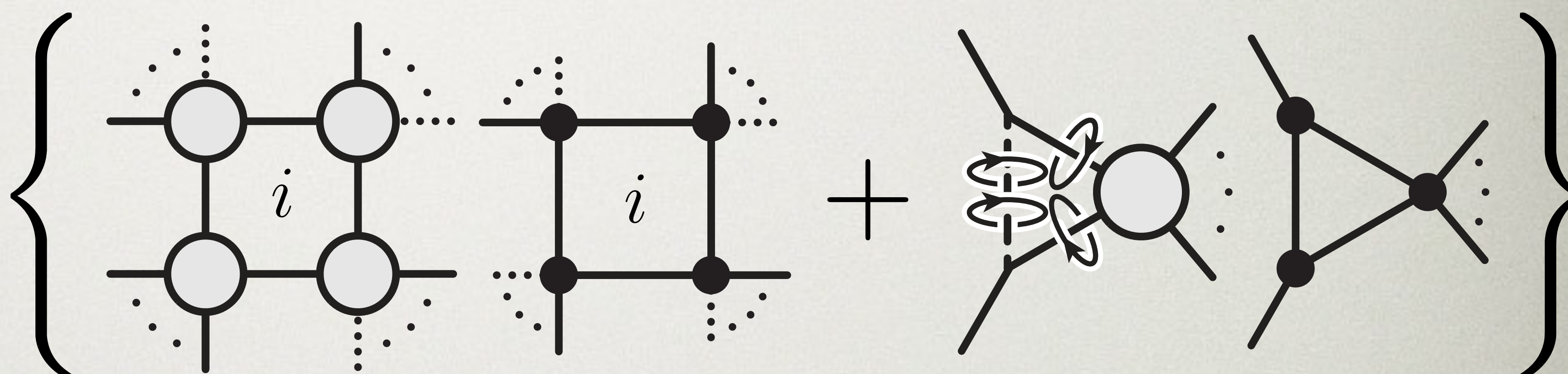

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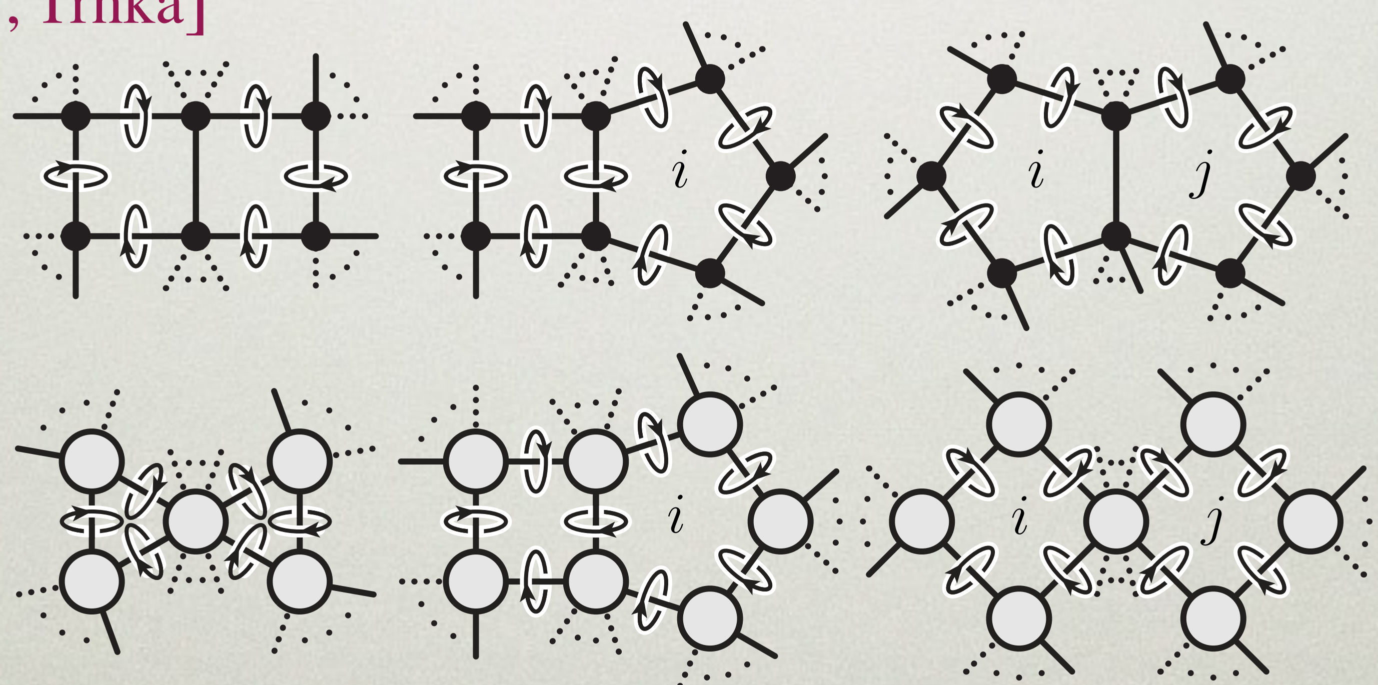

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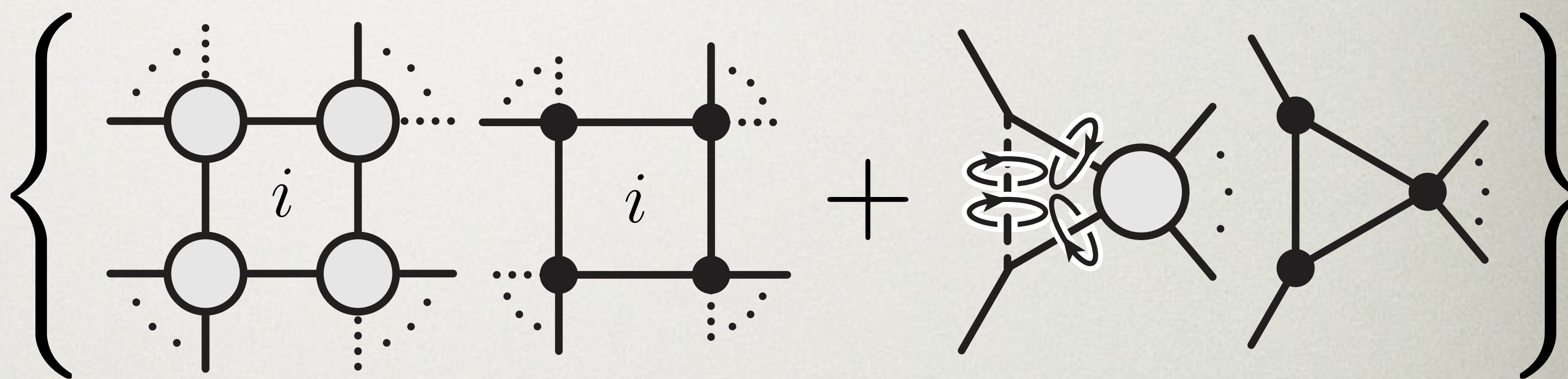

