

**Lepton Dipole Moments**  
**Overview**

**William J. Marciano**  
**December 3, 2018**  
**UCLA**

**Schwingerfest 2018**

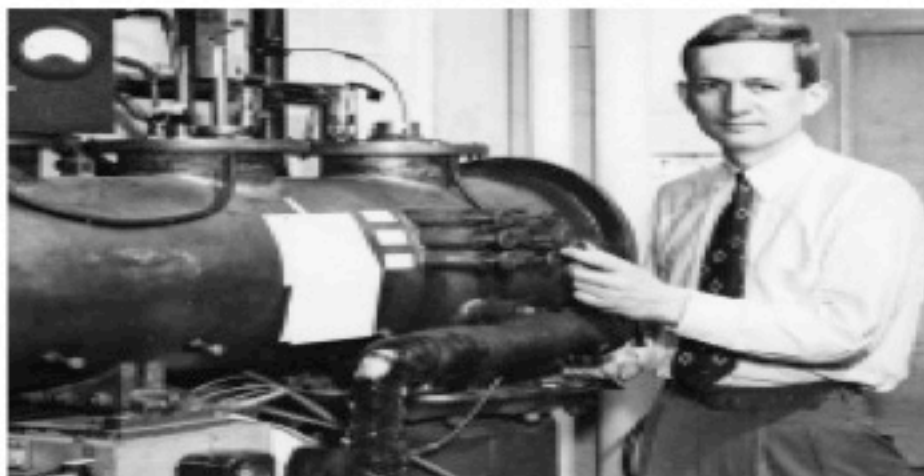
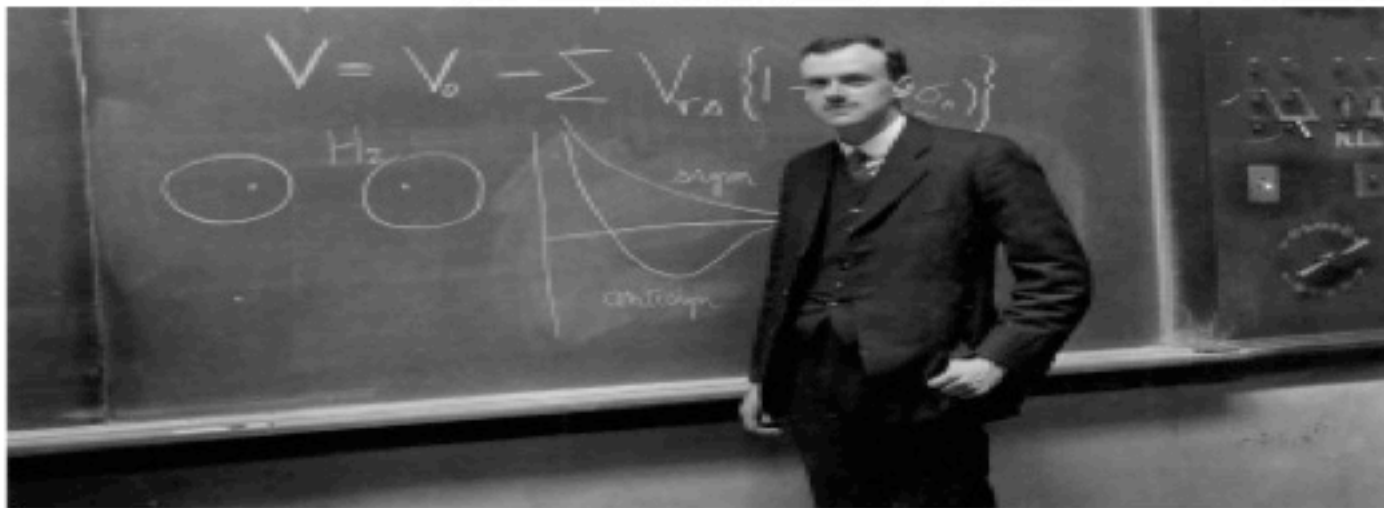
**2018 Schwinger Centennial Year**



Clockwise:

Julian Schwinger,  
Polykarp Kusch,  
Paul Dirac,  
Norman Ramsey and  
Edward Purcell

Courtesy AIP Emilio  
Segrè Visual Archives  
(full credits overleaf)



# Dirac-Schwinger Connections

## The Dirac Equation (1928)

90<sup>th</sup> Anniversary

Unified

QM+Special Rel.+Spin+EM Gauge Invariance

First Order Equation

$$i(\partial_\mu - ieA_\mu(\mathbf{x}))\gamma^\mu\psi(\mathbf{x}) = m_e\psi(\mathbf{x}),$$

$$4\times 4 \gamma^\mu \text{ (Dirac) matrices: } \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}I$$

**$g_e=2$  Success & Antimatter Predicted!  
Foundation of Quantum Electrodynamics**

## Post WWII Physics Boom (1947-48)

### *Julian Schwinger Master of QED*

1947 Small Anomalous Atomic Fine Structure Effects

1948 Schwinger Calculates:  $a_e = (g_e - 2)/2 = \alpha/2\pi \approx \underline{0.00116}$

$$(\alpha = e^2/4\pi = 1/137)$$

Agreed with measurement of Kusch & Foley!

