First to the Top: Julian Schwinger's Calculation of the Anomalous Magnetic Moment of the Electron

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$$\frac{g}{2} - 1 = \frac{\alpha}{2\pi}$$

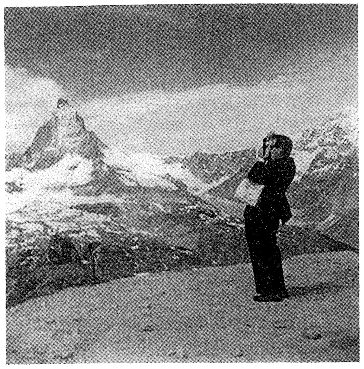
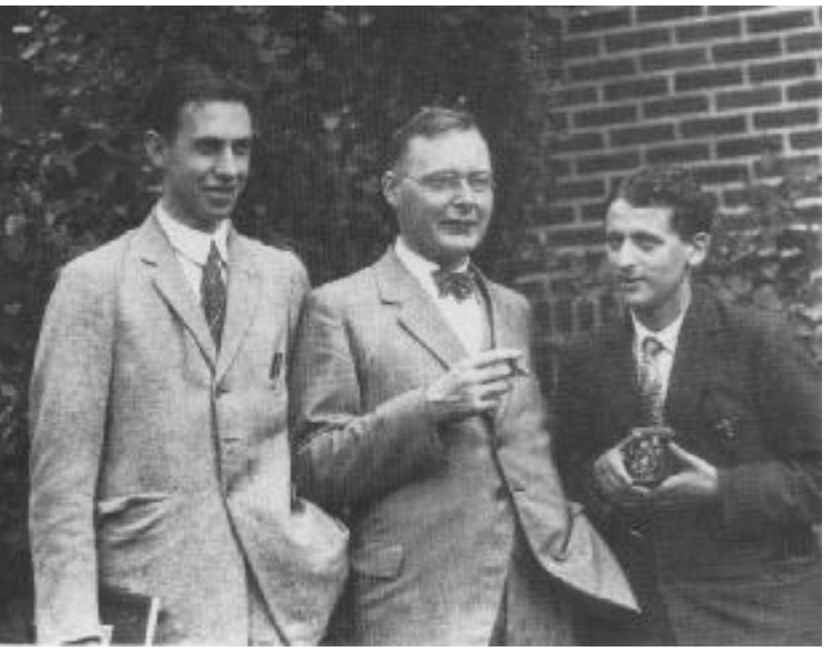


Plate 30
photogra
Zermatt,
1949.

The spin of the electron

Goudsmit and Uhlenbeck proposed that the electron had spin-½ (1925), carrying a magnetic moment of one Bohr magneton, $e\hbar/2mc$.

Factor of 2 discrepancy explained by Thomas precession.



Robert Kronig can come up with idea before, but was shot down by his advisor Pauli! Thomas: "The infallibility of the Deity does not extend to his self-styled vicar on earth."

George Uhlenbeck, Hendrik Kramers, Sam Goudsmit, 1928, Ann Arbor. All students of Ehrenfest.

Dirac Equation, 1928

$(\gamma\, p+m)\psi=0$

Dirac proposed a first-order equation consistent with relativity to describe the electron: It predicted that the electron had spin $S = \hbar/2$ and had a g-factor of 2.

$$\mu = g\, S\, (e/2m)$$

The Dirac theory predicted that the 2s_1/2 and 2p_1/2 levels would be degenerate, since the energies depended on principal quantum number and j, the total angular momentum quantum number.

Early hints that the Dirac equation w incomplete:

illman and Kusch (1940) obtained results for clear magnetic moments based on *g* for the ectron being exactly 2, but these nuclear *g*-ctors were consistently 0.12% higher than those und in previous work.

Polykarp Kusch

ouston, Williams, and Pasternack (1937-38) ggested that the Balmer doublets in H, D, and e+ were split:

S_1/2 was higher than 2^2 P_1/2 by 0.03 cm.

t there was no consensus about these small fects, and they were apparently not taken riously by the community.

Goudsmit and Simon Pasternack

Shelter Island (June 1947) and the Lamb shift and g-2

Lamb, Darrow, Weisskopf, Uhlenbeck, Marshak, Schwinger, Bohm. Seated: Oppenheimer, Pais, Feynman, Feshbach

xperiments of Nafe, Nelson, and Rabi; Nagel, lian, and Zacharias; and of Kusch and Foley stablished that $g \neq 2$, and Lamb and Retherford howed that the "Pasternack effect" existed as he "Lamb shift.'

he latter was known to the participants eforehand, but the former was a surprise.

Hans Bethe’s nonrelativistic calculation of Lamb shift.

Schwinger, Weisskopf, and Oppenheimer had suggested at Shelter Island that the energy displacements were due to the interaction of the electron with the electromagnetic radiation field.