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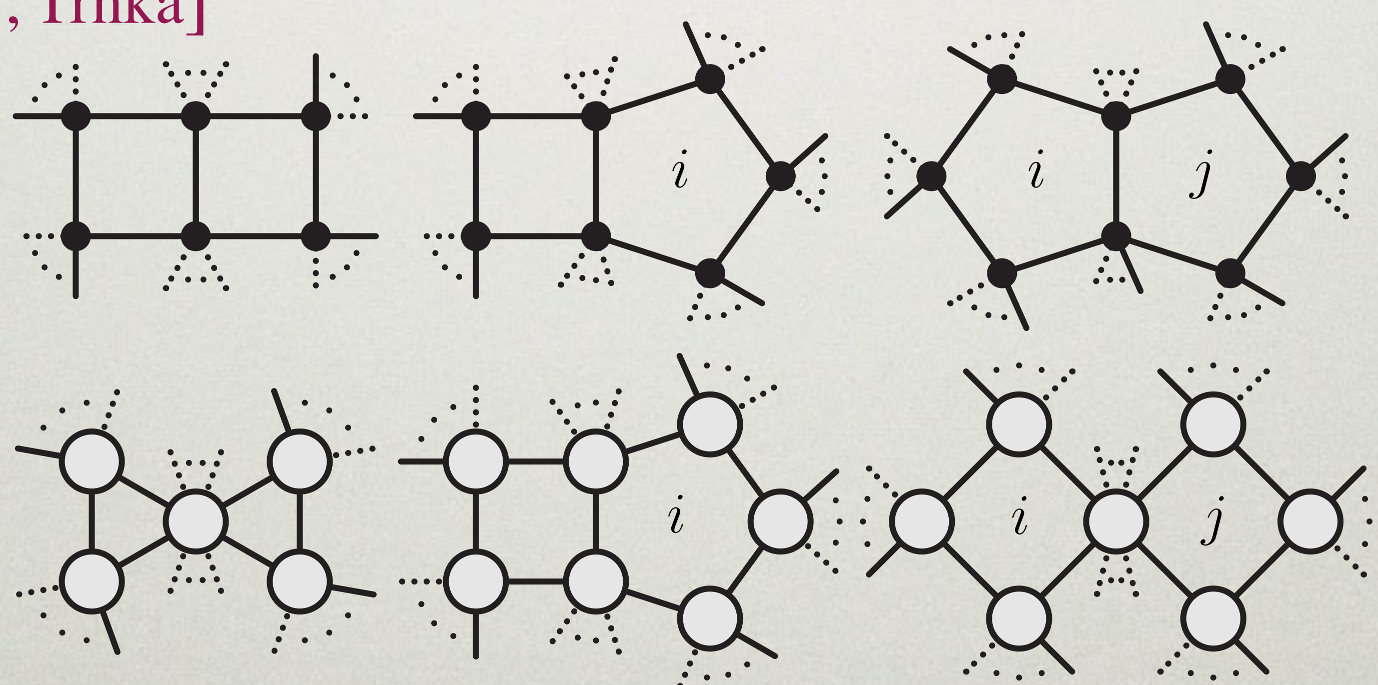

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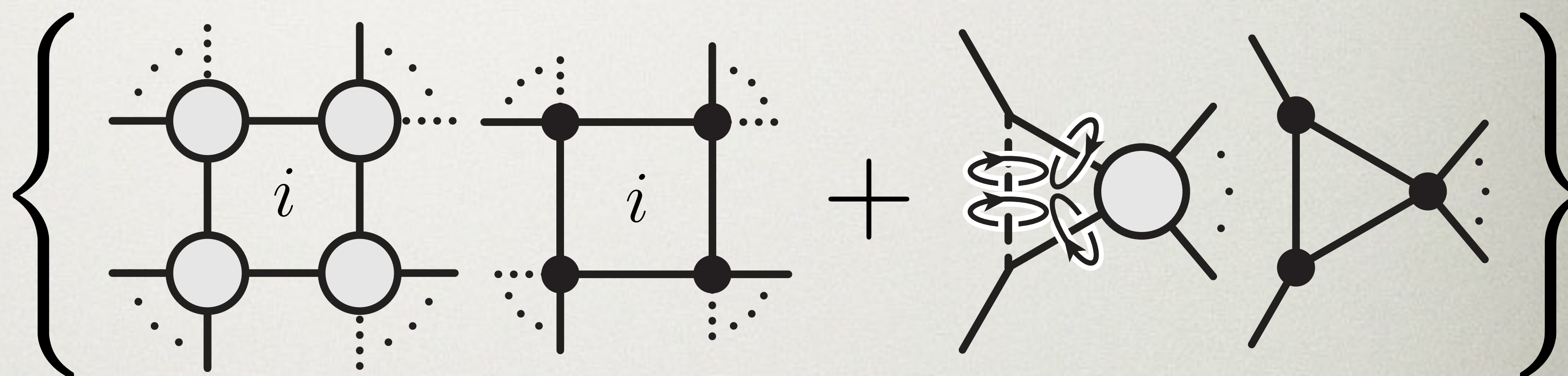

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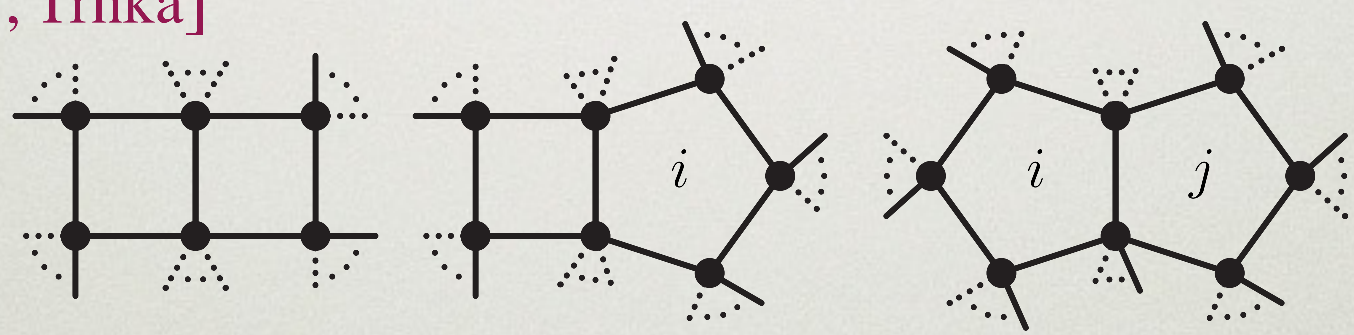

