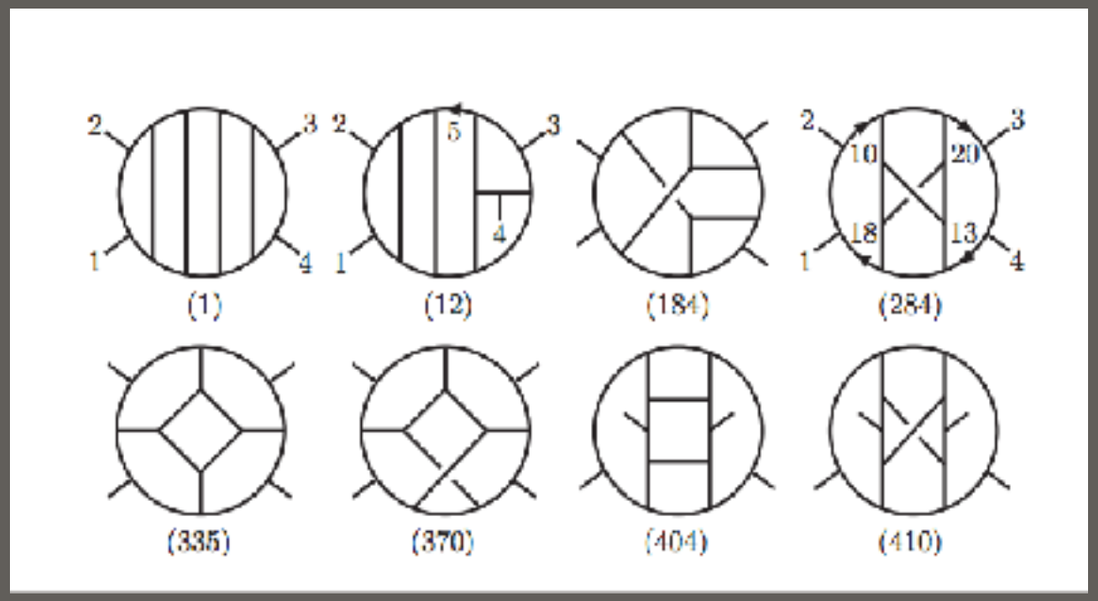
A SURPRISING OBSERVATION FOR 5-LOOP INTEGRAND IN $\mathcal{N}$=8 SUPERGRAVITY



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work in progress

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MOTIVATION

- More examples for loop amplitudes

- UV property of $\mathcal{N}$=8 SUGRA

- critical dimension: the lowest dimension where UV divergence first appears

$$D_c = 4 + \frac{6}{L}, \qquad L=2,3,4$$

- $D_c = \frac{26}{5}$ in 5-loop?

ANSATZ APPROACH

- if it works, simplest way to do it: BCJ with master diagrams
- various ansatz for master diagrams w/ parameters 1k ~120k
- attempts for high power of loop momenta in the same spirit of 1511.06652 [Mogull, O'Connell]
  2-loop 5 gluons YM
- not succeeded yet, despite considerable effort

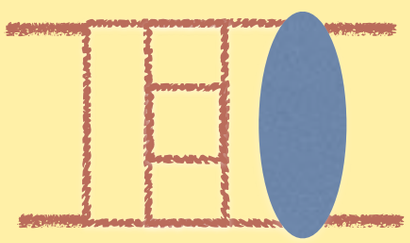
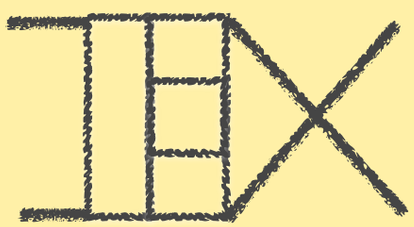
ANSATZ APPROACH

- building ansatz with or without BCJ involves guesswork
- large linear system needed to be solved

LESSON:
We want method
without
guesswork & no large linear system

CONTACT TERM APPROACH

- a constructive way to build the integrand, no guesswork, no large linear system
- contact terms approach: add contact terms to incomplete gravity amp to satisfy every cut

cut  contact 

- if amplitude ~ $\frac{(stuM_4^{tree})s^2 f(l)}{(l_5^2 l_6^2 \ldots l_{20}^2)}$, $[f(l)]=[M]^{12}$

  hundred thousand cuts, through N^6-max needed to be checked
- However, miracle happens so contacts are either relatively simple or zero and have a new structure.