Bethe on the train back to Schenectady wrote his famous paper, in which he argued that the cutoff to the logarithmic divergence was the electron mass; the result was about a 1000 Mc shift.

Mass and charge renormalization were explicit; however, the finite parts of the self-energy were not covariant.

Weisskopf was unhappy: he felt he should have been a co-author on the paper on the ``Pasternack effect.’’

Relativistic calculations proved more challenging

Both Feynman and Schwinger made the same mistake; they failed to correctly patch the relativistic and nonrelativistic parts of the contribution of longitudinal photons.

Dyson: Schwinger first detected the incorrect insertion of photon mass.

In fact, French and Weisskopf had it right first, but the reputation of Feynman and Schwinger caused a delay in their publication until after that of Kroll and Lamb in 1949. [FW, PR **75,** 1240 (1949);KL, PR **75,** 388 (1949)]

Feynman apologized for this delay in his famous Footnote 13, while Schwinger did not; but in fact, Schwinger never published his results at the time.

After Shelter Island

- Schwinger returned to Cambridge ill, but this did not delay his marriage to Clarice Carrol. (He stopped smoking because of his illness.)
- They then took off on a two month road-trip honeymoon to the West Coast.
- Only on their return did Julian start to seriously think about QED calculations.
- Feynman, French, Weisskopf, and others concentrated on the Lamb shift, but Julian fixed on the anomaly in the magnetic moment.

Washington Conference, November 1947

Schwinger described his yet uncompleted work on *g-2,* and on level shifts.

Feynman was there and said “the meeting … was very poor. … The only interesting thing was something that Schwinger said at the end of the meeting. … He did point out that … the discrepancy in the hyperfine structure of hydrogen noted by Rabi can be explained on the same basis as that of electromagnetic self-energy, as can the line shift of Lamb.”

On Quantum-ElectrodynamIcs and the MagnetIc Moment of the Electron
. Schwinger, Harvard University, Cambridge, Massachusetts, December 30, 1947
Physical Review **73**, 416

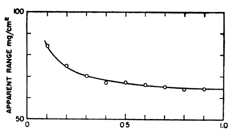


FIG. 2. Feather plot for Ca[45].

12,000 counts per minute, and the contribution due to gamma-rays and other unabsorbed contaminants was less than one part in 3000 with the strongest source, thus indicating the absence of any appreciable amount of gamma-radiation. The absorption curve obtained with the strongest source is shown in Fig. 1. The Feather plot, shown in Fig. 2, gives a range of 64 ± 1 mg/cm².

Glendenin[4] has shown that a reliable range-energy curve for the low energy region can be derived from the data of Marshall and Ward[5] for monoenergetic electrons and beta-ray spectrograph data on low energy beta-emitters. Glendenin's curve is identical with that of Marshall and Ward below 0.5 Mev. Using this range-energy curve, we have found that the Ca^{45} beta-radiation has a maximum energy of 260 ± 5 kev. We have found no evidence of any harder beta-radiation, or of any gamma-radiation at all in the course of this investigation.[6]

Acknowledgments.—This work has been supported with funds from the Office of Naval Research. The authors wish to express their appreciation to Miss Jacqueline Becker for her assistance in making the counts.

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[2] Solomon, Gould, and Anfinsen, Phys. Rev. **72**, 1097 (1947).
[3] Feather, Proc. Camb. Phil. Soc. **35**, 599 (1938).
[4] Glendenin, Nucleonics, in press for January, 1948.
[5] Marshall and Ward, Can. J. Research **15**, 29 (1939).
[6] This result is in good agreement with a value of 250 kev, given in *Radioisotopes, Catalog and Price List No. 2*, revised September, 1947, distributed by Isotopes Branch, United States Atomic Energy Commission. Unfortunately, the Atomic Energy Commission's result is not supported by any published experimental evidence.

On Quantum-Electrodynamics and the Magnetic Moment of the Electron

JULIAN SCHWINGER
Harvard University, Cambridge, Massachusetts
December 30, 1947

ATTEMPTS to evaluate radiative corrections to electron phenomena have heretofore been beset by divergence difficulties, attributable to self-energy and vacuum polarization effects. Electrodynamics unquestionably requires revision at ultra-relativistic energies, but is presumably accurate at moderate relativistic energies. It would be desirable, therefore, to isolate those aspects of the current theory that essentially involve high energies, and are subject to modification by a more satisfactory theory, from aspects that involve only moderate energies and are thus relatively trustworthy. This goal has been achieved by transforming the Hamiltonian of current hole theory electrodynamics to exhibit explicitly the logarithmically divergent self-energy of a free electron, which arises from the virtual emission and absorption of light quanta. The electromagnetic self-energy of a free electron can be ascribed to an electromagnetic mass, which must be added to the mechanical mass of the electron. Indeed, the only meaningful statements of the theory involve this combination of masses, which is the experimental mass of a free electron. It might appear, from this point of view, that the divergence of the electromagnetic mass is unobjectionable, since the individual contributions to the experimental mass are unobservable. However, the transformation of the Hamiltonian is based on the assumption of a weak interaction between matter and radiation, which requires that the electromagnetic mass be a small correction ($\sim(e^2/\hbar c)m_0$) to the mechanical mass m_0.