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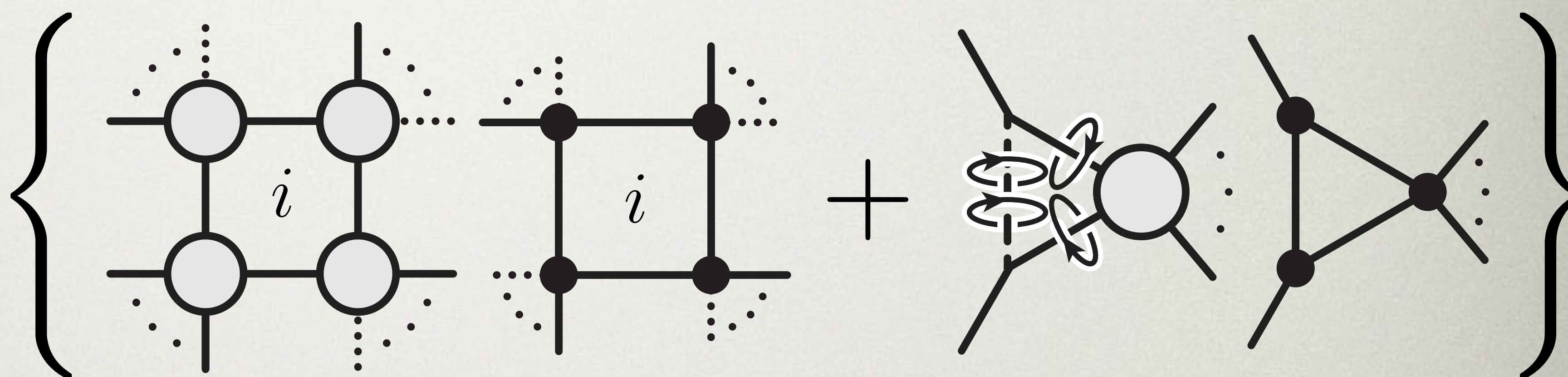

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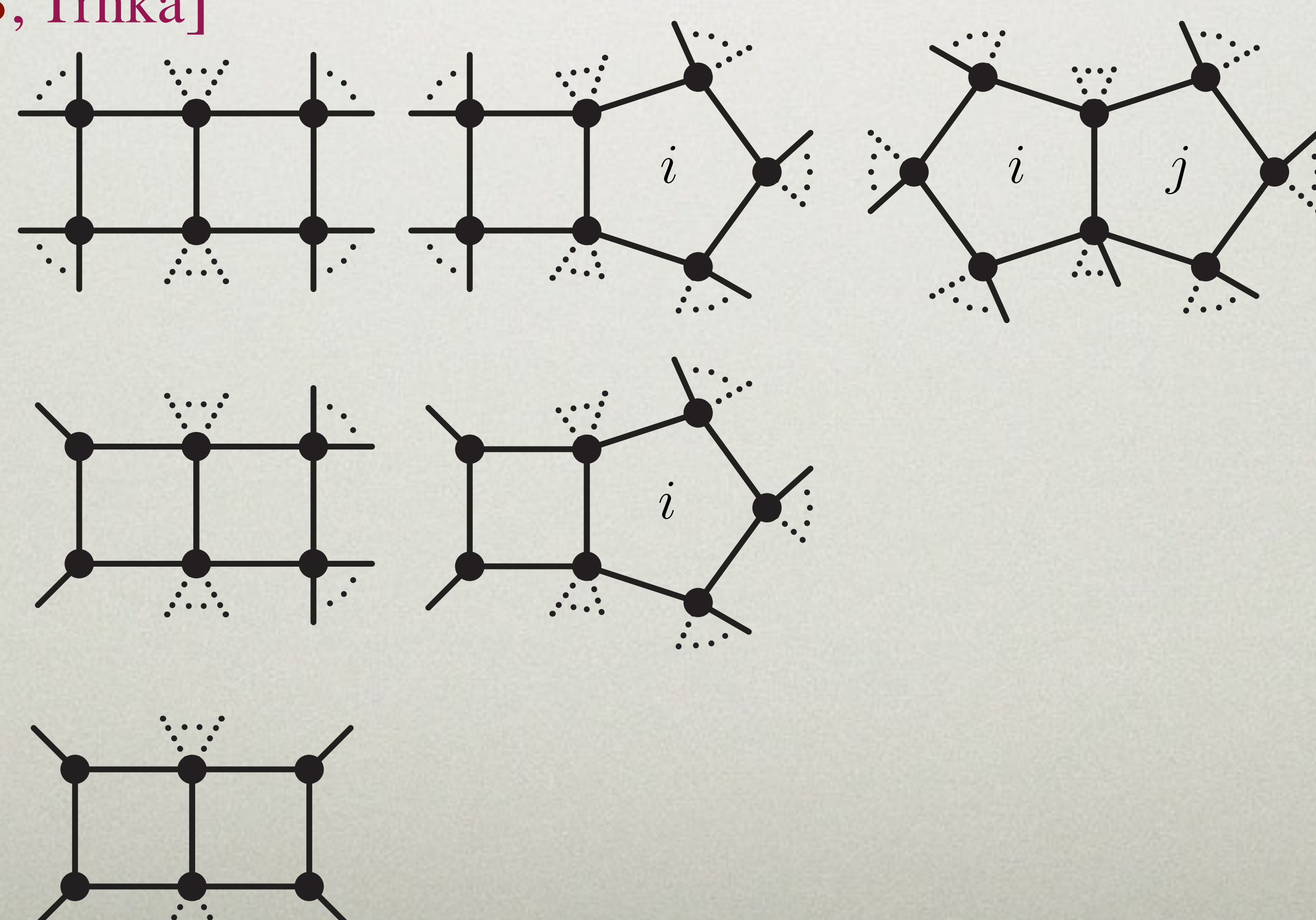

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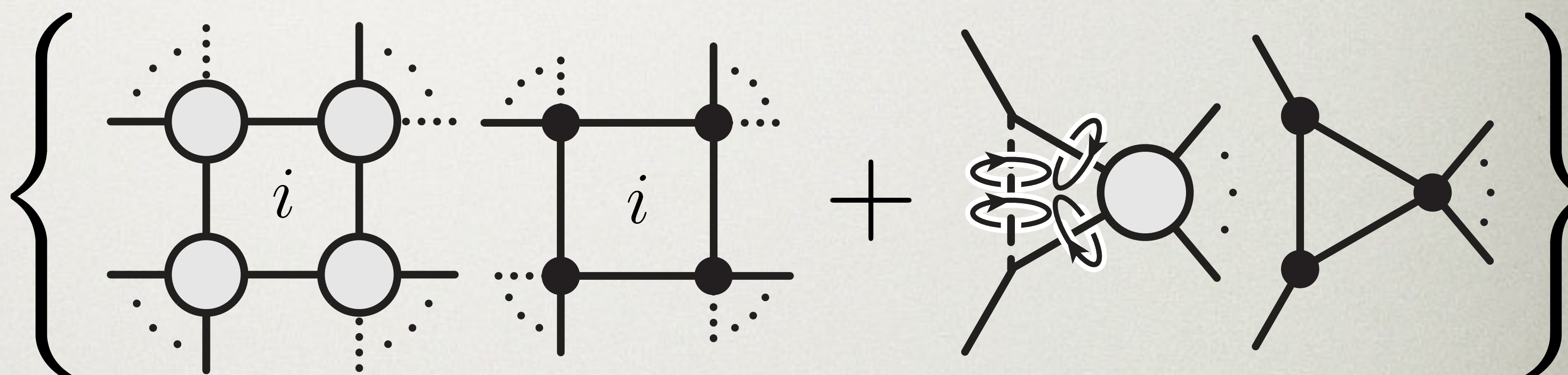

