The HVP contribution from lattice QCD - an update from the Fermilab/HPQCD/ MILC collaborations

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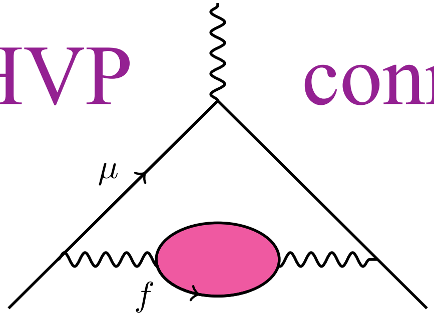
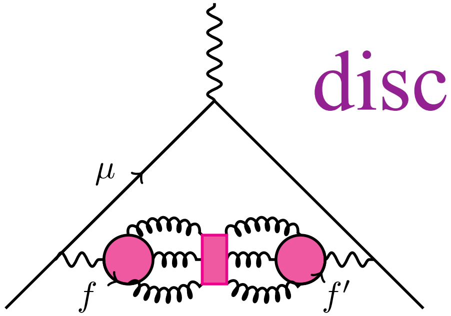
The story so far

LOHVP conn.



- HPQCD, 1403.1778 - first results for s and c HVP (and b - 1408.5768)

disc.



- HPQCD+Hadspec - 1512.03270 - first estimate of disconnected contribution from lattice results with a signal
- HPQCD + RdV - 1601.03071 - first u/d (av.) connected HVP using physical u/d quarks . Total conn. HVP obtained.

x 10^{-10}	u/d	599(6)(8)
	s	53.4(6)
	c	14.4(4)
LO total		667(13)

inc. uncty est. for disc.

est. of QED+isospin-breaking

- Fermilab/HPQCD/MILC : 1710.11212

$$\Delta a_\mu^{\rm conn} \equiv a_\mu^{\rm phys} - a_\mu^{\rm symm} = +1.5(7)\%$$

- first LQCD HOHVP: 1806.08190 $= -9.3(1.3) \times 10^{-10}$

Updates

- New connected u/d HVP results on finer (a=0.09, 0.06 fm) physical-$m_{u/d}$ lattices + very high stats for a=0.15fm
- Some changes in fit methodology - physical $m_{u/d}$ only, dropped rescaling with ρ mass, inc. t* where data $\rightarrow$ fit. Further tests of fit methodology confirm it is robust.

NEW

x 10^-10

u/d	622(8)	2σ up on 1601.03071
s	53.4(6)	
c	14.4(4)	
LO total	690(13) ←	PRELIMINARY allowing 0(10) for disc., IB and QED

- Results for disconnected contribution and first study of isospin-breaking effects on disc. contribution
- Tests of current normalisation using MOM schemes

Ensembles for connected u/d HVP

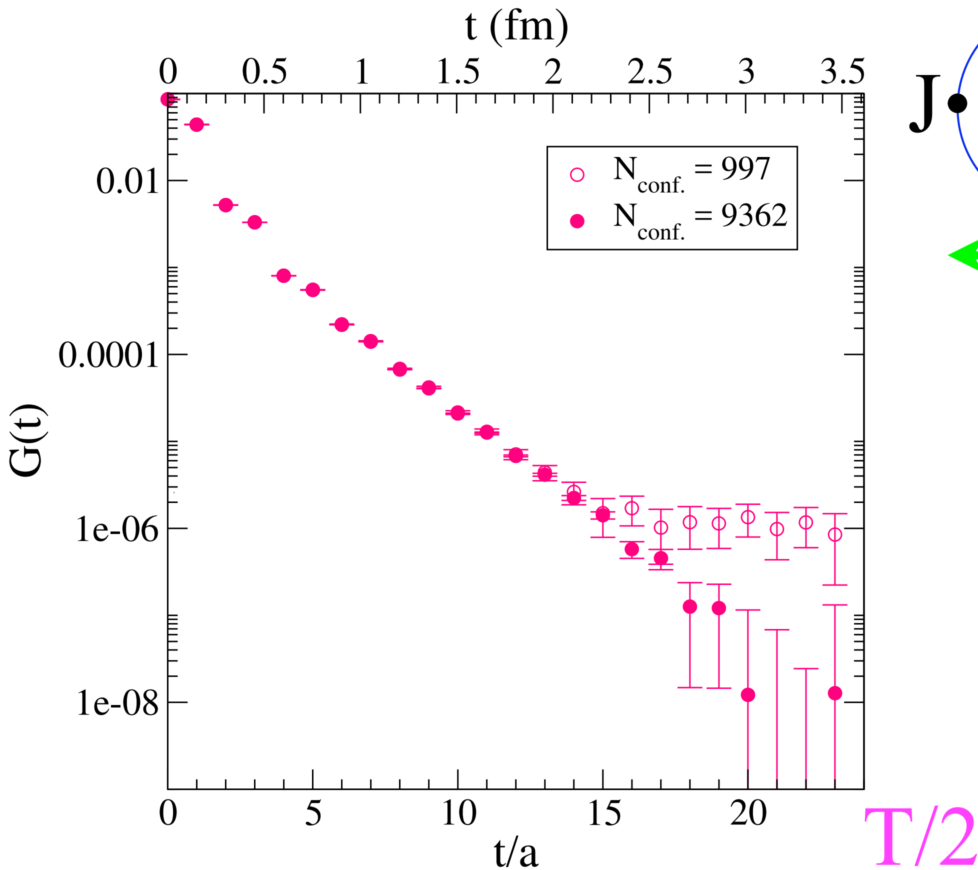
$n_f = 2+1+1$ u/d, s, c quarks in sea HISQ

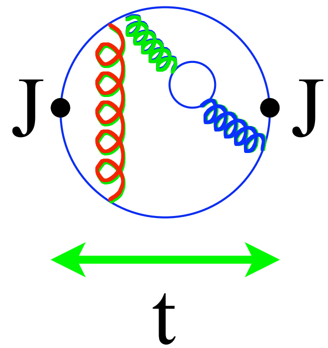
$\approx a$ (fm)	$am_l^{\rm sea}/am_s^{\rm sea}/am_c^{\rm sea}$	w_0/a	M_{π_5} (GeV)	$\left(\frac{L}{a}\right)^3 \times \left(\frac{T}{a}\right)$	$N_{\rm conf.}$	$N_{\rm wall\ src}$	NEW
0.15	0.00235/0.0647/0.831	1.13670(50)	0.13304(70)	$32^3 \times 48$	997	16	
0.15	0.002426/0.0673/0.8447	1.13215(35)	0.13473(71)	$32^3 \times 48$	9362	48(TSM)	*
0.12	0.00184/0.0507/0.628	1.41490(60)	0.13273(70)	$48^3 \times 64$	998	16	
0.09	0.00120/0.0363/0.432	1.95180(70)	0.12834(68)	$64^3 \times 96$	1557	16 (TSM)	*
0.06	0.0008/0.022/0.260	3.0170(23)	0.13495(72)	$96^3 \times 192$	1230	16 (TSM)	*

- physical u/d quark masses and range of factor 6 in a^2
- large stats set at a=0.15 fm - can test impact of noise
- calculate vector-vector correlators for l (=(u+d)/2) quarks and average over gluon field configs $J_V = \gamma_\mu \otimes \gamma_\mu$
- include smearing in 2x2 matrix of vector-vector correlators for improved fit. HVP calculated from loc-loc.

