Super-Gauss-Bonnet and other Evanescent Operators in Half-Maximal SUGRA

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QCD Meets Gravity

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Gravity Counterterms and Evanescence

Evanescent object: object that is zero in 4D, but not in other dimensions

- One-loop pure gravity is finite: only counterterm is Gauss-Bonnet, vanishes for 4D states ['t Hooft, Veltman]
- First pure gravity divergence is at two loops

  [Goroff, Sagnotti; van de Ven]
- This divergence has contributions from GB subdivergence and depends on choice of duality transformations

  [Bern, Cheung, Chi, Davies, Dixon, Nohle; Duff, van Nieuwenhuizen]

SUGRA Divergences

Ultraviolet properties of gravity are more complex than previously thought

- "Enhanced cancellations": no known symmetry arguments for lack of divergence [Bern, Davies, Dennen, Huang]
- Vanishing $\log \mu^2$ dependence in two-loop SUGRA, can have non-vanishing divergence

  [Bern, Cheung, Chi, Davies, Dixon, Nohle; Bern, Chi, Dixon, AE to appear]
- Half-Maximal SUGRA explored at high loop order, but limited understanding of evanescence [Bern, Davies, Dennen]

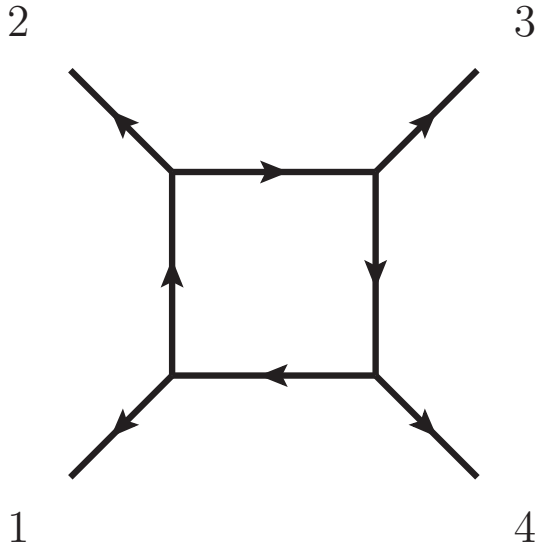
Evanescent effects might contribute to $\mathcal{N} = 4$ four-loop divergence

One-Loop Half-Maximal SUGRA via Double Copy

- $\mathcal{N} = 4 \otimes \mathcal{N} = 0$
- SUSY numerator non-zero only for box
- BCJ double copy gives

$$s\,t\,A^{\text{tree}}(1,2,3,4)\boxed{\int \frac{n_{\mathcal{N}=0,\text{box}}}{\prod_i D_i}}$$

- Evaluate using standard techniques



Can find contributions to gravity evanescence with just a YM calculation

Gauge Invariant Tensor Bases

- $T_i = a_i(s,t)(\epsilon_1 \cdot \epsilon_3)(\epsilon_2 \cdot \epsilon_4) + b_i(s,t)(\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_3)(\epsilon_3 \cdot \epsilon_4) + \ldots$
- Organize amplitude using tensor bases
  [Glover, Tejeda-Yeomans]
  - Map to gravity
  - Explore evanescence
- Project onto tensors using

$$m_{ij} = \sum_{\text{states}} T_i^* T_j$$

$$P_i = (m^{-1})_{ij} T_j^*$$

F Tensor Basis

- Define a set of gauge invariant tensors

$$F_i^{\mu\nu} \equiv i(k_i^\mu \epsilon_i^\nu - k_i^\nu \epsilon_i^\mu)$$

$$F_i^{\mu\nu} F_{j\nu\rho} F_k^{\rho\sigma} F_{l\sigma\mu} \equiv (F_i F_j F_k F_l) \quad F_i^{\mu\nu} F_{j\mu\nu} F_k^{\rho\sigma} F_{l\rho\sigma} \equiv (F_i F_j)(F_k F_l)$$

$$F_{st}^4 = (F_1 F_2 F_3 F_4) \qquad F_{tu}^4 = (F_1 F_4 F_2 F_3) \qquad F_{us}^4 = (F_1 F_3 F_4 F_2)$$

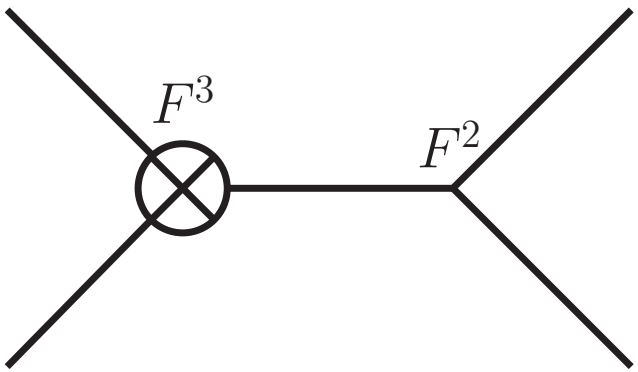
$$F_s^{2,2} = (F_1 F_2)(F_3 F_4) \quad F_t^{2,2} = (F_1 F_4)(F_2 F_3) \quad F_u^{2,2} = (F_1 F_3)(F_4 F_2)$$

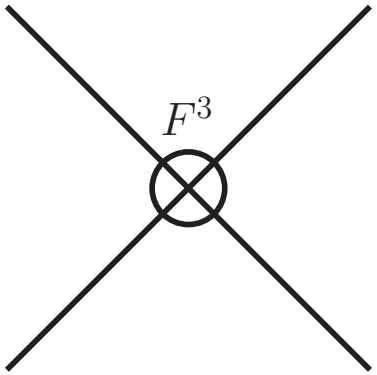
- Need seventh tensor to match one loop coeff.