$$a_e^{\text{exp}} \approx \underline{0.00119(5)}$$

**Triumph of Quantum Field Theory**

**Precision Quantum Electrodynamics (QED)**

Mount Auburn Cemetery

Courtesy Lee Roberts



## **Fine Structure Constant $\alpha=1/137$**

1931 Dirac introduces magnetic monopoles

Finds  $eM/4\pi = n/2$   $n=1,2,\dots$  **charge quantization!**

$M^2/4\pi = 137/4 \approx 34$  strong magnetic coupling

**Failed to derive  $\alpha=1/137$  Goal!**

1957 SO(3) EW Unification Schwinger-(Georgi-Glashow 1972)

1965 Schwinger revisits magnetic monopoles & dyons

$eM/4\pi = n$   $n=1,2,\dots$

1974 't Hooft-Polyakov SO(3) Magnetic Monopole Soliton

1975 B. Price Monopole Candidate = Schwinger Charge!

**“Monopole Candidate Retracted”**

## Alpha, Grand Unification & Monopoles

M. Goldhaber & WJM (1987)

*in "Reminiscences about a great physicist"*

Renormalization group invariant  $e(\mu)M(\mu)/4\pi=3/8$

Duality/Unification scale  $e(\mu_0)=M(\mu_0)$   $\mu_0 \approx 10^{18} \text{GeV}$

$\alpha^{-1}(\mu_0)=8/3$  vs  $\alpha^{-1}(0)=137$   $\alpha_i^{-1}(\mu_0)=1, i=1,2,3$

*Observed Change due to vacuum polarization by  $S=0, 1/2, 1$  particles*

$137=8/3+\sum c_i Q_i^2 \ln(\mu_0/m_i)+\dots$  sum over all charged particles!

**Example: Supersymmetric  $E_6$  with 78 Gauge Fields and three generation relatively light matter 27plets**

## Anomalous Magnetic Moments After Schwinger

$$a_l = (g_l - 2)/2 \quad l = e, \mu$$

$$a_e(\text{exp}) = 0.00115965218073(28) \quad \text{unc. } \pm 28 \times 10^{-14}!$$

[\(Hanneke, Fogwell, Gabrielse: PRL 2008\)](#)

$$g_e = 2.00231930436146(56)$$

*Most precisely known dimensionless physical quantity!*

***Future improvement Goal? See Gerry Gabrielse***

$$a_e(\text{SM}) = \alpha/2\pi - 0.328478444002546(\alpha/\pi)^2$$

$$+ 1.181234016(\alpha/\pi)^3 - 1.912245764(\alpha/\pi)^4$$

$$+ \underline{6.675(192)(\alpha/\pi)^5} \dots + 169(1) \times 10^{-14}(\text{had}) + 3 \times 10^{-14}(\text{EW})$$

[Aoyama, Kinoshita, & Nio 2018, Laporta 4 loop Analytic](#)

[Spectacular Achievements](#)

***Uncertainty  $\pm 1.3 \times 10^{-14}$  (QED theory)  $\pm 1.2 \times 10^{-14}$  (Hadronic)***



## New Alpha Measurement

$$\alpha^{-1}(^{133}\text{Cs})=137.035999046(27) \text{ Parker et al. 2018}$$

earlier

$$\alpha^{-1}(^{87}\text{Rb})=137.035998995(85)$$

$$\alpha^{-1}(a_e)=137.035999149(33)_{\text{exp}}$$

$$\Delta a_e = a_e^{\text{exp}} - a_e^{\text{SM}} = -87(28)_{\text{exp}}(23)_{\alpha}(2)_{\text{th}} \times 10^{-14}$$

**(2.4 $\sigma$ ) Very Large Effect! Expect  $O(6 \times 10^{-14}) \sim m_e^2/m_\mu^2 \Delta a_\mu$**

Central Value  $\sim 30 \times a_e(\text{EW})$

reconcile:  $\Delta a_e \rightarrow -5539(\alpha/\pi)^6$  seems unlikely

$-13(\alpha/\pi)^5$  large part missing?

experiments? Atomic Physics?

“new physics” eg light (pseudo)scalar?

# Alpha determination using Rydberg Constant

$$R_\infty = 1.0973731568527(73) \times 10^7 \text{m}^{-1}$$

$$\frac{1}{2}m_e\alpha^2 = 13.60569253(30)\text{eV}$$

$$\alpha^{-1}(\text{Cs}) = 137.035999046(27)$$

R. Parker et al. Science (2018) major improvement!

$$\Delta a_e = a_e(\text{exp}) - a_e(\text{theory}) = -87(36) \times 10^{-14} \text{ Note Sign}$$