Tests of noise

$$G(t) = \langle \sum_{\vec{x}} j(\vec{x},t) j(0,0) \rangle$$





$$\hat{\Pi}(Q^2)$$

via time-moments + [3,3] Padé

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha_{EM}}{\pi}\right)^2 \times \int_0^\infty dQ^2 K_E(Q^2) \hat{\Pi}(Q^2)$$

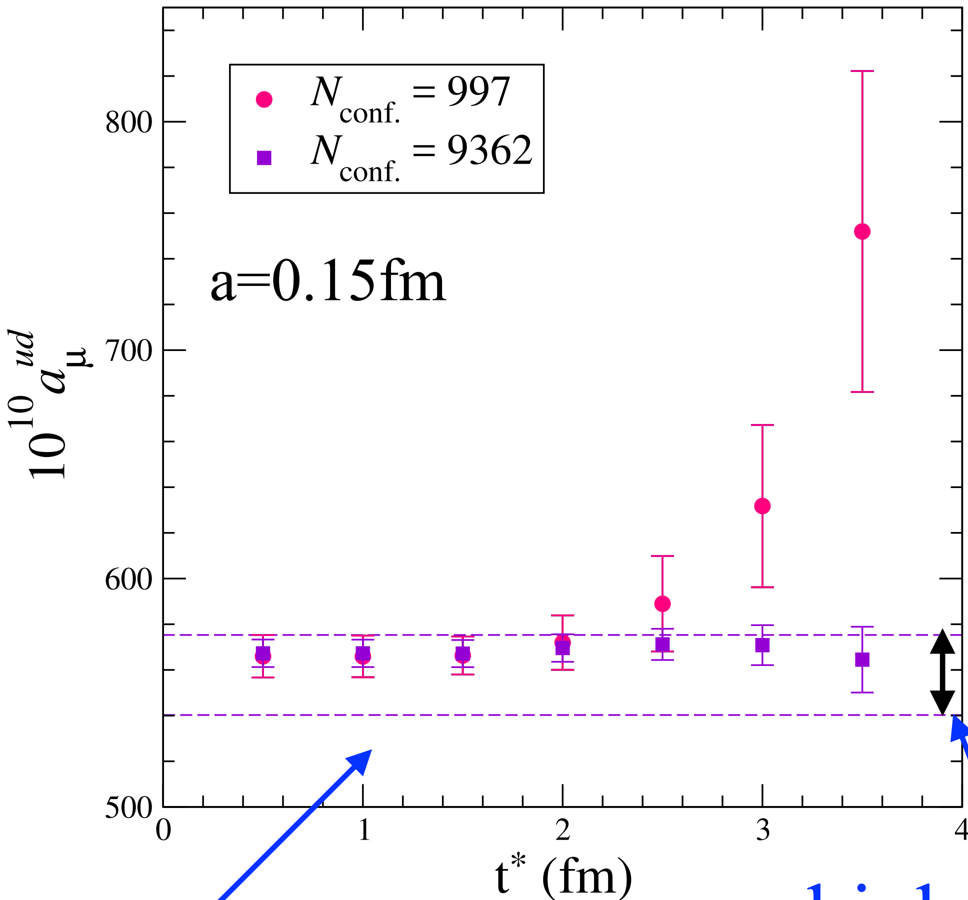
For high stats G(t) reliable across full t range.

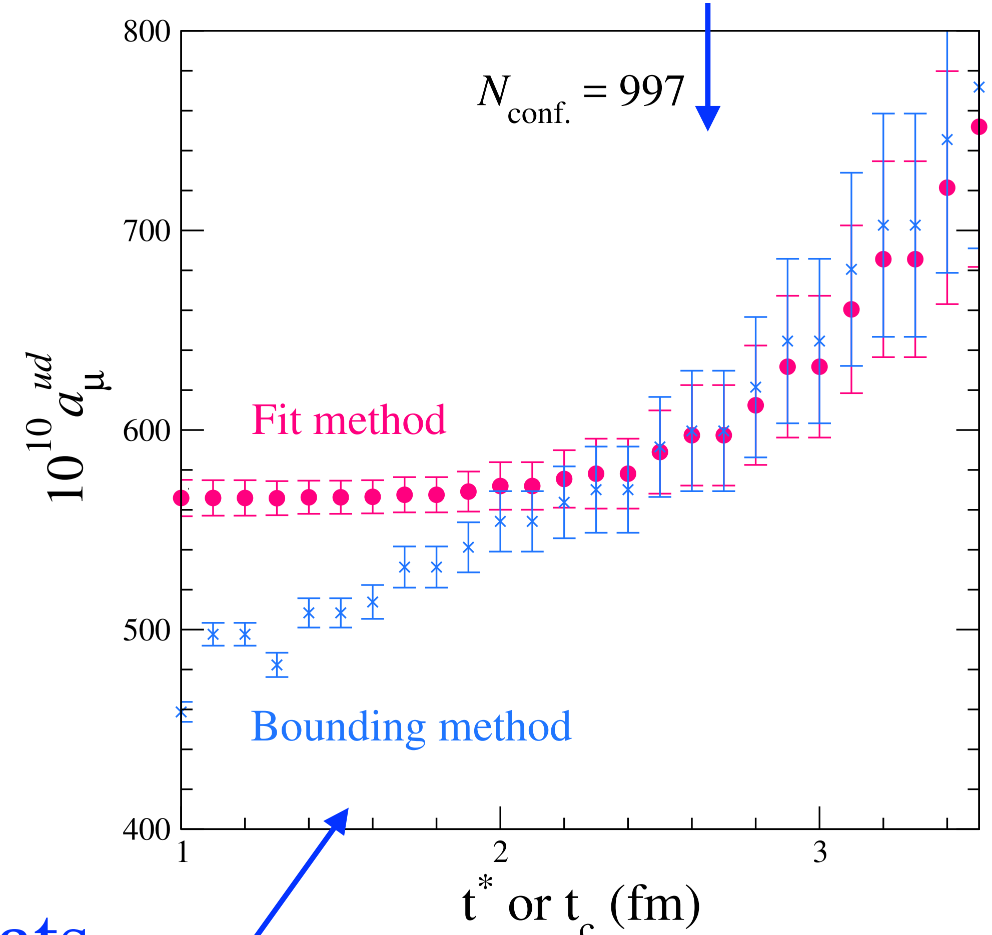
For lower stats non-Gaussianity often visible - fixes tmax for multi-exponential fit to G(t). $G(t) = \sum_i A_i e^{-M_i t}$