CONTACT TERM APPROACH

- **starting with the known 5-loop SYM amps**

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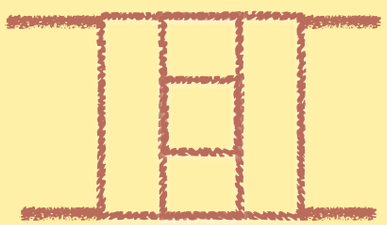
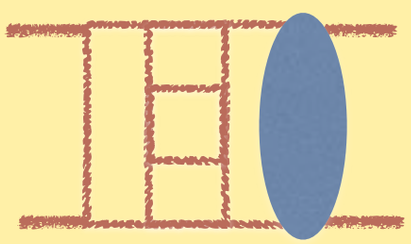
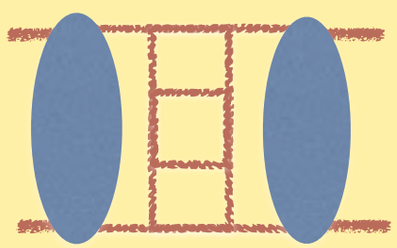
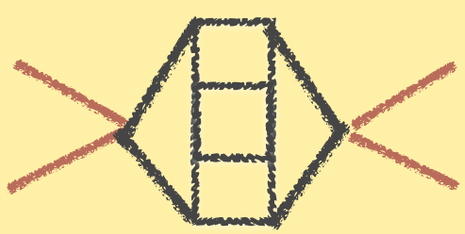
$$\sum_i \int dl_5 \ldots dl_9 \frac{n_i^{\rm SYM}}{\prod_{\alpha_i} P_{\alpha_i}} \Rightarrow \hat{M} = \sum_i \int dl_5 \ldots dl_9 \frac{(n_i^{\rm SYM})^2}{\prod_{\alpha_i} P_{\alpha_i}}$$

max-cut  **works by construction**

N¹max-cut  **works by BCJ**

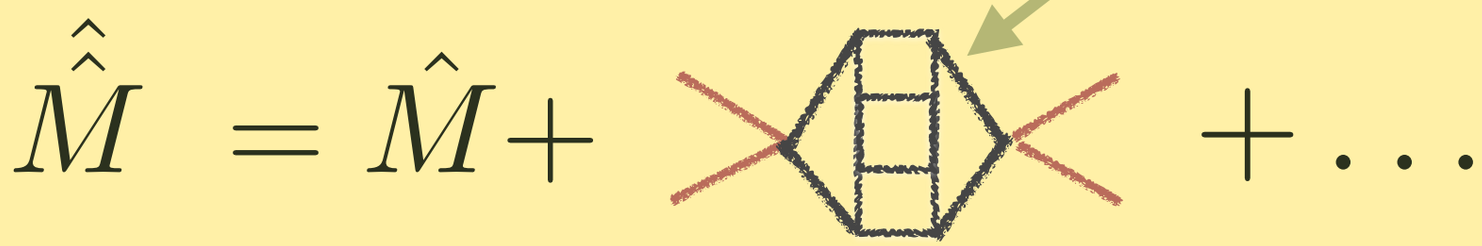
N²max-cut  **does not work in general**

 **add contact term to make it work**

CONTACT TERM APPROACH

- no contact terms in max-cut and N-max-cut
- non-zero contacts start to appear at N^2-max-cut
- how to obtain a contact

define an off-shell contact from the on-shell result



- $\hat{\hat{M}}$ satisfies all N^2-max-cuts
- iteratively check cuts and construct amp through N^6-max-cut

CONTACT TERM APPROACH

Computational complexity of cuts

$$\mathbf{N}^2 \quad \frac{(stuM_4^{tree})g(s,t)f_1(l)}{(l_7^2 \ldots l_{20}^2)} \quad [f_1(l)]=[M]^8$$

$$\mathbf{N}^3 \quad \frac{(stuM_4^{tree})g(s,t)f_2(l)}{(l_8^2 \ldots l_{20}^2)} \quad [f_2(l)]=[M]^6$$

$$\mathbf{N}^4 \quad \frac{(stuM_4^{tree})g(s,t)f_3(l)}{(l_9^2 \ldots l_{20}^2)} \quad [f_3(l)]=[M]^4$$

$$\mathbf{N}^5 \quad \frac{(stuM_4^{tree})g(s,t)f_4(l)}{(l_{10}{}^2 \ldots l_{20}^2)} \quad [f_4(l)]=[M]^2$$

$$\mathbf{N}^6 \quad \frac{(stuM_4^{tree})g(s,t)f_5(l)}{(l_{11}{}^2 \ldots l_{20}^2)} \quad [f_5(l)]=[M]^0$$

$$g(l)=a_1 s^2+a_2 st+a_3 t^2$$

analytically

numerically

CONTACT TERM APPROACH

- easy and fast way to obtain SUGRA cut by applying KLT to known 5-loop SYM cuts

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HOW TO OBTAIN SUGRA CUT

- **SUGRA cuts are expressed by color-ordered SYM cuts**

$$(\text{SUGRA cut}) = \sum_{\substack{N=8\\ \text{states}}} M_{n_1}^{tree} M_{n_2}^{tree} \ldots M_{n_k}^{tree}$$

1. **KLT relation** $\quad M_n^{tree} = \sum_{i,j} K_{ij} (A_n^{tree})_i (\tilde{A}_n^{tree})_j$
2. **(state sum of N=8)**