**Error Budget:  $\pm 23 \times 10^{-14}$ (alpha)  $\pm 28 \times 10^{-14}$ (exp.)  $\pm 2 \times 10^{-14}$ (th.)**

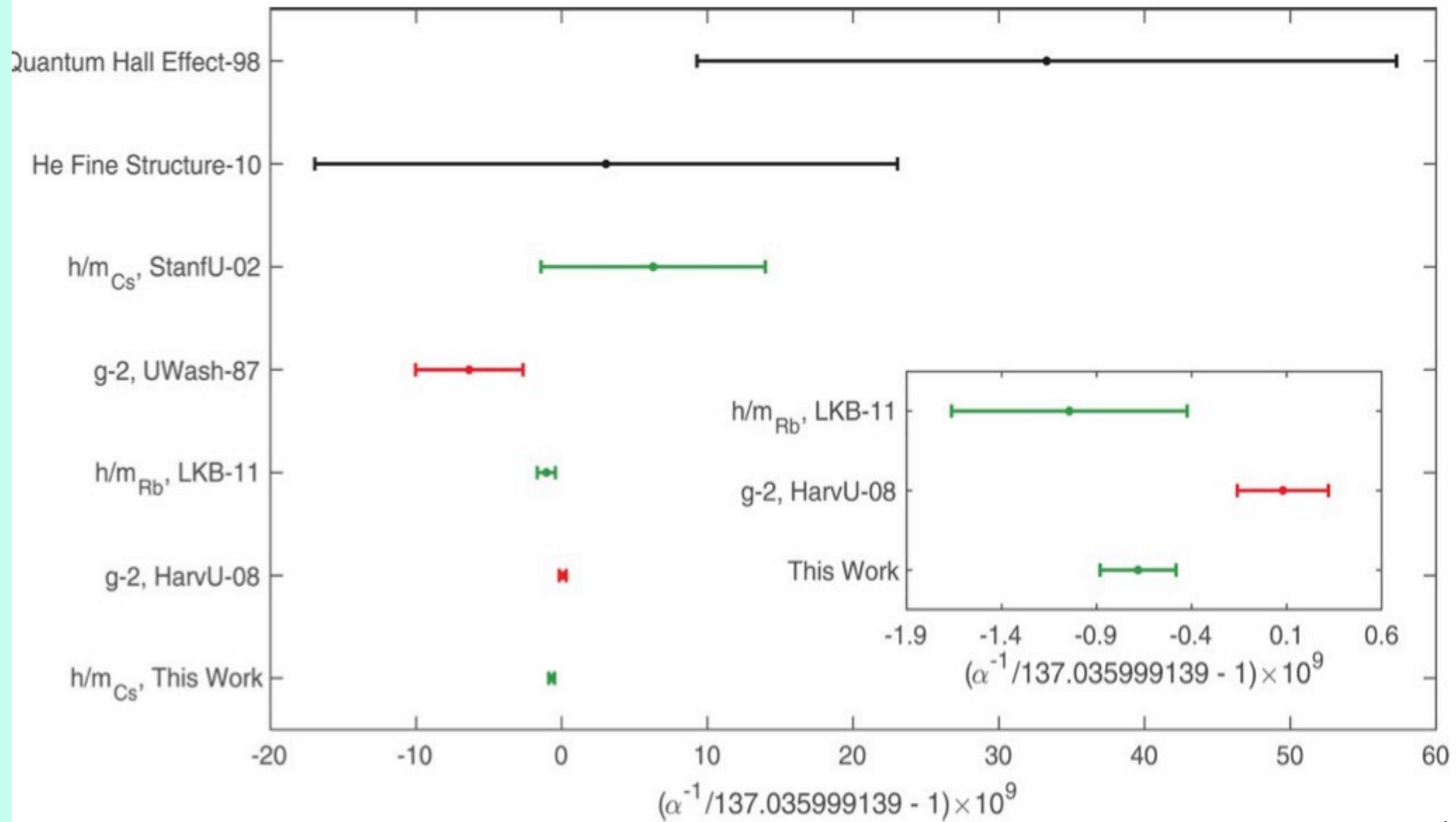
Ongoing Cs exp. Goals see H. Mueller

Future  $a_e(\text{exp})$  Improvement see G. Gabrielse

Is  $\Delta a_e$  a harbinger of “New Physics”?

eg. Light Scalar loop effect see H. Davoudiasl

New Measurement of alpha:  
 From Parker et al. Science 2018  
 $\alpha^{-1}(^{133}\text{Cs})=137.035999046(27)$



# Muon Standard Model Prediction

$$a_{\mu}^{\text{SM}} = a_{\mu}^{\text{QED}} + a_{\mu}^{\text{EW}} + a_{\mu}^{\text{Hadronic}} \text{ (quark/gluon loops)}$$

## QED Contributions:

- $a_{\mu}^{\text{QED}} = 0.5(\alpha/\pi) + 0.765857425(\alpha/\pi)^2 +$   
 $24.05050996(\alpha/\pi)^3 +$   
 $130.8796(\alpha/\pi)^4 +$   
 $752.173(\alpha/\pi)^5 + \dots$

## 2018 Update: Aoyama, Kinoshita, & Nio

$$\alpha^{-1}(^{133}\text{Cs}) = 137.035999046(27) \text{ from } h/m_{\text{Cs}}$$

$$a_{\mu}^{\text{QED}} = \underline{116584718.92(3)} \times 10^{-11} \text{ Very Precise!}$$

## Electroweak Loop Effects

$a_{\mu}^{EW}(1 \text{ loop}) = \underline{194.8 \times 10^{-11}}$  original goal of E821