- Use data for t < t*; fit for t > t* - test dependence on t*

Tests of noise/fit

compare our fit method to bounding method





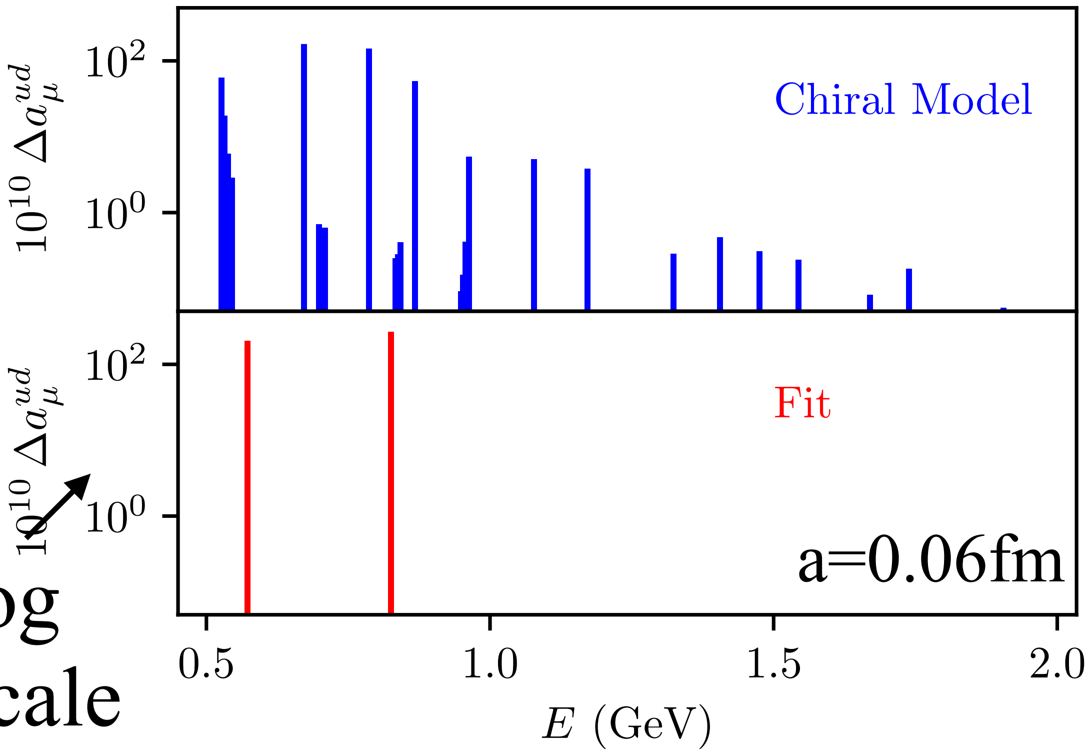
any t* up to 2.5fm gives same result - use t* = 2 fm - 90% HVP from data

high stats data for all t

For t > t_cut set G(t) = 0 or $\propto e^{-E_{\pi\pi} t}$
Average the two results

t* method is correct for larger range.

Test with fake data

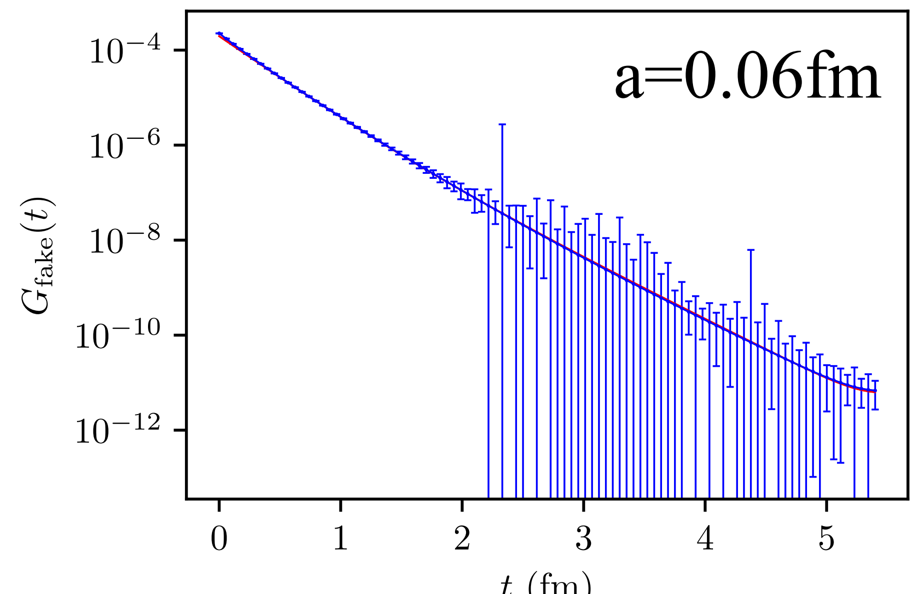


log scale

Use chiral EFT with $\rho, \gamma, \pi\pi$ interactions

Generate fake G(t) from finite-volume energy levels up to 2 GeV

Add covariance from real data and fit.