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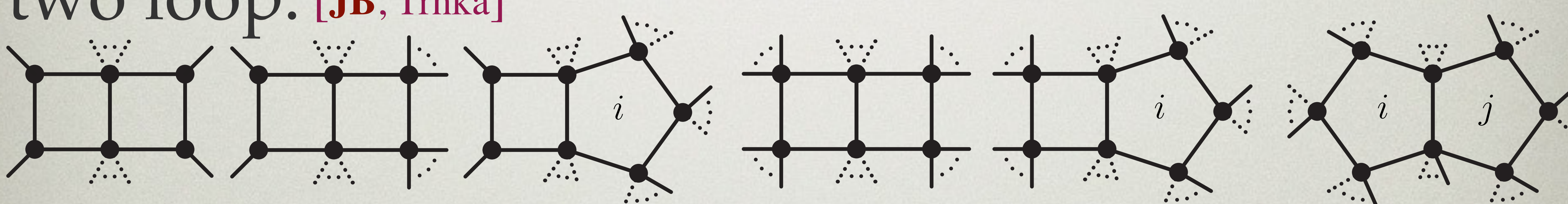

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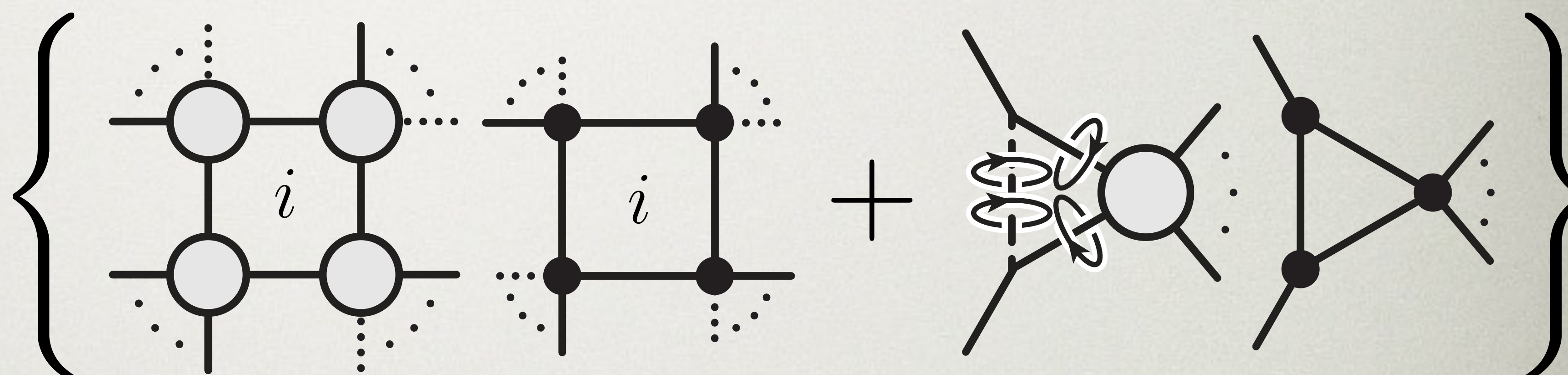

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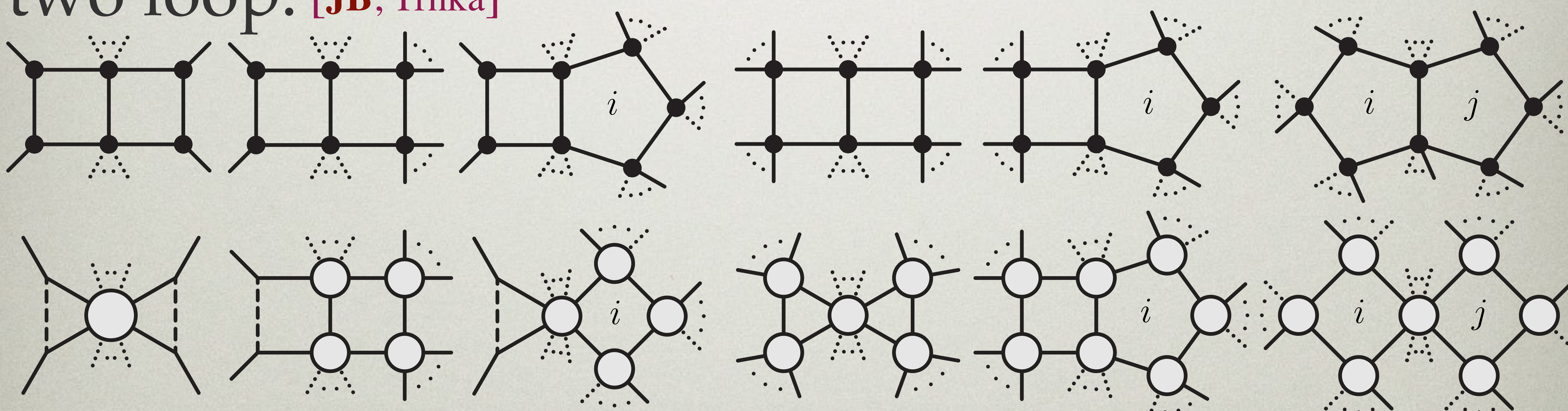

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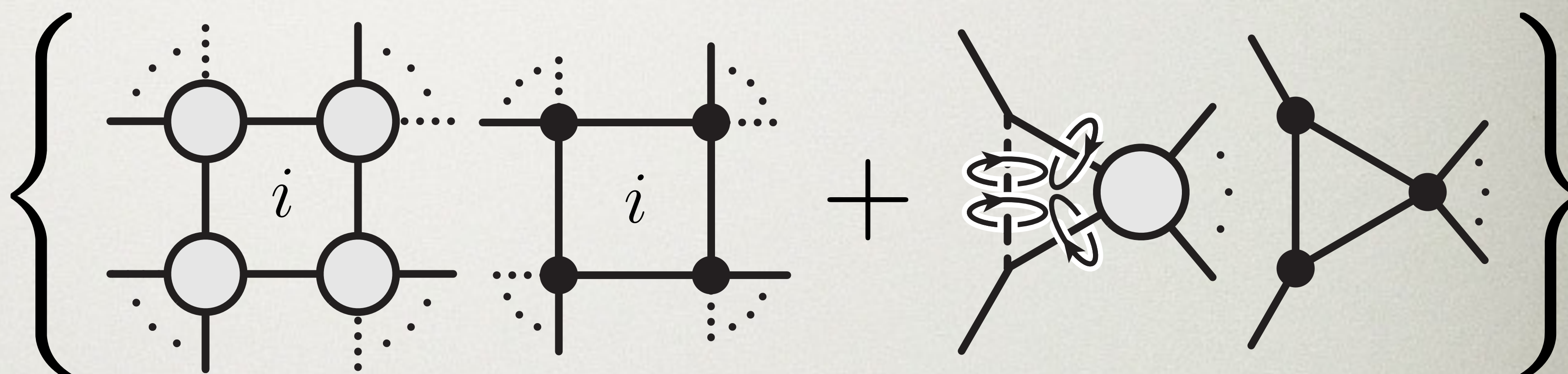