The new Hamiltonian is superior to the original one in essentially three ways: it involves the experimental electron mass, rather than the unobservable mechanical mass; an electron now interacts with the radiation field only in the presence of an external field, that is, only an accelerated electron can emit or absorb a light quantum;* the interaction energy of an electron with an external field is now subject to a *finite* radiative correction. In connection with the last point, it is important to note that the inclusion of the electromagnetic mass with the mechanical mass does not avoid all divergences; the polarization of the vacuum produces a logarithmically divergent term proportional to the interaction energy of the electron in an external field. However, it has long been recognized that such a term is equivalent to altering the value of the electron charge by a constant factor, only the final value being properly identified with the experimental charge. Thus the interaction between matter and radiation produces a renormalization of the electron charge and mass, all divergences being contained in the renormalization factors.

The simplest example of a radiative correction is that for the energy of an electron in an external magnetic field. The detailed application of the theory shows that the radiative correction to the magnetic interaction energy corresponds to an additional magnetic moment associated with the electron spin, of magnitude $\delta\mu/\mu = (\frac{1}{2}\pi)e^2/\hbar c = 0.001162$. It is indeed gratifying that recently acquired experimental data confirm this prediction. Measurements on the hyperfine splitting of the ground states of atomic hydrogen and deuterium[1] have yielded values that are definitely larger than those to be expected from the directly measured nuclear moments and an electron moment of one Bohr magneton. These discrepancies can be accounted for by a small additional electron spin magnetic moment.[2] Recalling that the nuclear moments have been calibrated in terms of the electron moment, we find the additional moment necessary to account for the measured hydrogen and deuterium hyperfine structures to be $\delta\mu/\mu = 0.00126 \pm 0.00019$ and $\delta\mu/\mu = 0.00131 \pm 0.00025$, respectively. These values are not in disagreement with the theoretical prediction. More precise conformation is provided by measurement of the g values for the $^2S_{\frac{1}{2}}$, $^2P_{\frac{1}{2}}$, and $^2P_{3/2}$ states of sodium and gallium.[3] To account for these results, it is necessary to ascribe the following additional spin magnetic moment to the electron, $\delta\mu/\mu = 0.00118 \pm 0.00003$.

The radiative correction to the energy of an electron in a Coulomb field will produce a shift in the energy levels of hydrogen-like atoms, and modify the scattering of electrons in a Coulomb field. Such energy level displacements have recently been observed in the fine structures of hydrogen,[4] deuterium, and ionized helium.[5] The values yielded by our theory differ only slightly from those conjectured by Bethe[6] on the basis of a non-relativistic calculation, and are, thus, in good accord with experiment. Finally, the finite radiative correction to the elastic scattering of electrons by a Coulomb field provides a satisfactory termination to a subject that has been beset with much confusion.

A paper dealing with the details of this theory and its applications is in course of preparation.

* A classical non-relativistic theory of this type was discussed by H. A. Kramers at the Shelter Island Conference, held in June 1947 under the auspices of the National Academy of Sciences.
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[5] J. E. Mack and N. Austern, Phys. Rev. **72**, 972 (1947).
[6] H. A. Bethe, Phys. Rev. **72**, 339 (1947).

Excitation Curves of (α, n); $(\alpha, 2n)$; $(\alpha, 3n)$ Reactions on Silver

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January 5, 1948

SILVER bombarded with α-particles from the 60-in. cyclotron produces radioactive substances with the following three half-lifes: 65 min., 5.2 hr., and 2.7 d. All of these activities have been chemically attributed to indium and have been assigned by mass-spectrograph separation to In^{110}, In^{109}, and In^{111}, respectively. Tendam and Bradt[1] recently announced similar activities. Their assignment of 65-min. and 2.7-d activities agrees with ours. The 23-min. activity found by them was not looked for in the present experiment.

The excitation curves for the isotopes reported above have been determined for α-energies up to 37 Mev and are reproduced in Fig. 1. The abscissae give the energy in Mev, the ordinates the cross sections in arbitrary units. The ordinate units are, however, the same for reactions leading to the formation of the same isotope. Evaluation of absolute cross sections has not yet been possible due to lack of knowledge regarding the efficiencies of the different radiations for the ionization chamber used.