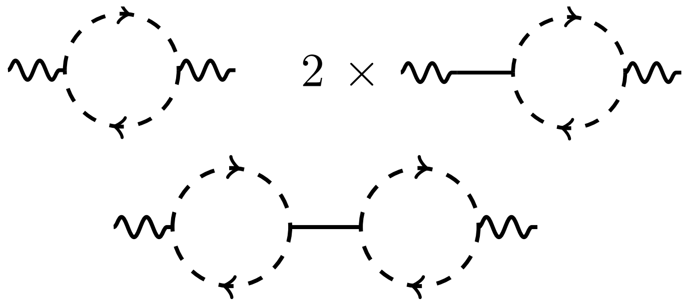


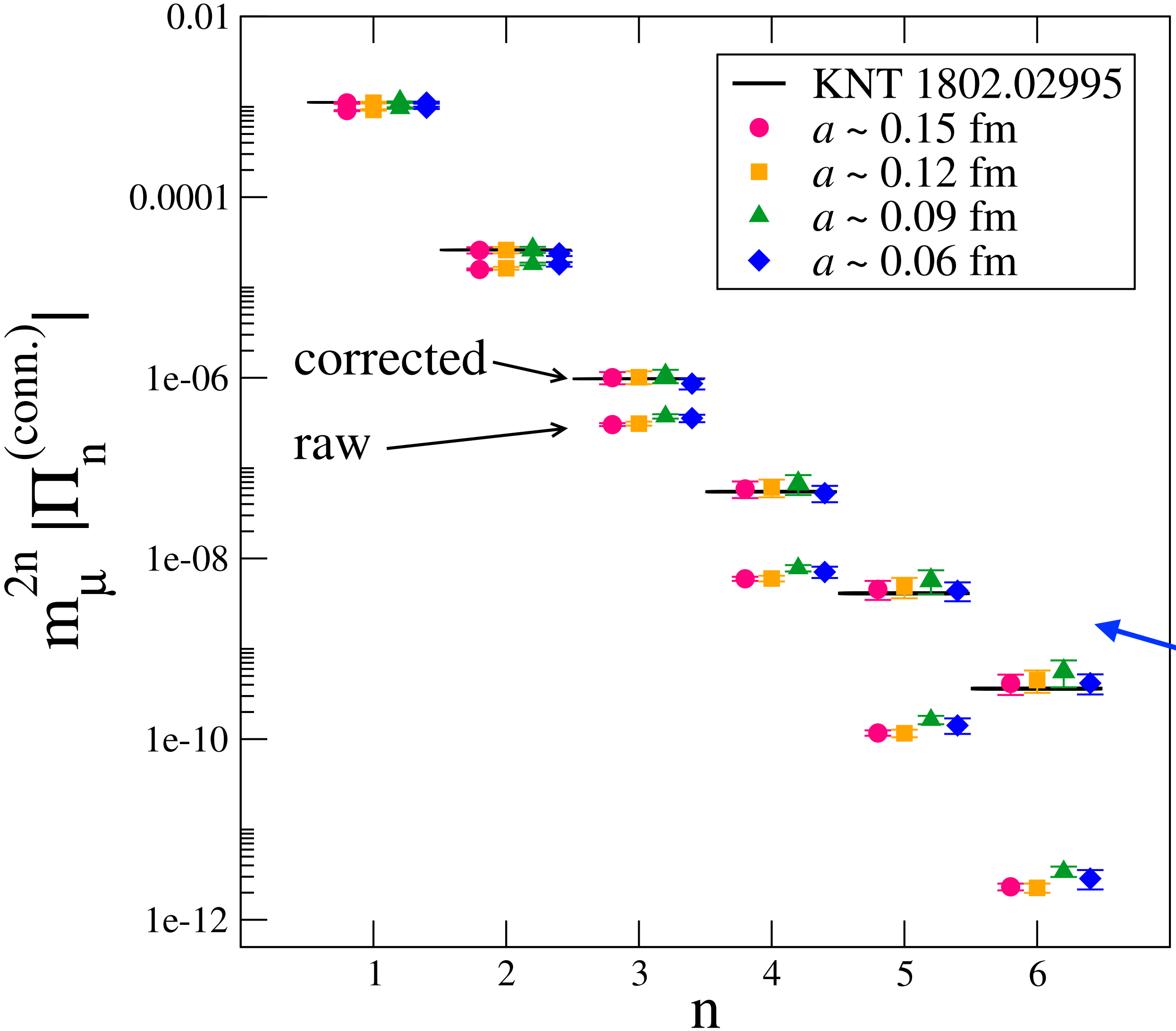
2-exp fit *WORKS* - gives correct result to ~$1\text{x}10^{-10}$ (within fit error, all cases)

NOTE: key need is to track G(t), *not* necessary to resolve all the energy levels

- See AEK talk for $\pi\pi$ plans

Correct for finite volume and staggered pion masses





$$\hat{\Pi}(Q^2) = \sum_j Q^{2n} \Pi_n$$

correct Π_n by adding (continuum-lattice) from chiral EFT

Test vs R_{e+e-} - large n good test since corrn large, impact on HVP small

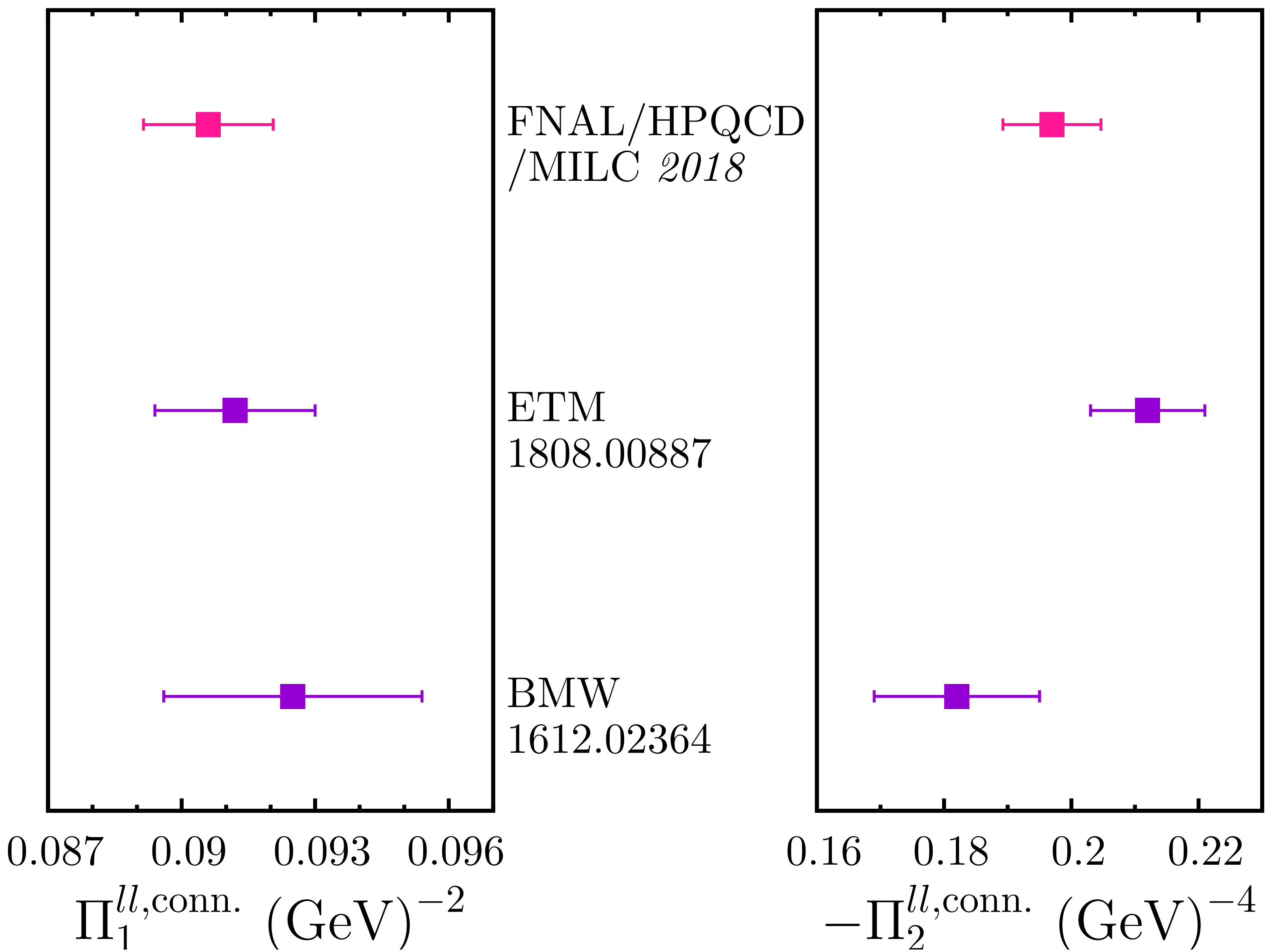
Our results for Π_i:

$\Pi_1^{l,\mathrm{conn}} = 0.0906(15)_{\mathrm{fit}}(13)_{\mathrm{QED,IB}}\ \mathrm{GeV}^2$ $\Pi_2^{l,\mathrm{conn}} = -0.1968(77)_{\mathrm{fit}}(28)_{\mathrm{QED,IB}}\ \mathrm{GeV}^4$

Comparison of Π_i results from lattice QCD

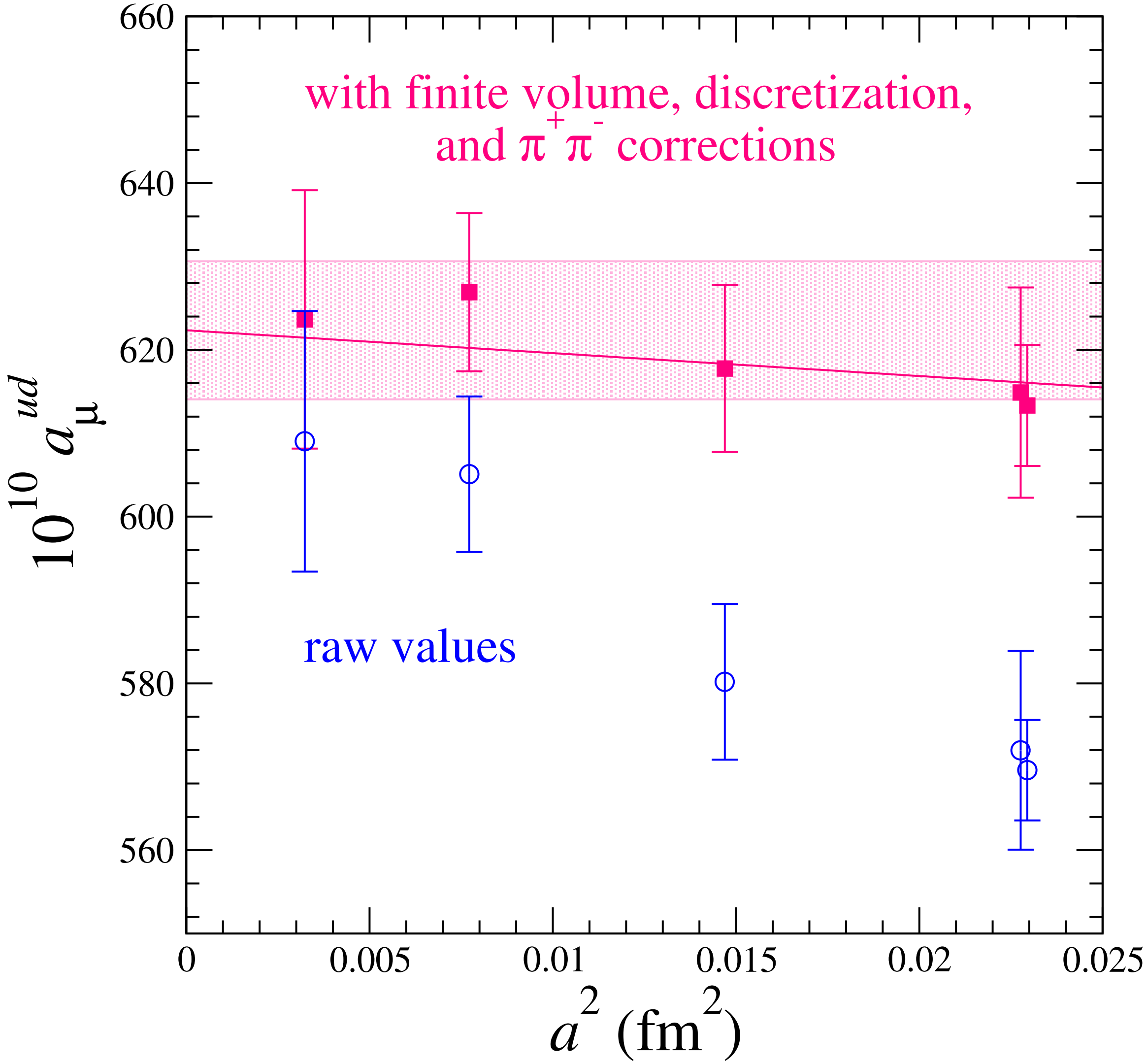
isospin-symmetric, no QED

All $N_f = 2+1+1$



Result in continuum limit

all results at physical $am_{u/d}$



After corrections, see ~no dependence on a lattice spacing

$$a_\mu(a) = a_\mu^{l,LO} \times \left(1 + c_s \sum_f \frac{\delta m_f}{\Lambda} + c_{a^2}\frac{(a\Lambda)^2}{\pi^2}\right)$$

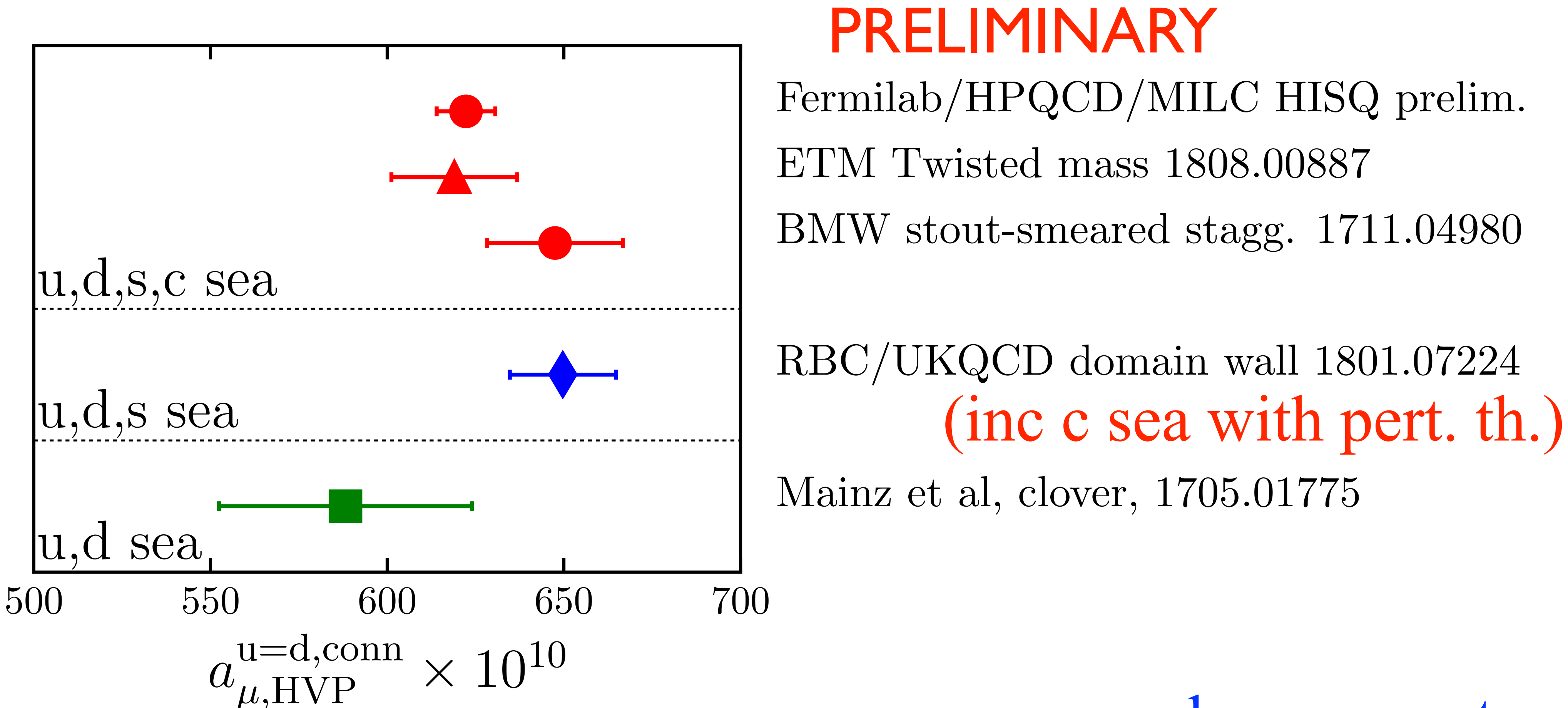
$$a_\mu^{l,LO} = 622(8) \times 10^{-10}$$