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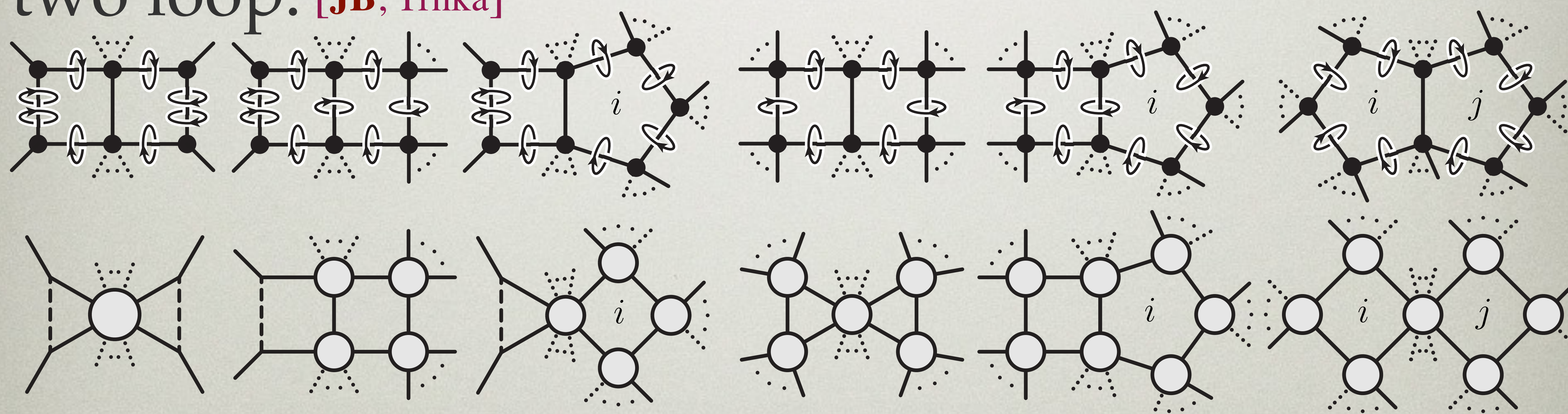

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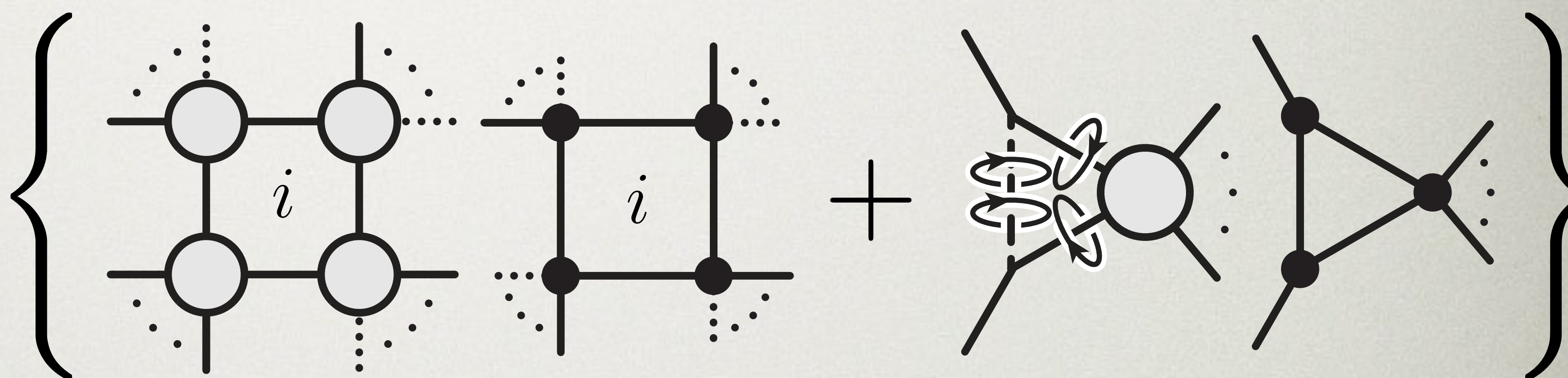

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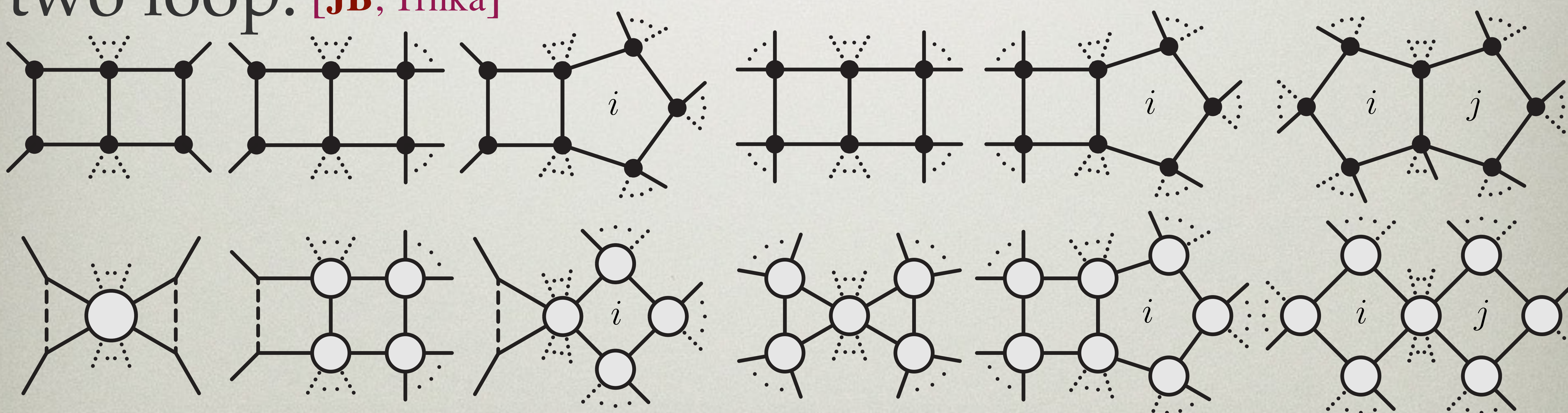

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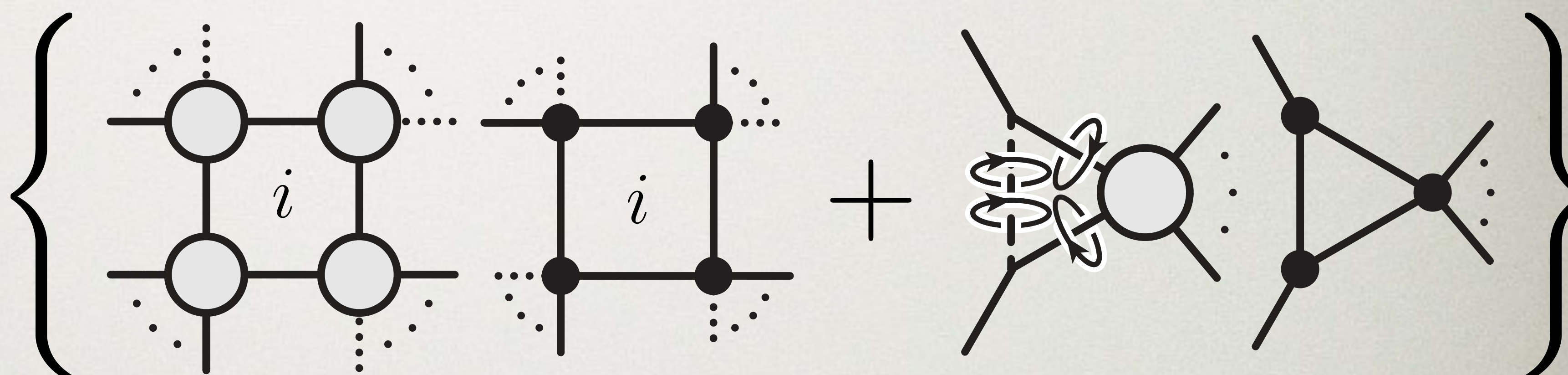