$a_{\mu}^{EW}(2 \text{ loop}) = \underline{-41.2(1.0) \times 10^{-11}}$  (Higgs Mass = 125 GeV)

Approx. ~1780 diagrams

3 loop EW leading logs very small  $O(10^{-12})$

- $a_{\mu}^{EW} = \underline{154(1) \times 10^{-11}}$  *Non Controversial? See A. Czarnecki*

- Hadronic Contributions (HVP & HLBL)

$a_{\mu}^{\text{Had}}(\text{V.P.})^{\text{LO}} = \underline{6923(37) \times 10^{-11}}$

$a_{\mu}^{\text{Had}}(\text{V.P.})^{\text{NLO+NNLO}} = -86(1) \times 10^{-11}$

$a_{\mu}^{\text{Had}}(\text{LBL}) = 105(26) \times 10^{-11}$  (Consensus?) 3 loop  $(\alpha/\pi)^3$  QCD

$a_{\mu}^{\text{SM}} = \underline{116591815(37) \times 10^{-11}}$  *(Future Improvement?)*

*Goal: Factor 2 Reduction*

*see many Following Speakers*

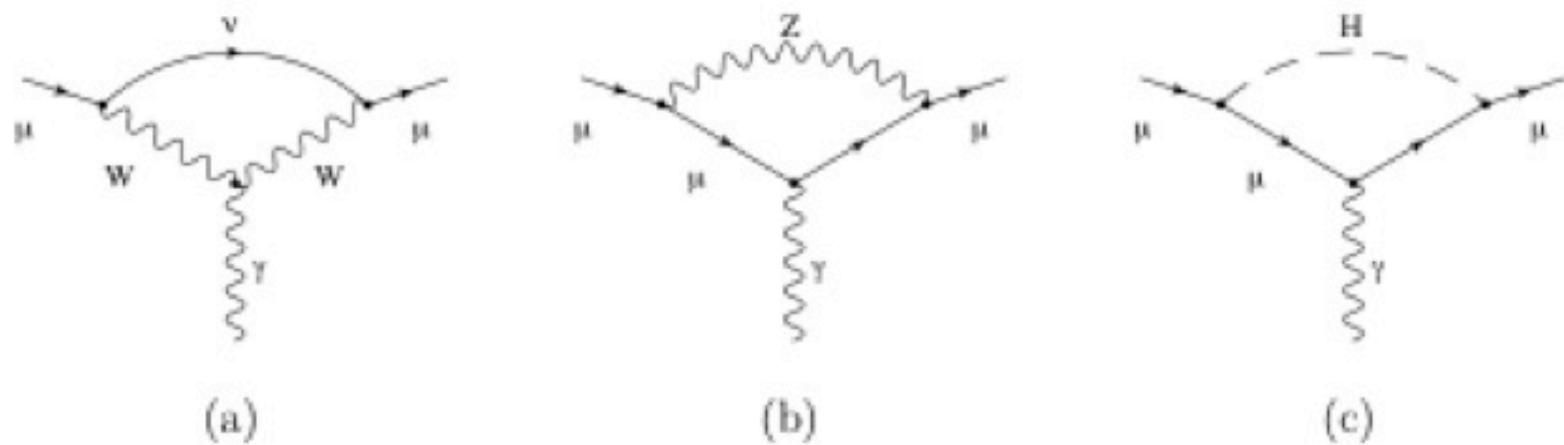


Figure 2: One-loop electroweak radiative corrections to  $a_\mu$ .

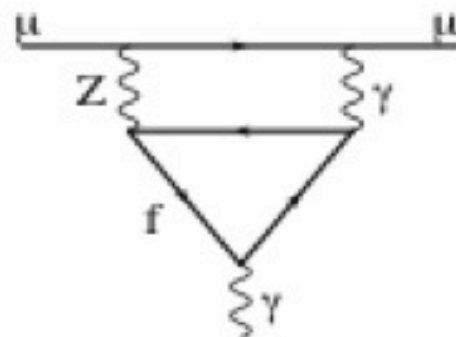
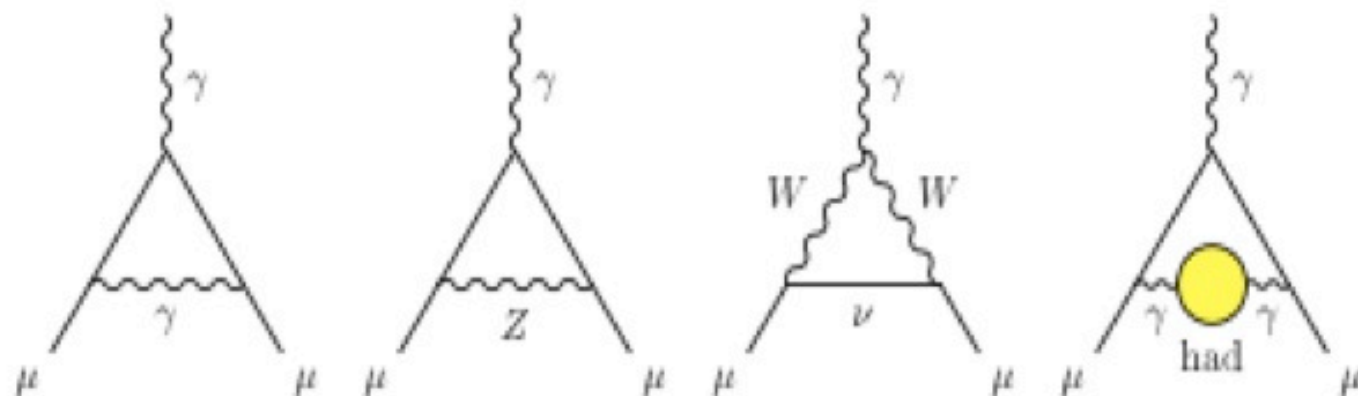