Errors dominated by a^{-1}, $a \to 0$, stats

disc. pieces, QED, isospin-breaking not yet inc….

Comparison of lattice HVP(ud, conn)

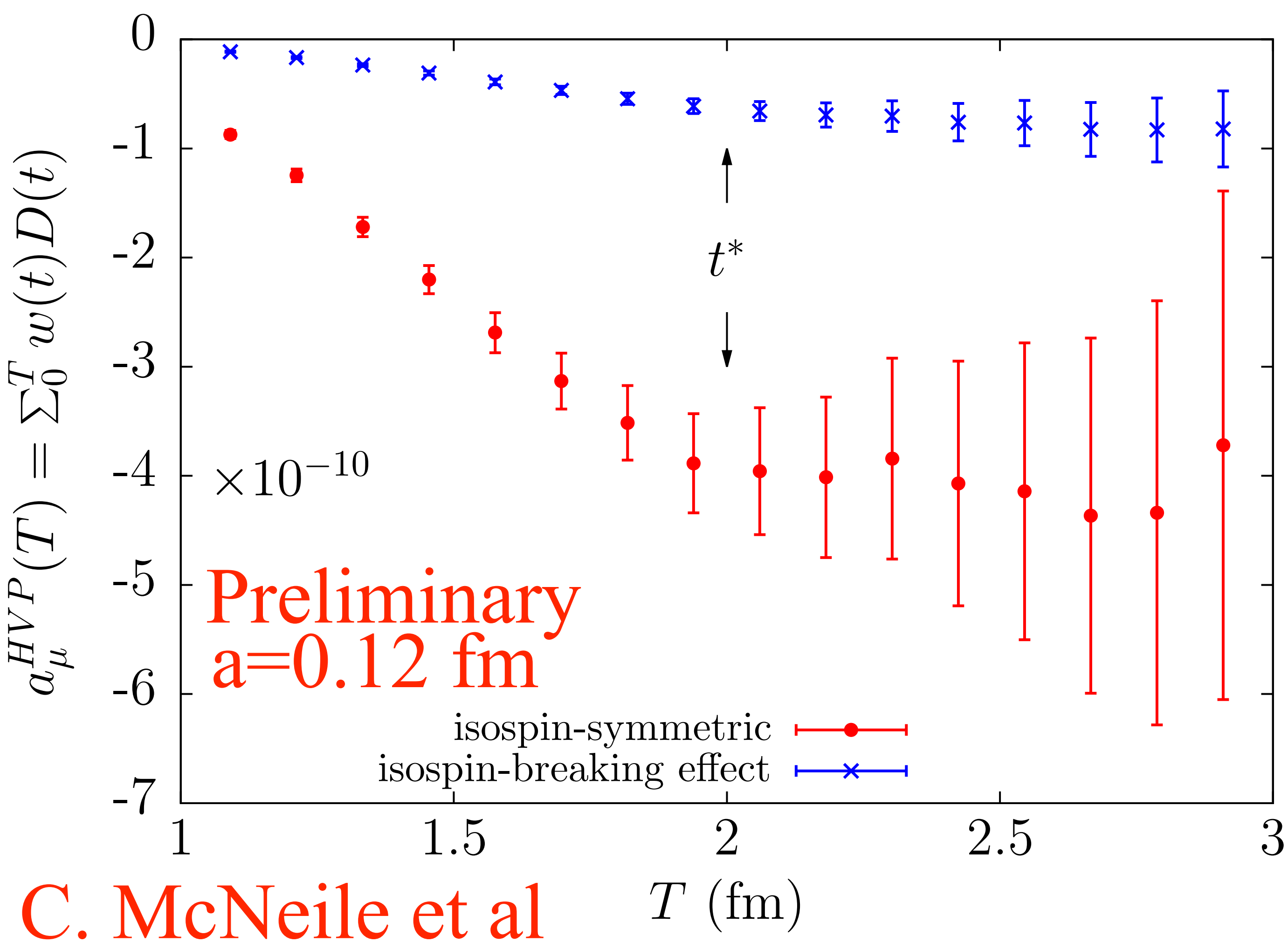
isospin-symmetric, no QED



good agreement

‘Disconnected’ contribution $D(t) = \langle j_d(t) j_d(0) \rangle$

$$j_d = \gamma_i \otimes 1$$



C. McNeile et al

$$j_d = \frac{1}{3}(\bar{l}\gamma_i l - \bar{s}\gamma_i s)$$

for isospin-symmetric case

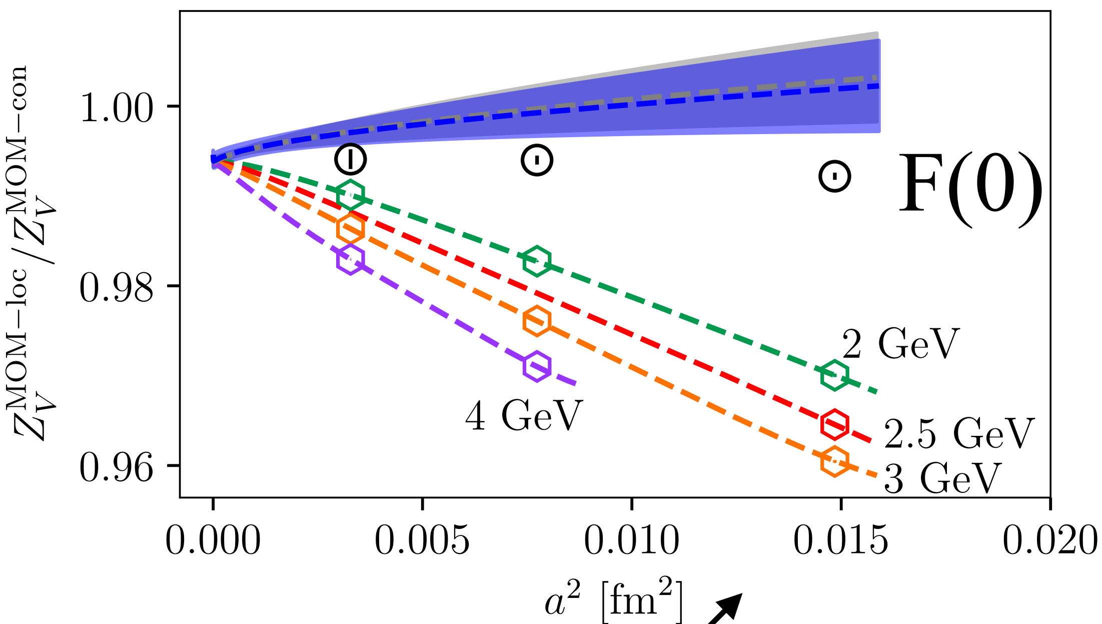
$$j_d = \frac{1}{3}(\bar{u}\gamma_i u - \bar{d}\gamma_i d)$$
$$+\frac{1}{3}(\bar{u}\gamma_i u - \bar{s}\gamma_i s)$$

for physical case

- Isospin-breaking is a large (~20%) effect for disc. because $u\bar{u}/d\bar{d}$ ππ pieces must cancel with connected IB.