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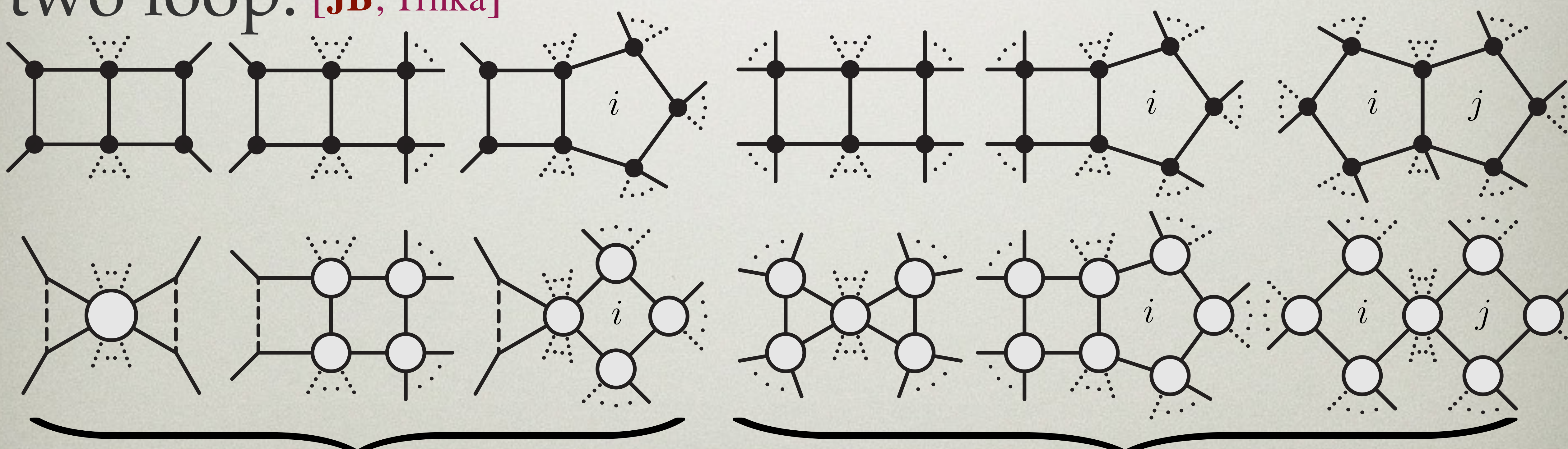

$$\mathcal{A}^0\left(\mathcal{I}_{\text{div}}\bigotimes\mathcal{I}_{\text{div}}\right)+\left(\mathcal{A}^1_{\text{fin}}\bigotimes\mathcal{I}_{\text{div}}\right) \qquad \mathcal{A}^2_{\text{fin}}$$

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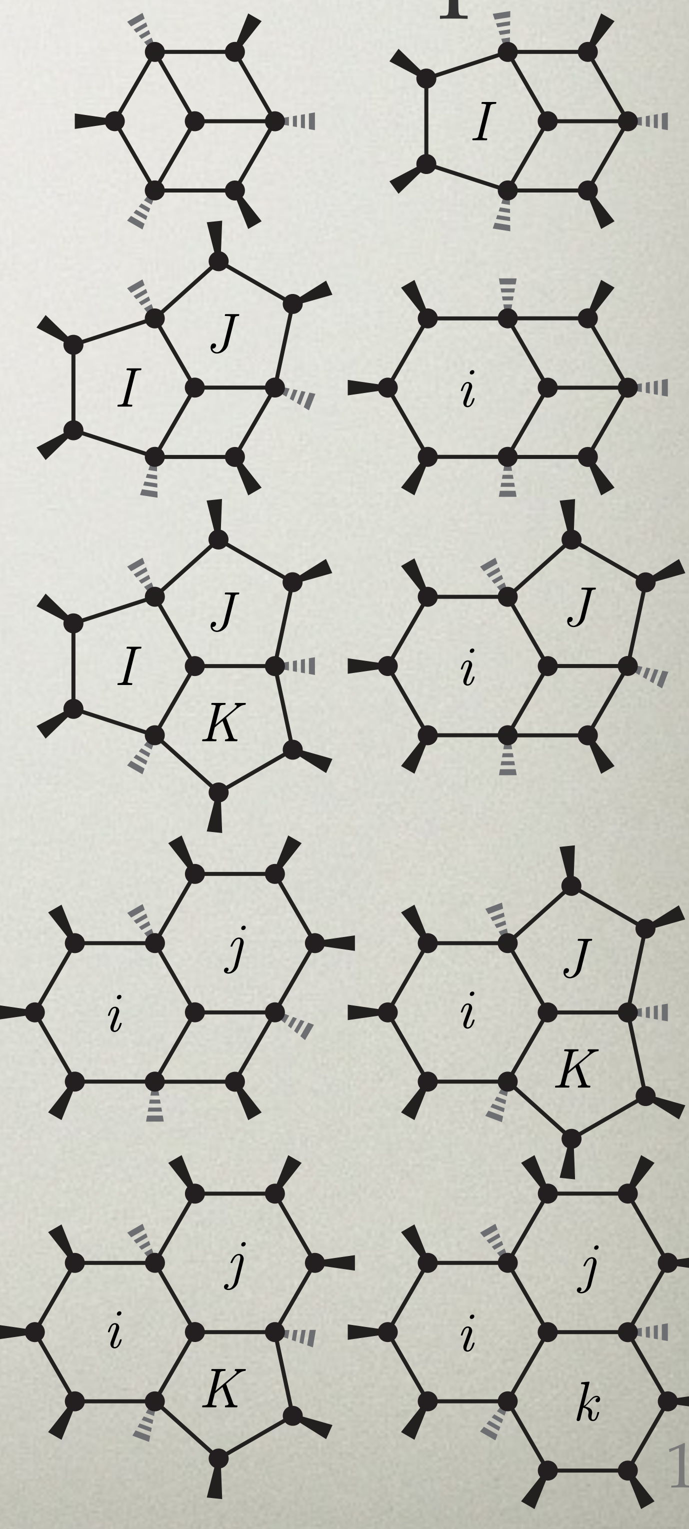