FIG. 3: Effective  $Z\gamma\gamma^*$  coupling induced by a fermion triangle, contributing to  $a_\mu^{\text{EW}}$ .



**Figure 1:** Representative diagrams contributing to  $a_{\mu}^{\text{SM}}$ . From left to right: first order QED (Schwinger term), lowest-order weak, lowest-order hadronic.

## Experimental E821 at BNL (2004 Final)

$$a_{\mu}^{\text{exp}} \equiv (g_{\mu}-2)/2 = 116592091(54)_{\text{stat}}(33)_{\text{sys}} \\ = \underline{116592091(63) \times 10^{-11}}$$

Future factor of 4 improvement expected  
see Lee Roberts

## ***Muon*** Discrepancy

$$\Delta a_{\mu} = a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = 274(63)_{\text{exp}}(37)_{\text{theory}} \times 10^{-11} \quad (\underline{3.7\sigma!})$$

***(Hadronic effects?, Experiment? or New Physics?)***

Lattice QCD comes of Age: HVP & LBL

Goal: Factor of 2 Further Improvement



## Comparison of Experiment and Theory

- $\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 274(63)(37) \times 10^{-11}$  ( $3.7\sigma!$ ) see **C. Lehner Talk**

**This is a very large deviation!**

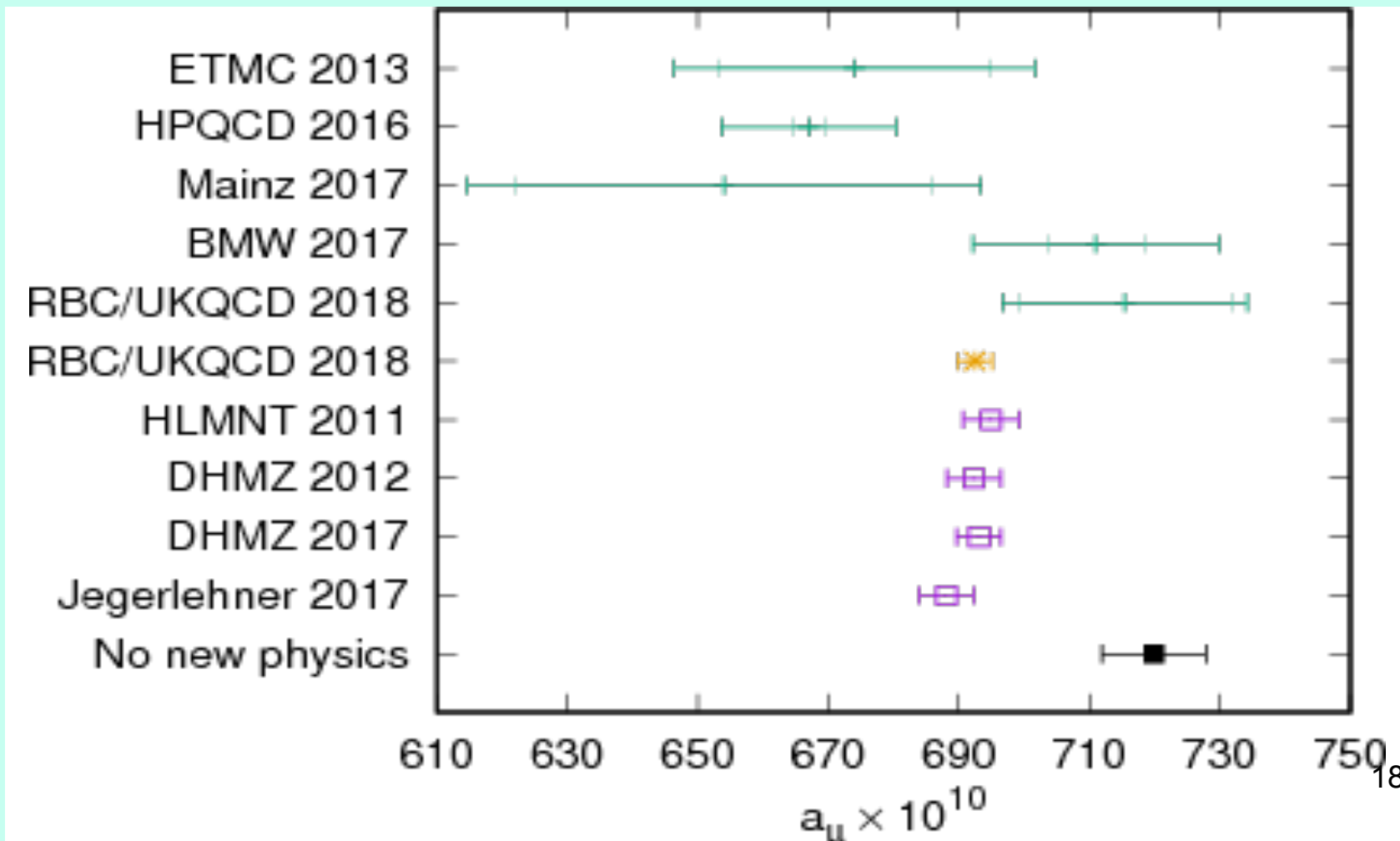
Remember, the EW contribution is only  **$154 \times 10^{-11}$**