From the figure it is seen that the 65-min. activity belonging to In^{110} (emitting positrons of 1.7 Mev), a product of $Ag^{107}(\alpha, n)$ reaction, has a threshold of 11 Mev.* The yield after attaining a peak at 17.5 Mev drops rapidly to low values when the $(\alpha, 2n)$ process appears as a competing process. After attaining a minimum, the 65-min. activity again increases and does not reach saturation even at 37 Mev. Apparently this part of the curve is due to $Ag^{109}(\alpha, 3n)In^{110}$. The sharpness of the peak at 17.5 Mev is also interesting. The difference of 4 Mev between (α, n) and $(\alpha, 2n)$ thresholds is much smaller than that between $(\alpha, 2n)$ and $(\alpha, 3n)$ thresholds ($\sim$8 Mev). This difference seems to be due to the Coulomb barrier which cuts off the production of any alpha-reaction below 11 Mev.

The 2.7-d activity belonging to In^{111} has a threshold of about 15 Mev, which is in agreement with that found by Tendam and Bradt.[1] This activity is produced by the $Ag^{109}(\alpha, 2n)$ process, and emits a γ-ray of about 0.2 Mev (no positrons). After attaining a peak around 27 Mev, the yield begins to drop and reaches about 16 percent of maximum at 37 Mev.

The 5.2-hr. period is produced by $Ag^{107}(\alpha, 2n)In^{109}$ reaction. The excitation curve is similar to the excitation curve of In^{111}, as is expected, since both are products of $(\alpha, 2n)$ reactions. The threshold of In^{109} is about 13.5 Mev, slightly lower than that of In^{111}. At higher energies, however, the two curves differ widely. Instead of decreasing, the 5.2-hr. curve goes on increasing even beyond 30 Mev, after which it drops slightly, the yield at 37 Mev being 80 percent of the maximum.

This suggests the production of a different isotope at higher α-energies having a very similar half-life. A comparison with the $Ag^{109}(\alpha, 3n)In^{110}$ curve and with the $Ag^{109}(\alpha, 2n)In^{111}$ curve suggests that this new activity is probably due to $Ag^{107}(\alpha, 3n)In^{108}$. The possibility of its being due to $Ag^{109}(\alpha, 3n)In^{110}$ (an isomer of 65-min. period) is ruled out by the fact that the threshold and low energy part of the curve is similar to the other $(\alpha, 2n)$ curve and not to the (α, n) curve.

To verify this conclusion, two foils were bombarded, one with 37-Mev alphas (foil 1) and the other with 20-Mev alphas (foil 2). The latter is not likely to have any In^{108} in it, while the former should mostly contain In^{108} with little In^{109}. The absorption curves for the radiations from the two foils, corrected for In^{111}, showed marked differences. Foil 1 showed a γ-ray of about 0.65 Mev, while foil 2 showed a γ-ray of about 0.5 Mev. No positrons were detected. These conclusions were also corroborated by

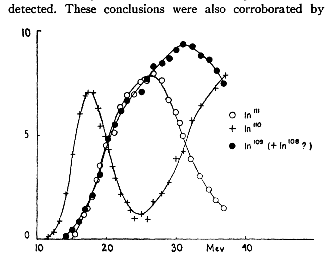


FIG. 1. The abscissa represents energy of the bombarding α-particles in Mev. The ordinate represents cross section in arbitrary units. The curve with open circles represents the cross section for the formation of In^{111}. The one with crosses represents the cross section for the formation of In^{110}, while the curve with solid circles represents the cross section for the formation of In^{109}, and at the higher energies probably of In^{108} also.

As stated in the Letter to the Editor:

`The detailed application of the theory shows that the radiative correction to the magnetic interaction energy corresponds to an additional magnetic moment associated with the electron spin, of magnitude $\delta\mu/\mu = (½ \pi)e^2/\hbar c = 0.001162$. It is indeed gratifying that recently acquired experimental data confirm this prediction."

Note the misprint: it should be $1/(2\pi)$.

Schwinger goes on to compare with the hyperfine splittings seen in hydrogen, deuterium, and most precisely for Na and Ga.

He notes that in the process of obtaining these results, mass and charge renormalization must be effected.

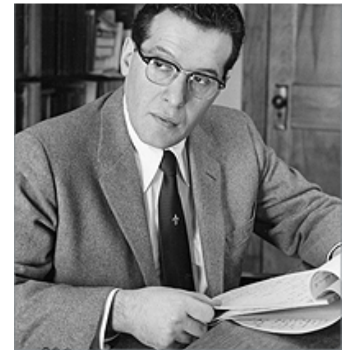
Lamb shift and electron scattering

He ends the note by remarking that he has calculated the shift of energy levels in hydrogen-like atoms, and the modification of scattering of electrons in a Coulomb field.

Agreement with the Lamb-Retherford experiment and with Bethe's calculation is noted, but no results are given: "A paper dealing with the details of this theory and its applications is in course of preparation."