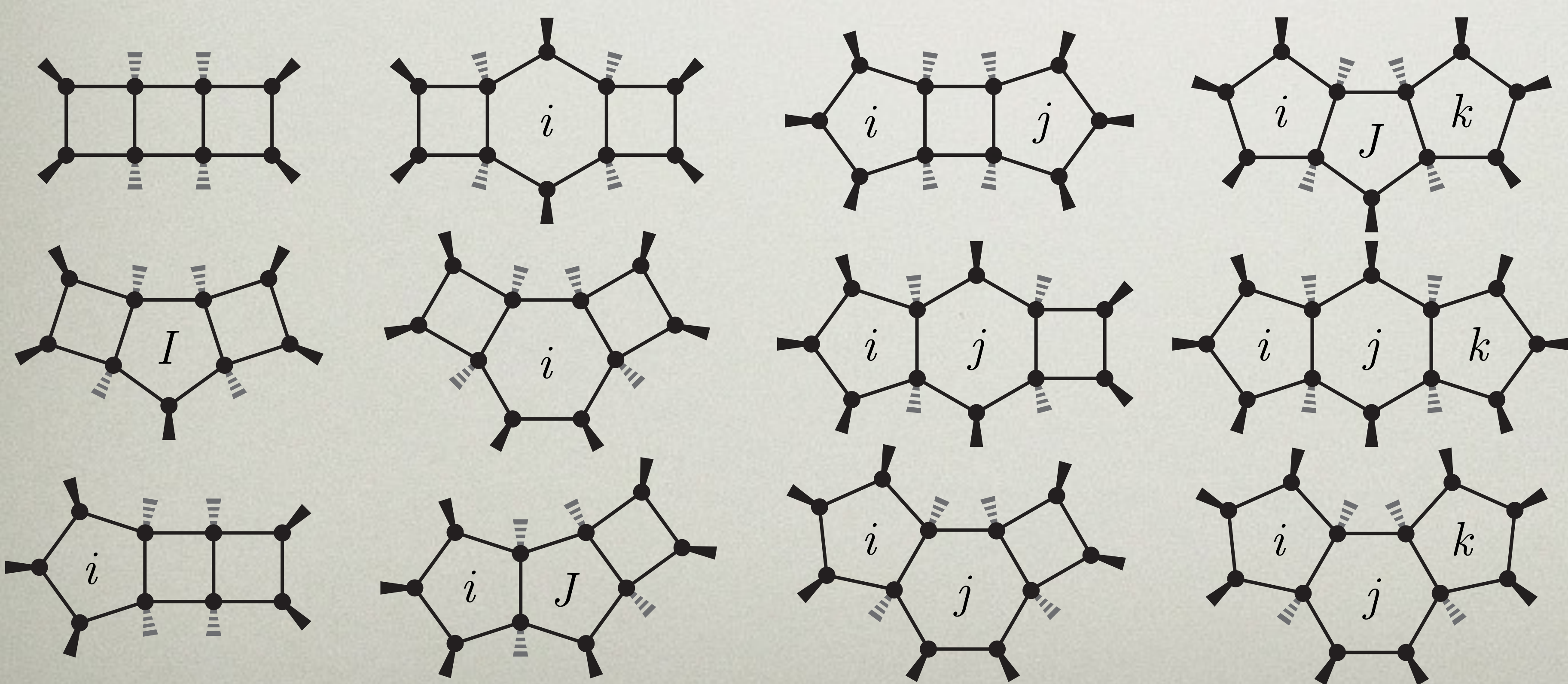

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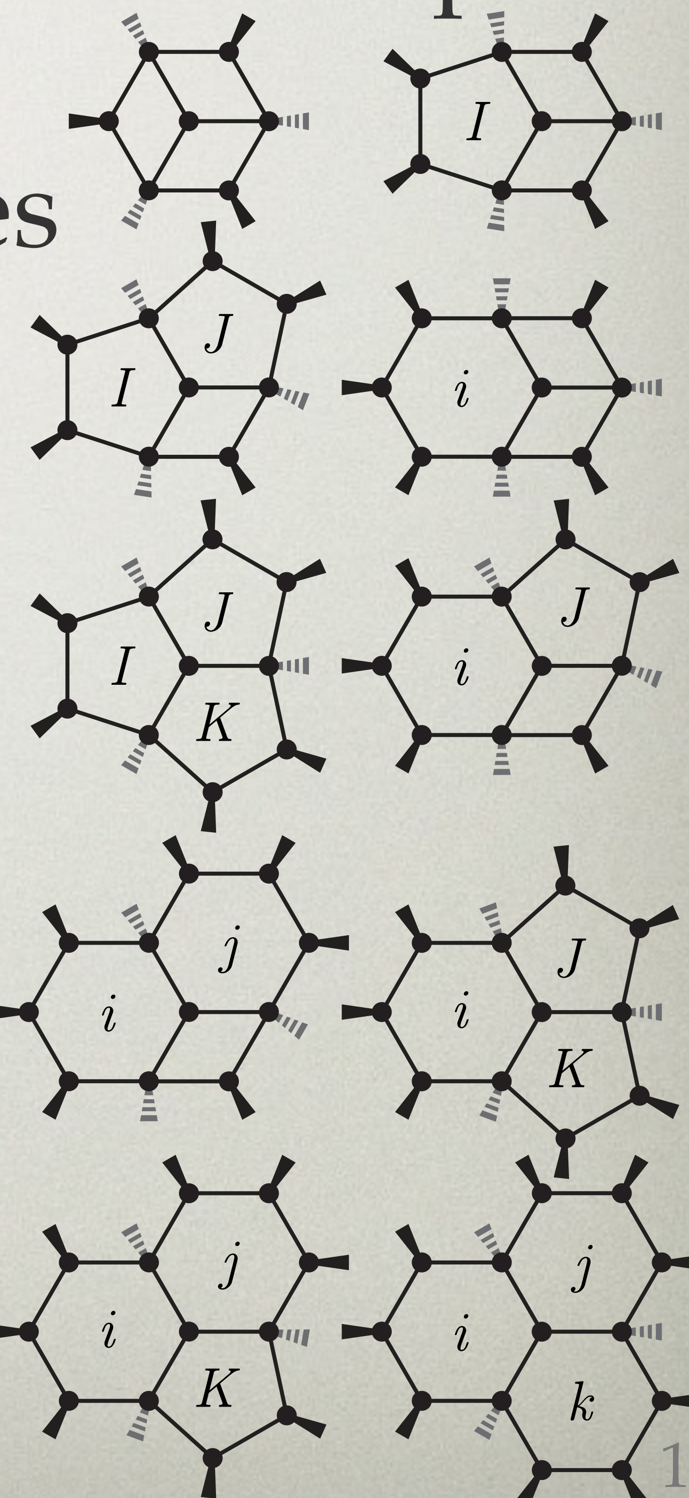