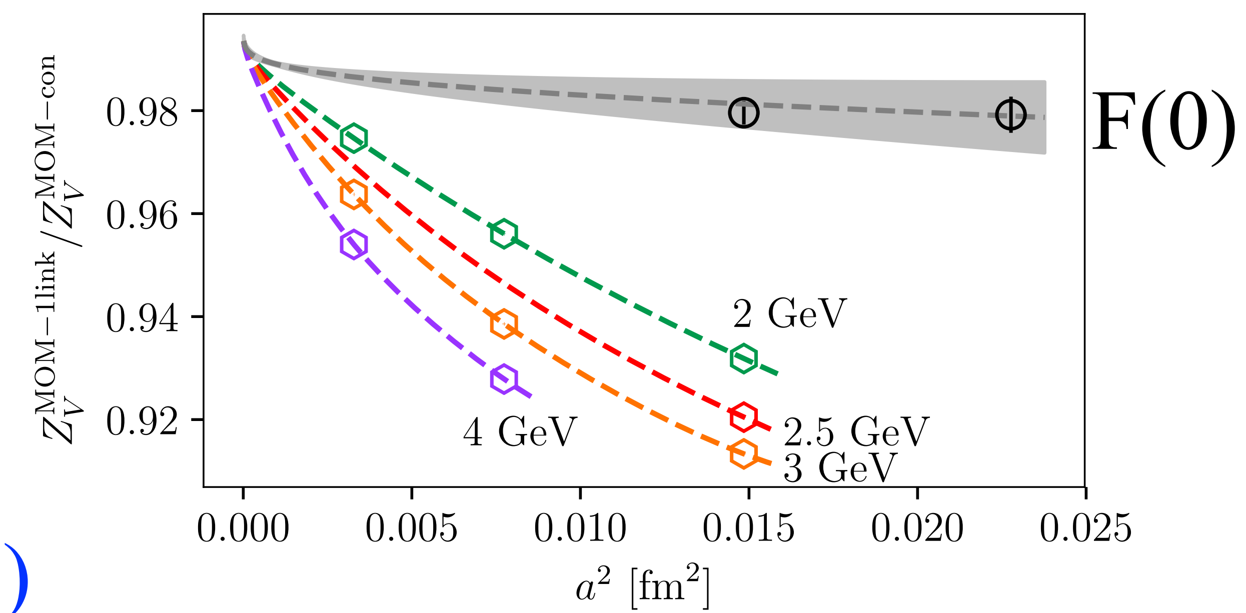
Tests of current renormalisation

D. Hatton



Our Z_V determined fully nonpert. from F(0), but numerically expensive. Lattice MOM schemes cheaper but have systematic errors from disc. and condensates.

We find that MOM schemes reproduce F(0) results, if carefully defined.



Fit = (pert. th. + disc.) find same pert. th. as F(0)

Determination of total lattice LO HVP

x 10^{-10}

u/d	622(8)
s	53.4(6)
c	14.4(4)
LO total	690(13)

PRELIMINARY

allowing 0(10) for disc. IB and QED

Additional arithmetic

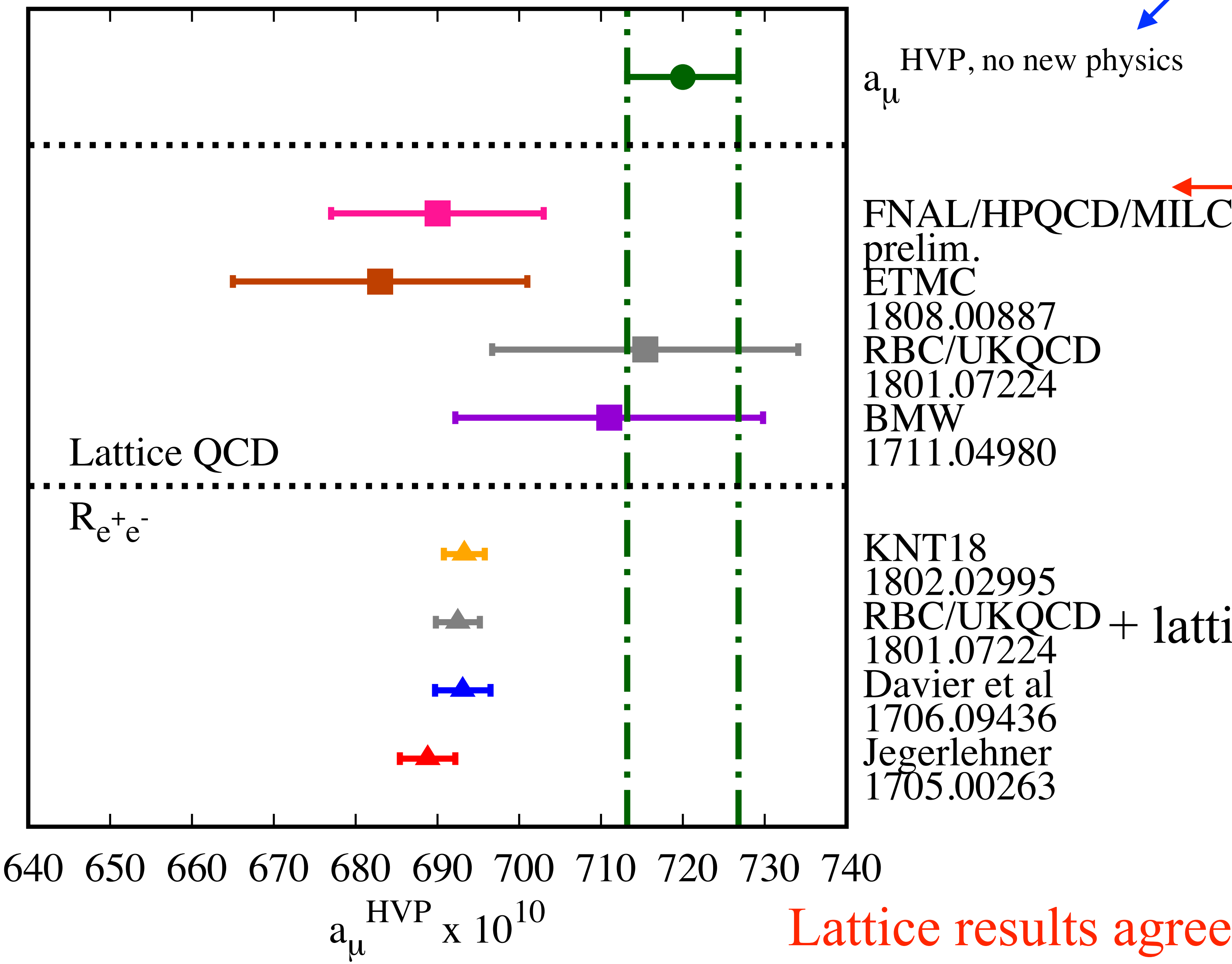
QED	+5	pheno
disc.	-13(2)	BMW
IB	+8(4)	FNAL/HPQCD /MILC + ETM
Total	0(5-10)	