**New Physics Nearly 2x Electroweak?**

Why don't we see it in other measurements?

“New Physics” Scale Implied  $< m_\mu / |\Delta a_\mu|^{1/2} < \mathbf{O}(\text{TeV})$  LHC?

From Blum et al. PRL 2018  
Green = Pure Lattice QCD  
Purple =  $e^+e^-$  data  
Gold = Mixed LQCD &  $e^+e^-$  data



## Some $g_{\mu}-2$ “New Physics” Scenarios

see talks by D. Stockinger & H. Davoudiasl

i) Supersymmetry - LHC Tension?

\*ii) 2 Higgs Doublet Models (new favorite)

Relatively Light (Pseudo)Scalar (2 loop  $g_{\mu}-2$ )

$$10\text{GeV} < m_A < 25\text{GeV} \quad 30 < \tan\beta = v_2/v_1 < 80$$

iii) “Light” Dark Photon

iv) Light Scalar O(0.25-few GeV)

Other

## Many Other Scenarios

*$L_\mu - L_\tau$  ,  $B-L$  etc. gauged...*

*Leptoquarks &  $B$  anomalies*

*Dynamical Mass Generation*

$\Delta a_\mu = m_\mu^2/M^2$   $M \leq 1\text{TeV!}$  *Observable!*

etc.

**Challenge**: Find dual solution to  $\Delta a_\mu$  &  $\Delta a_e$ ?

Davoudiasl & WJM 1 & 2 loop scalar effects

Liu, Wagner & Wang Complex Scalar

Other?

## “Light” Higgs

CHEN, DAVOUDI ASL, MARCIANO, ZHANG (2016)

Batell, Lange, McKeen, Pospelov, Ritz (2016)

Light scalar  $\phi \sim O(\text{GeV})$  possible remnant of  $U(1)_d$  breaking

Coupling to  $\mu^+\mu^- \sim 10^{-3}$  solves  $g_\mu - 2$  discrepancy

Why such a relatively large coupling (Higgs size)?

Misalignment (Mass &  $H\mu\mu$ ) due to several sources of  $\mu$  mass!

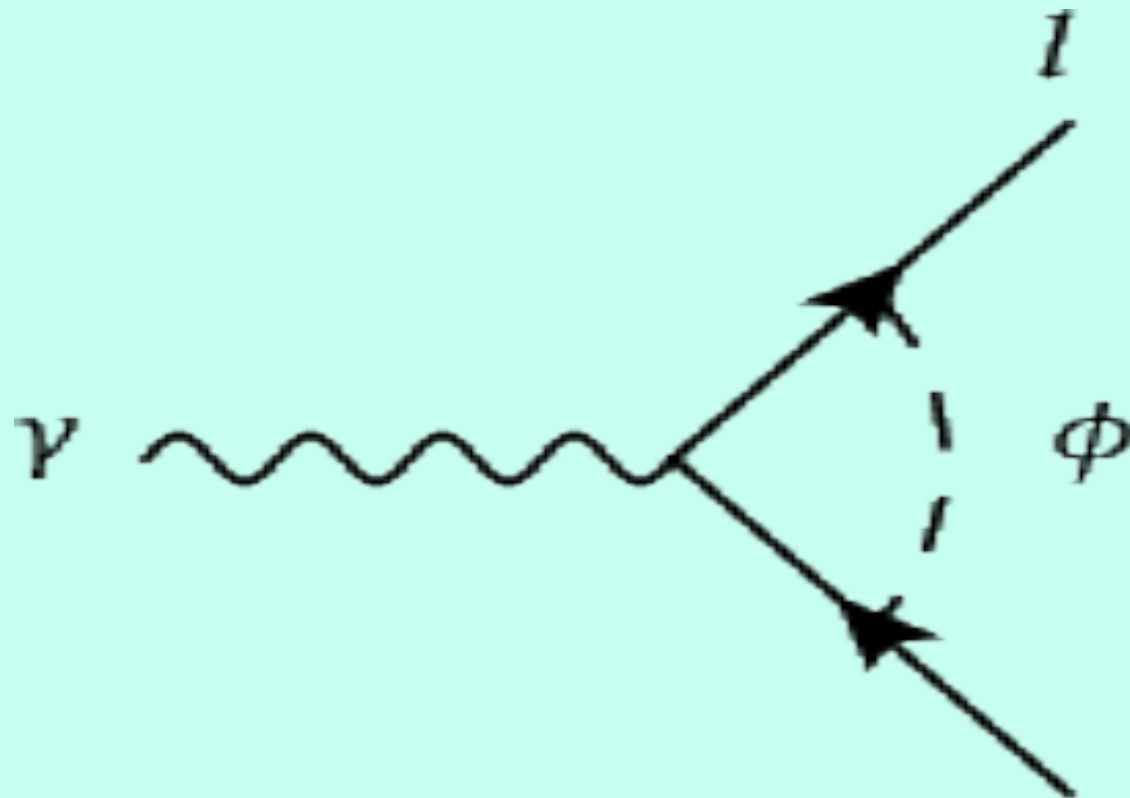
SM Higgs + Heavy charged lepton mixing effects + ...