But before such a paper could be written, Schwinger completely reformulated his theory.

nuary 1948 New York APS eeting (Columbia)



- Schwinger reported on his results in Quantum Electrodynamics in a one-hour lecture.
- The talk was met with such acclaim, but given in such a small room (K.K. Darrow, secretary of the APS, did not have much regard for theory), that it had to be repeated two more times, in successively larger venues.
- Feynman announced after Schwinger’s lecture that he had obtained all the same results, but that he found, unlike Schwinger, that the magnetic moment of an electron in an atom was the same as that of a free electron. (Schwinger had not done a covariant calculation.)
- There is some doubt Feynman had a complete calculation by that date, but Feynman was right.
- After the talk, Rabi encountered Victor LaMer in the Faculty Club elevator and started speaking about this sensational revolution. LaMer asked who was the speaker? Rabi: ``Oh, you know him, you gave him an F, Julian Schwinger!”

Schwinger had two advantages over Feynman

He knew from his work with Oppenheimer at Berkeley that vacuum polarization existed: “On pair creation in the proton bombardment of fluorine,” PR **56,** 1066 (1939). [Feynman initially doubted that “loop” corrections existed for the photon propagator, since his method did not seem to render them finite.]

He had extensively understood relativistic self-energy effects at the classical level from his work on radar and synchrotron radiation at the MIT Radiation Lab during the War, which would give him a head start in dealing with the problems of self-energy in quantum electrodynamics.

Pocono Conference, March 30-April 2, 1948

Schwinger presented in the morning, Feynman in the afternoon.

In spite of questions and objections from Bohr, Teller, Dirac, etc., Schwinger proceeded on for hours, saying “perhaps it will become clearer if I proceed.’’

The same trick did not work for Feynman. “What about the exclusion principle?” Feynman: “It doesn’t make any difference for intermediate states.” Teller: “It is fundamentally wrong!”

But afterwards, Schwinger and Feynman compared notes, and although they couldn’t understand each other’s methods, they always got the same answers, so both knew they were on the right track!

Schwinger's lecture

Shin'inchiro Tomonaga

- Schwinger presented a covariant approach based on the Tomonaga-Schwinger equation

$$i\,\hbar c\;\delta\Psi(\sigma)/\delta\sigma(\boldsymbol{x}) = H(\boldsymbol{x})\Psi(\sigma)$$

where $\sigma(\boldsymbol{x})$ is a spacelike surface, and $H(\boldsymbol{x})$ is the interaction Hamiltonian density.

- Through a series of canonical transformations, he was able to calculate the Lamb shift and the anomalous magnetic moment of the electron.
- This was the genesis of his "Quantum Electrodynamics" series: PR **74,** 1439 (1948**); 75,** 651 (1949); **76,** 790 (1949).

Reaction in Chicago

Fermi, Teller, and Wentzel went to Pocono; for once Fermi took voluminous notes.

After they returned, they met with graduate students, Chew, Goldberger, Rosenbluth, and Yang, and spent 6 weeks trying to digest Fermi's notes of Schwinger's lecture.

Yang: afterwards "we were all very tired, and none of us felt we had understood what Schwinger had done. We only knew he had done something brilliant."

``Didn't Feynman also talk?" All that could be remembered was his strange notation, *p* with a slash through it.

Michigan Summer School, July-August 1948

- Schwinger’s lectures, notes of which are extant in the UCLA archives, largely parallel the Quantum Electrodynamics papers.
- There he gave his (incorrect) result for the Lamb shift:

$$\Delta E \sim \mathit{(Bethe\ log)} - \ln 2 + 3/8 - 1/5 + 1/2$$

vhere the first term is the Bethe logarithm, -1/5 corresponds to acuum polarization, and the ½ is the magnetic moment effect, ow correctly incorporated. 3/8 should have been 5/6, as rench and Weisskopf first found. Schwinger found his error a ew months later.

- That was where he first encountered Dyson, who would go on to explicitly demonstrate the equivalence of Feynman’s and Schwinger’s approaches.

"Gauge Invariance and Vacuum Polarization"

Before he reformulated QED a third time, Schwinger published his most cited paper, PR **82,** 664 (1951).

Besides setting forth QED in external electromagnetic fields, including the "Schwinger effect," and giving the first clear statement of the axial-vector anomaly (anticipating Adler, Bell, and Jackiw by 2 decades), he gave what he thought was the shortest derivation of the anomalous magnetic moment of the electron in an Appendix. He uses a proper-time method to give a less than one-page derivation of $\alpha/2\pi$.

APPENDIX B

An electron in interaction with its proper radiation field, and an external field, is described by the modified Dirac equation,[10]

$$\gamma_\mu(-i\partial_\mu - eA_\mu(x))\psi(x) + \int (dx')M(x,x')\psi(x') = 0. \quad (B.1)$$

To the second order in e, the mass operator, $M(x,x')$, is given by

$$M(x,x') = m_0\delta(x-x') + ie^2\gamma_\mu G(x,x')\gamma_\mu D_+(x-x'). \quad (B.2)$$

Here $G(x,x')$ is the Green's function of the Dirac equation in the external field, and $D_+(x-x')$ is a photon Green's function, expressed by

$$D_+(x-x') = (4\pi)^{-2}\int_0^\infty dt t^{-2} \exp[i\tfrac{1}{4}(x-x')^2/t]. \quad (B.3)$$