FUTURE :

- further improve stats
- disc. inc. IB.
- further conn. IB
- full QED effects
- $\pi\pi$ analysis

Comparison of lattice LO HVP

based on BNL expt



2σ from "no new physics"

+ lattice QCD

prelim. Mainz (Wittig talk) : 710(21)

Lattice results agree-
uncertainties need to be reduced

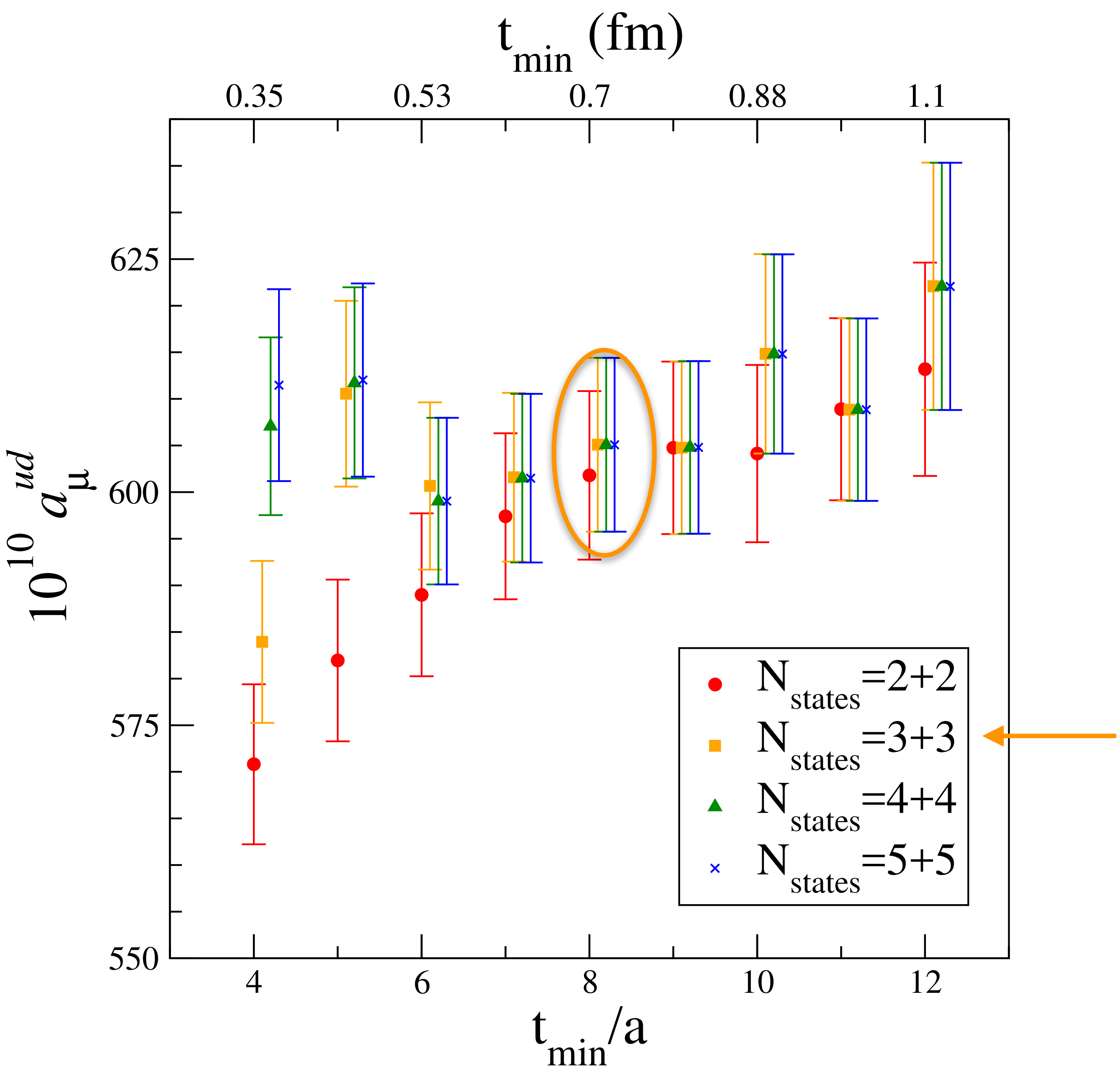
Spares

Correlator fit stability

a=0.09fm, 64^3 x 96

t_max/a = 30

t*/a = 22 = 2.0 fm



Fits are stable as we change t_min if enough states are included in fit

Preliminary error budget

source	$a_\mu^{ud,\mathrm{conn.}}$ (%)	$\Pi_1^{ud,\mathrm{conn.}}$ (%)
Lattice-spacing (a^{-1}) uncertainty	0.8	0.9
Monte Carlo statistics	0.7	0.8
Continuum ($a \to 0$) extrapolation	0.7	0.7
Finite-volume & discretization corrections	0.3	0.7
Current renormalization (Z_V)	0.1	0.1
Chiral (m_l) extrapolation/interpolation	0.1	0.1
Sea (m_s) adjustment	0.1	0.1
Lattice pion mass uncertainties	0.0	0.0
Total	1.3%	1.6%