or Multi-Higgs Mixing (2 doublets + singlet)

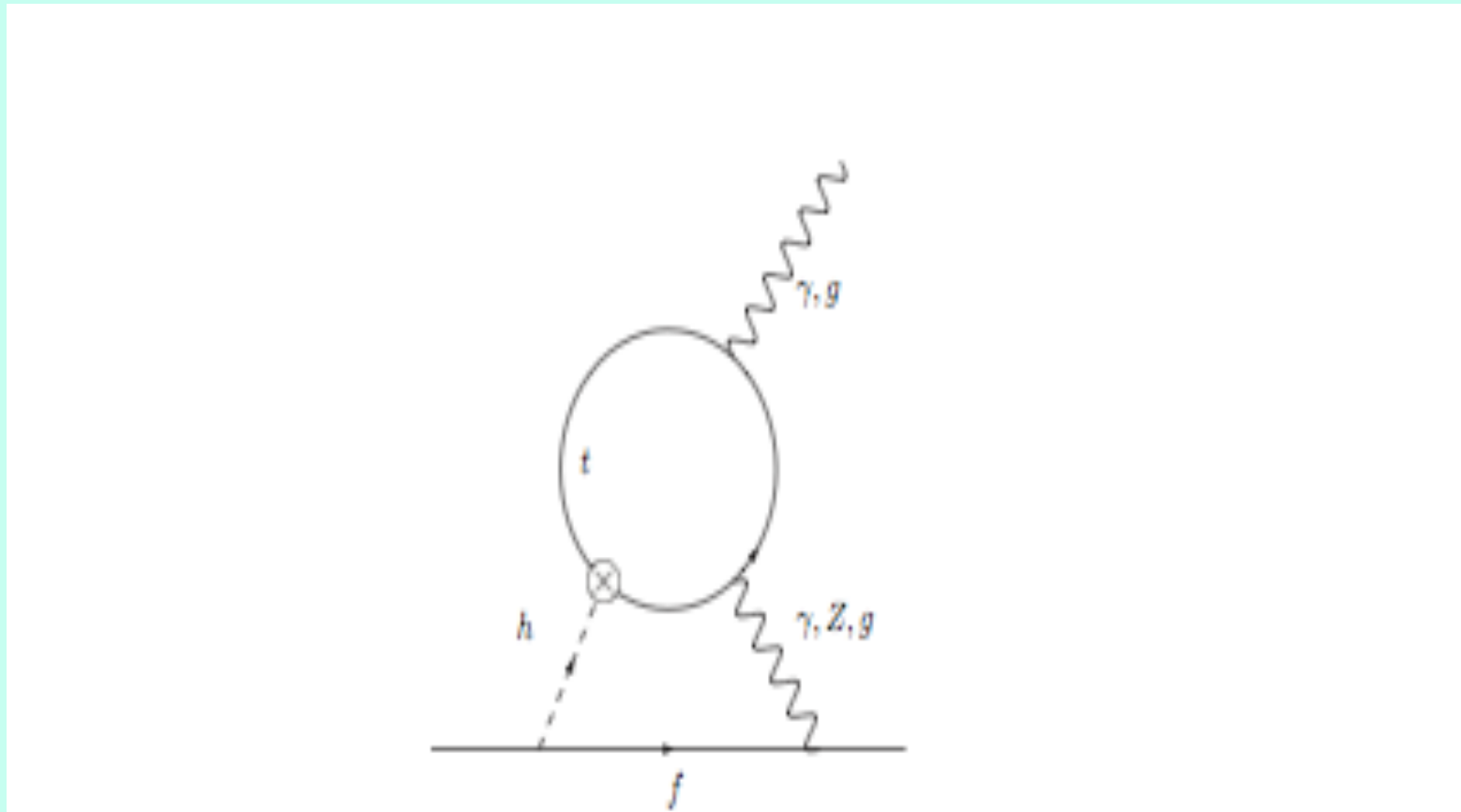
Signatures: tau decays +  $\phi \rightarrow \mu^+\mu^-$  or  $e^+e^-$  BR <  $10^{-6}$

$e^+e^- \rightarrow \tau^+\tau^-$  +  $\phi \rightarrow \mu^+\mu^-$  or  $e^+e^-$  (see Batell et al.)