We shall suppose the external field to be weak and uniform. Under these conditions, the transformation function $(x(s)|x(0)')$, involved in the construction of $G(x,x')$, may be approximated by

$$(x(s)|x(0)') \simeq -i(4\pi)^{-2}\Phi(x,x')s^{-2} \times \exp[i\tfrac{1}{4}(x-x')^2/s]\exp(i\tfrac{1}{2}e\sigma F); \quad (B.4)$$

that is, terms linear in the field strengths enter only through the Dirac spin magnetic moment. The corresponding simplification of the Green's function, obtain by averaging the two equivalent forms in Eq. (3.21), is

$$G(x,x') \simeq (4\pi)^{-2}\Phi(x,x')\int_0^\infty ds s^{-2}\exp(-im^2 s) \times \exp[i\tfrac{1}{4}(x-x')^2/s]\tfrac{1}{2}\left\{\frac{-\gamma(x-x')}{2s}+m, \exp(i\tfrac{1}{2}e\sigma F)\right\}. \quad (B.5)$$

The mass operator is thus approximately represented by

$$M(x,x') = m_0\delta(x-x') + [ie^2/(4\pi)^4]\Phi(x,x')\int_0^\infty ds s^{-2}\int_0^\infty dt t^{-2} \times \exp(-im^2 s)\exp\left[i\tfrac{1}{4}(x-x')^2\left(\frac{1}{s}+\frac{1}{t}\right)\right] \times \gamma_\lambda\tfrac{1}{2}\left\{\frac{-\gamma(x-x')}{2s}+m, \exp(i\tfrac{1}{2}e\sigma F)\right\}\gamma_\lambda, \quad (B.6)$$

or

$$M(x,x') = m_0\delta(x-x') + [ie^2/(4\pi)^4]\Phi(x,x') \times \int_0^\infty ds s^{-2}\exp(-im^2 s)\int_0^s dw w^{-2}\exp[i\tfrac{1}{4}(x-x')^2/w] \times [-4m - s^{-1}\gamma(x-x') + \tfrac{1}{2}i\{\gamma(x-x'), \tfrac{1}{2}e\sigma F\}], \quad (B.7)$$

in which we have replaced t by the variable w,

$$w^{-1} = s^{-1} + t^{-1}, \quad (B.8)$$

and employed properties of the Dirac matrices, notably

$$\gamma_\lambda\sigma_{\mu\nu}\gamma_\lambda = 0. \quad (B.9)$$

We shall also write

$$\begin{aligned}(x-x')_\mu\Phi(x,x')\exp[i\tfrac{1}{4}(x-x')^2/w] &= 2w(-i\partial_\mu - eA_\mu(x) - \tfrac{1}{2}eF_{\mu\nu}(x-x')_\nu)\Phi(x,x')\exp[i\tfrac{1}{4}(x-x')^2/w] \\ &\simeq [2w(-i\partial_\mu - eA_\mu(x)) - 2w^2 eF_{\mu\nu}(-i\partial_\nu - eA_\nu(x))] \times \Phi(x,x')\exp[i\tfrac{1}{4}(x-x')^2/w], \quad (B.10)\end{aligned}$$

which gives

$$\begin{aligned}M(x,x') = m_0\delta(x-x') + [e^2/(4\pi)^2]\int_0^\infty ds s^{-2}\exp(-im^2 s) &\times \int_0^s dw[2m(2-w/s) + (2w/s)(\gamma(-i\partial - eA)+m) \\ &- 2mw(1-w/s)i\tfrac{1}{2}e\sigma F - iw(1+w/s) \\ &\times\{\gamma(-i\partial - eA)+m, \tfrac{1}{2}e\sigma F\}](x(w)|x(0)'), \quad (B.11)\end{aligned}$$

in virtue of the relation

$$[\gamma(-i\partial - eA), \tfrac{1}{2}\sigma F] = 2i\gamma F(-i\partial - eA). \quad (B.12)$$

We now introduce a perturbation procedure in which the mass operator assumes the role customarily played by the energy. To evaluate $\int(dx')M(x,x')\psi(x')$, we replace $\psi(x')$ by the unperturbed wave function, a solution of the Dirac equation associated with the mass m (we need not distinguish, to this approximation, between the actual mass m and the mechanical mass m_0). The x' integration can be effected immediately,

$$\int (x(w)|x(0)')(dx')\psi(x') = \int (x|U(w)|x')(dx')\psi(x') = \exp(im^2 w)\psi(x), \quad (B.13)$$

since $\psi(x)$ is an eigenfunction of $\mathcal{H}$, with the eigenvalue $-m^2$. Therefore, on discarding all terms containing the operator of the Dirac equation, which will not contribute to

$$\int (dx)(dx')\bar\psi(x)M(x,x')\psi(x'),$$

we obtain

$$[\gamma(-i\partial - eA) + m - \mu'\tfrac{1}{2}\sigma F]\psi = 0, \quad (B.14)$$