   = (state sum of N=4) X (state sum of N=4)

$$(\text{gravity cut}) = \sum_{\substack{i_1 \ldots i_k \\ j_1 \ldots j_k}} \tilde{K}_{\substack{i_1 \ldots i_k \\ j_1 \ldots j_k}} \left[\sum_{\substack{N=4\\ \text{states}}} (A_{n_1}^{tree} \ldots A_{n_k}^{tree})_{i_1 \ldots i_k} \sum_{\substack{N=4\\ \text{states}}} (\tilde{A}_{n_1}^{tree} \ldots \tilde{A}_{n_k}^{tree})_{j_1 \ldots j_k} \right]$$

color-ordered SYM cuts can be obtained from known 5-loop SYM amps

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MIRACLES OF SIMPLICITY

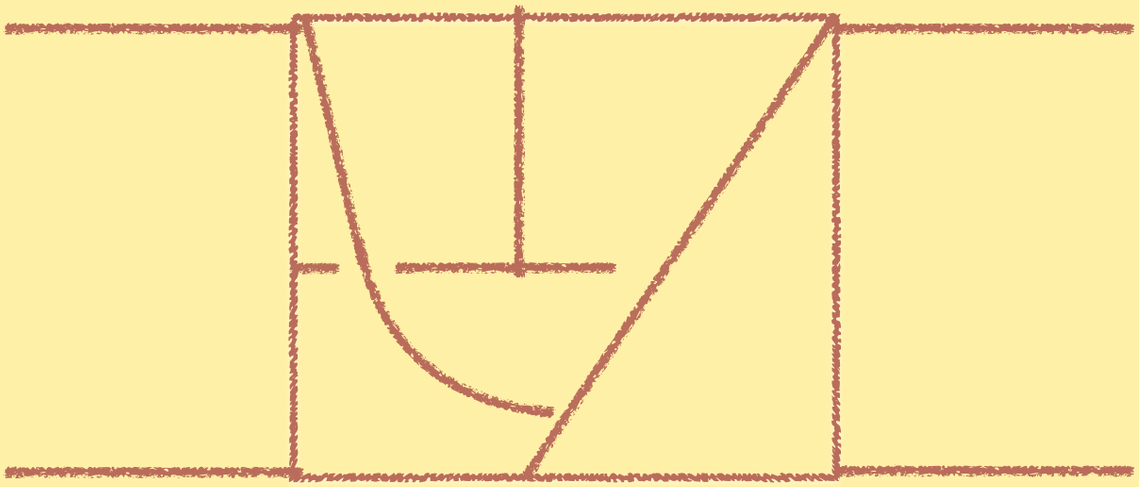
(gravity cut) - (incomplete amp on cut) –> contact

- Most of contacts are zero

N^2 and N^3 contacts most complicated.

- file sizes for non-zero cuts are mostly few kB
- contacts for cuts w/ only three or four-point blob always factorized
- sample results for contact at N^2 level

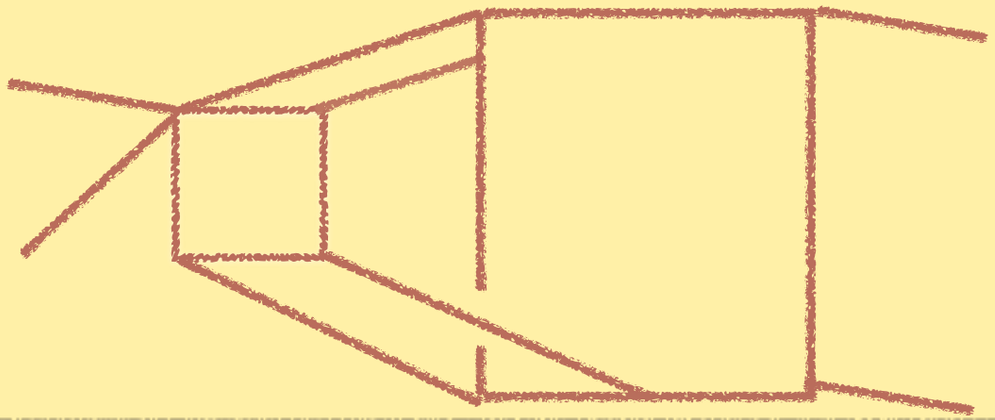
$$\left[2s^3 - 2s^2u - 3su^2 + 4s^2(2k_1 \cdot l_6) + \ldots\right]\left[s^2u + 2su^2 + 3u^3/2 - s^2(2k_1 \cdot l_6) + \ldots\right]$$



MIRACLES OF SIMPLICITY

- nontrivial, $\mathrm{KLT} - \sum_i \frac{(n_i^{\mathrm{SYM}})^2}{p_i^2}$. Why should it factorize?
- Clearly there is a double copy-like formula for contacts.
- For 5 or hight point blobs, double copy-like formula does not imply factorization but still simple.

$$-s^6 - 2s^5u - 2s^4u^2 - 2s^5(2k_1 \cdot l_5) - 4s^4u(2k_1 \cdot l_5) - 4s^4(2k_1 \cdot l_5)^2 + \ldots$$



SUMMARY

- use any known SYM representation to build the integrand

- Naive double copy is almost right up to contact terms

- contact terms miraculously seem to also obey double copy-like formulae

SUMMARY

- **miracle happens in our interesting 5-loop case**

  (gravity cut) - (incomplete amp on cut) –> contact

- **What are these formulae? How to derive them?**

- **See J.J.'s talk**