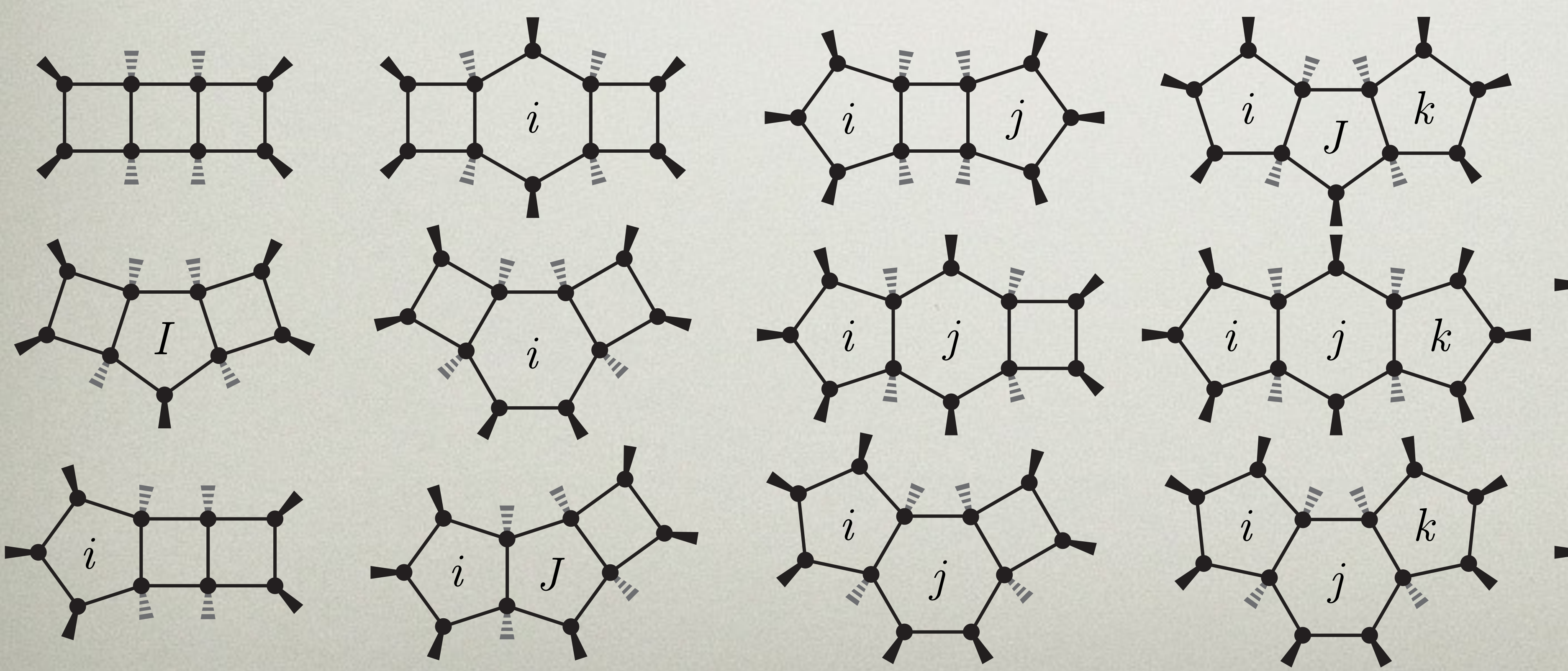

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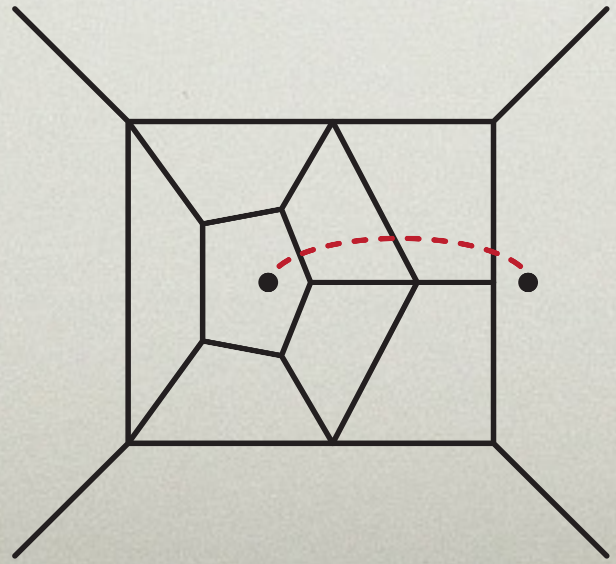

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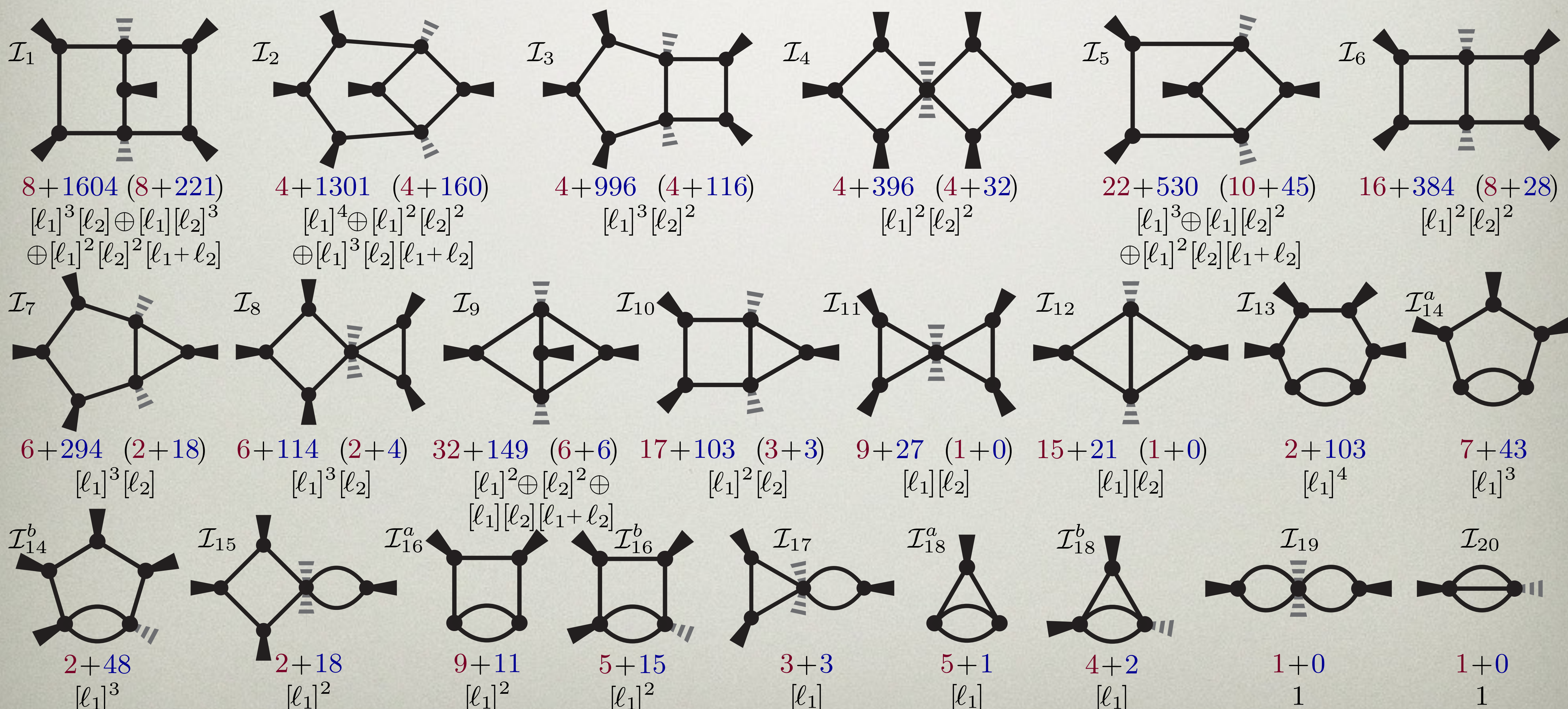

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If so, a triangle power-counting representation would be (asymptotically, arbitrarily) better than BCJ in the UV!

Questions?