where

$$m = m_0 + (\alpha/2\pi)m\int_0^\infty ds s^{-1}\int_0^s dw s^{-1}(2-w/s) \times \exp[-im^2(s-w)] \quad (B.15)$$

represents the mass of a free electron, and

$$\mu' = (\alpha/2\pi)emi\int_0^\infty ds\int_0^s (dw/s)(w/s)(1-w/s) \times \exp[-im^2(s-w)] \quad (B.16)$$

describes an additional spin magnetic moment. Both integrals are conveniently evaluated by introducing

$$u = 1 - w/s, \quad (B.17)$$

and making the replacement $s \to -is$, which yields

$$\begin{aligned}m &= m_0 + (\alpha/2\pi)m\int_0^\infty ds s^{-1}\int_0^1 du(1+u)\exp(-m^2 us) \\ &= m_0 + (3\alpha/4\pi)m\left[\int_0^\infty ds s^{-1}\exp(-m^2 s) + \tfrac{5}{6}\right], \quad (B.18)\end{aligned}$$

and

$$\begin{aligned}\mu' &= (\alpha/2\pi)em\int_0^\infty ds\int_0^1 du u(1-u)\exp(-m^2 us) \\ &= (\alpha/2\pi)(e/m)\int_0^1 du(1-u) = (\alpha/2\pi)(e\hbar/2mc). \quad (B.19)\end{aligned}$$

We thus derive the spin magnetic moment of $\alpha/2\pi$ magnetons produced by second-order electromagnetic mass effects.

[10] The concepts employed here will be discussed at length in later publications.

The Dynamical Action Principle

Schwinger's 3rd reformulation of QFT began before GIVP, in terms of his Quantum Action Principle, "the quantum transcription of Hamilton's action principle." This began right after the submission of QED III in May 1949.

$$\delta\langle \zeta',\sigma_1|\zeta'',\sigma_2\rangle = (i/\hbar)\langle \zeta',\sigma_1|\delta W_{12}|\zeta'',\sigma_2\rangle$$

in terms of observables defined on spacelike hypersurfaces, σ_1 and σ_2. The changes in the action may be either kinematical or dynamical. This principle supplies not only equations of motion, but the commutation relations between the dynamical variables.

The Theory of Quantized Fields

A series of five papers bearing this name appeared between 1951 and 1954, PR **82,** 914 (1951); **91,** 713 (1953); **91,** 728 (1953); **92,** 1283 (1953); **93,** 615 (1954).

Even more important were his “Green’s functions of quantized fields, I,II,” PNAS **37,** 452, 455 (1951).

Paul Martin stated: “During this period the religion had its own golden rule---the action principle---and its own cryptic testament---”On the Green’s Functions of Quantized Fields.” Mastery of this paper conferred on followers a high priest status. The testament was couched in terms tha could not be questioned, in a language whose elements were the values of real physical observables and their correlations.”

Further comments from High Priest Martin

"The language was enlightening, but the lectures were exciting because they were more than metaphysical. Along with structural insights, succinct and implicit self-consistent methods for generating true statements were revealed. To be sure, the techniques were perturbative, but they were sufficiently potent to work when power series in the coupling failed because, for example, the coupling was strong enough to produce bound states."

"The lectures continued through Harvard's reading period and then the examination period. In one course we attended, he presented his last lecture---a novel calculation of the Lamb shift---during Commencement Week. The audience continued coming and he continued lecturing."

Fourth-order correction to g-2

This was calculated first by Robert Karplus and Norman Kroll in 1950; Schwinger used this result in his own work, but suspected it might be erroneous. (Petermann reports there was a discrepancy in the Lamb shift.) Schwinger suggested to his student Charles Sommerfield that he re-do the calculation.

Schwinger: “Interestingly enough, although Feynman-Dyson methods were applied early [by Karplus and Kroll], the first correct higher-order calculation was done by Sommerfield using [my] methods.”

Sommerfield, PR **107,** 328 (1957); Ann. Phys. (N.Y.) **5,** 20 (1958).

A. Petermann, Helv. Phys. Acta **30,** 407 (1957); Nucl. Phys. **5,** 677 (1958).

Schwinger eventually calculated g-2 to
order himself

- He devoted more than 60 pages of the third volume of *Particles, Sources, and Fields* (written in 1973, published in 1989) to a detailed calculation of the anomalous magnetic moment to 4^{th} order, now using his 4^{th} reformulation of quantum field theory, source theory.