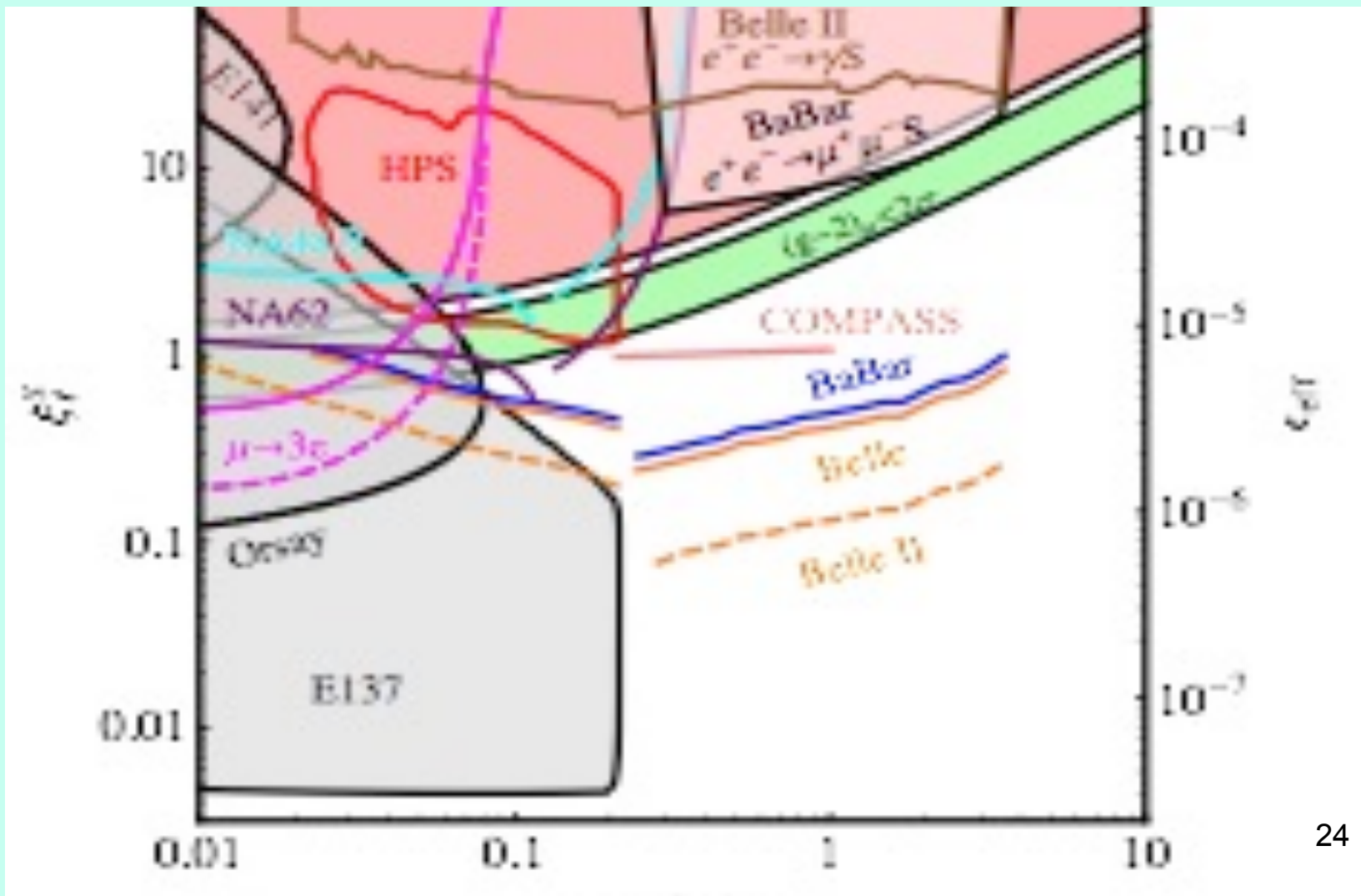
One Loop Dark Higgs ( $m_\phi < 1\text{ GeV}$ ) Contribution to Dipole Moments  
Scalar  $\Phi$  ( $0^{++}$ ) solves  $\Delta a_\mu$  discrepancy for  $\Phi\mu\mu$  coupling  $\sim 10^{-3}$



# Two Loop Higgs Contribution to fermion Dipole Moments Barr-Zee Diagrams



From Batell et al. (2016)  
Constraints on  $S_{\mu\mu}$  coupling and allowed muon  $g-2$  band  
Possible future bounds





# Outlook

Precision  $a_e$ , alpha and  $a_\mu$  test SM and Probe “New Physics”  
currently

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 274(63)(37) \times 10^{-11} \quad (3.7\sigma)$$

$$\Delta a_e = a_e^{\text{exp}} - a_e^{\text{SM}} = -87(36) \times 10^{-14} \quad (\text{Note Sign}) \quad (2.4\sigma)$$

$$\alpha^{-1}(^{133}\text{Cs}) = 137.035999046(27)$$

*All should be pushed as far as possible*

Crucial Improvements: Hadronic Corrections to  $g_\mu - 2$

Lattice + DR +  $e^+e^-$  data

$a_e^{\text{exp}}$

$\alpha$  exp (theory corrections?)

$a_\mu^{\text{exp}}$  FNAL Results 2019

*Continuing the Legacy of Schwinger*