F^3 Insertion

Will use color-ordered four-point tree with insertion of F^3 operator

$$F^3 \equiv \frac{1}{3}\mathrm{tr}F^{\mu}{}_{\nu}F^{\nu}{}_{\rho}F^{\rho}{}_{\mu}$$





Actual tensor will be

$$T^{F^3} \equiv stA^{\mathrm{tree}}_{F^3}(1,2,3,4)$$

Helicity basis

Basis of tensors that can be reduced to helicity configurations in 4D

Tensor	Helicities	$[m]$	Mapping from Other Basis
T_0	$--++$	8	$-\frac{i}{2}(F^4_{st}+F^4_{tu}+F^4_{us})+\frac{i}{8}(F^{2,2}_s+F^{2,2}_u+F^{2,2}_t)$
T_1	$++++$	8	$-iF^4_{st}+\frac{i}{4}(F^{2,2}_s+F^{2,2}_u+F^{2,2}_t))$
T_3	$-+++$	10	$\frac{i}{4}T^{F^3}-\frac{i}{4}((s+t)F^4_{st}+(s-t)F^4_{us}+(t-s)F^4_{tu})$ $+\frac{i}{16}(s+t)(F^{2,2}_s+F^{2,2}_t+F^{2,2}_u)$
T_4	$--++$	8	$\frac{i}{2}(F^4_{us}-F^4_{tu})+\frac{i}{4}(F^{2,2}_u-F^{2,2}_s)$
T_5	$-+-+$	8	$-iF^4_{st}+\frac{i}{4}(F^{2,2}_s-F^{2,2}_u+F^{2,2}_t)$
T_6	Evanescent	10	$\frac{i}{2}((s+t)F^4_{st}-(3s+t)F^4_{us}-(s+3t)F^4_{tu})$ $+\frac{3i}{8}(s+t)(F^{2,2}_s+F^{2,2}_u+F^{2,2}_t)$
T_7	Evanescent	10	$-\frac{i}{2}((s-t)F^4_{st}+(s+t)F^4_{us}-(s+t)F^4_{tu})$ $+\frac{i}{8}(s-t)(F^{2,2}_s+F^{2,2}_u+F^{2,2}_t)$

Use this basis to examine evanescent properties of gravity

Results of Projections

F basis projects nicely:

$$\boxed{\frac{T^{F^3}}{stu}} - F^4_{st}\left(\frac{4}{3st} + \text{logs} + \pi^2 \text{ terms}\right)$$
$$- F^{2,2}_{s}\left(\frac{1}{s^2} + \text{logs} + \pi^2 \text{ terms}\right) + \text{perms}$$

Notably the F^3 contribution has no transcendental part

Helicity basis projection not as clean

Mapping to Gravity

$$s\,t\,A^{\text{tree}}(1,2,3,4) \otimes X$$

- Special Combination

$$\begin{aligned} 16 s\,t\,A^{\text{tree}} &= t_{8\mu\nu\rho\sigma\alpha\beta\gamma\delta} F^{\mu\nu} F^{\rho\sigma} F^{\alpha\beta} F^{\gamma\delta} = t_8 F^4 \\ &= 8(F_{st}^4 + F_{tu}^4 + F_{us}^4) - 2(F_s^{2,2} + F_t^{2,2} + F_u^{2,2}) \end{aligned}$$

- Can map linearized F_i to linearized R_i

$$F_{i\mu\nu} F_{i\rho\sigma} \to -2 R_{i\mu\nu\rho\sigma}$$

- Gravity mappings

$$\begin{aligned} t_8 F^4 t_8 F^4 &\to t_8 t_8 R^4 \\ t_8 F^4 (F_i F_j F_k F_l) &\to t_8 (R_i R_j R_k R_l) \\ t_8 F^4 (F_i F_j)(F_k F_l) &\to t_8 (R_i R_j)(R_k R_l) \end{aligned}$$

Looking at F^3

- F^3 tensor doesn't map to gravity as simply as F^4
- KLT maps $F^3 \to R^2$ [Broedel, Dixon; Bjerrum-Bohr; He, Zhang]

$$s\,t\,A^{\text{tree}}(1,2,3,4)\,T^{F^3} = s\,u\,A^{\text{tree}}(1,2,4,3)s\,t\,A^{\text{tree}}_{F^3}(1,2,3,4)$$
$$\to s\,t\,u\,\mathcal{M}_{R^2}$$

- R^2 contribution to one-loop amplitude is trivial up to loop factors!

$$\frac{T^{F^3}}{stu} \to \mathcal{M}_{R^2}$$

Beginnings of Super-GB

- On YM side, F^3 is not supersymmetrizable, so only contribute from pure YM
- $\mathcal{N} = 4$ is highest SUGRA that can contain GB
- Helicity basis confirms GB is evanescent [He, Zhang]
- Ongoing work to get off-shell Super-GB
- Connection to string predictions [Green, Rudra]

Conclusions

- Evanescent operators present at one loop even in SUSY gravity
- GB contribution is trivial for one-loop $\mathcal{N} = 4$ SUGRA, no divergence and no log or π^2
- Potential effects at higher loops
- Re-examine four-loop $\mathcal{N} = 4$ divergence for evanescent contributions

Questions?