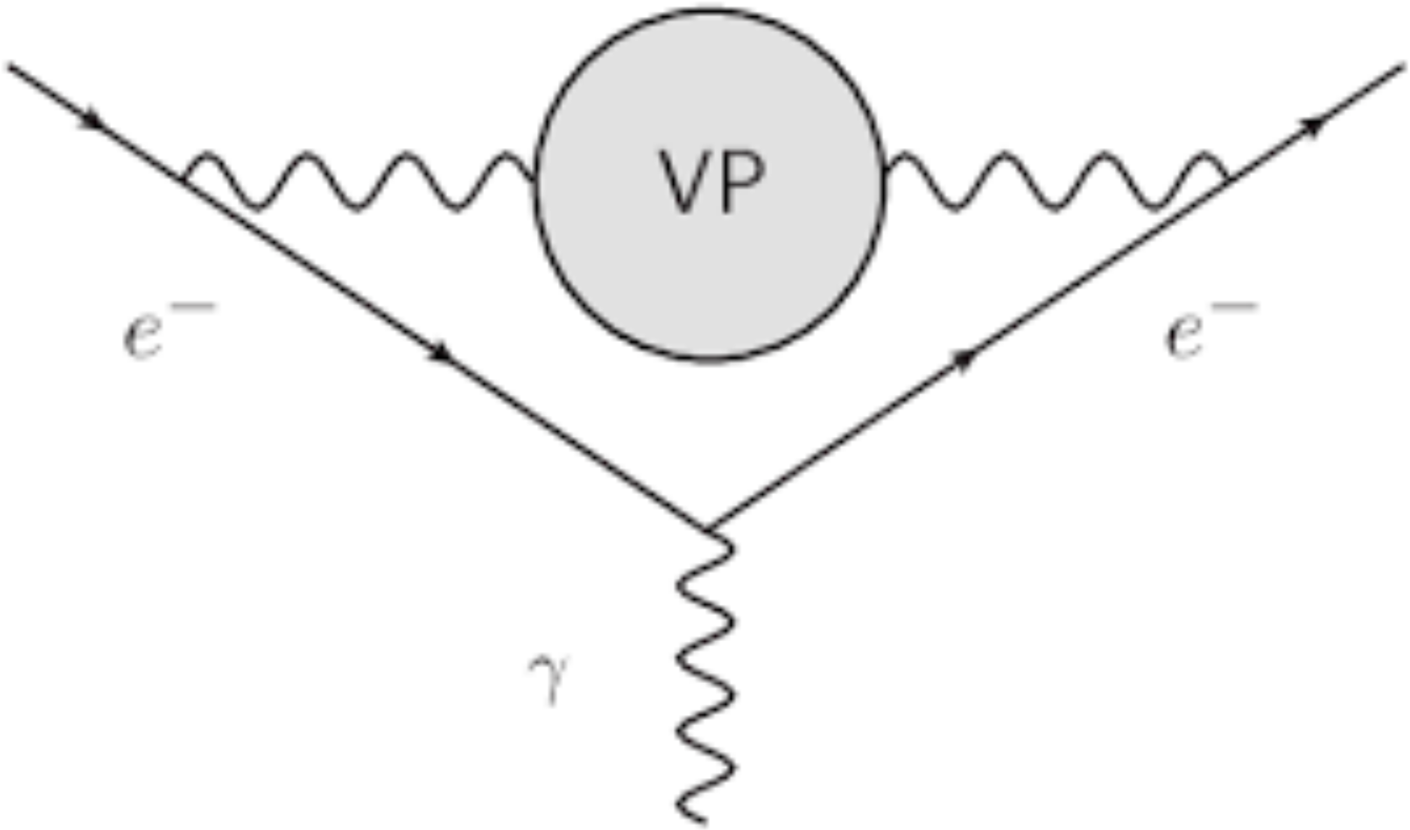
$$g/2-1 = \alpha/(2\pi)+[197/144+\pi^2/12-(\pi^2/2)\ln 2+3\,\zeta(3)/4](\alpha/\pi)^2+\ldots$$

$$= \alpha/(2\pi) - 0.328\,(\alpha/\pi)^2+ \ldots ,$$

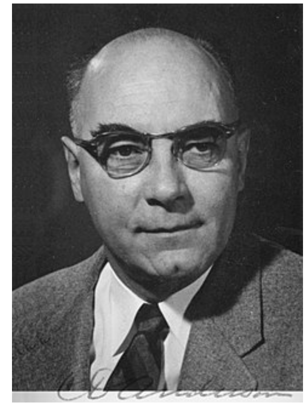
he second term being almost 10 times smaller than that found by K&K. The atter simply made an arithmetic error, but the heroism of their early alculation should not be minimized.

- Of course now the 6^{th} order and 8^{th} order are known analytically. (Remidd and Laporta). 10^{th} order is well estimated by Kinoshita's group.

Vacuum polarization comes in at 4th order



Discovery of the muon



- Discovered by Anderson and Neddermeyer in 1937, it was initially identified with Yukawa's mesotron, carrier of the strong force between nucleons. Of course, the interactions were all wrong!
- Not till 1947 was the pion discovered, and so the muon became just a heavy electron. Rabi: "Who ordered that?"

- We still don't know why the world is made of three replicas of the first generation of quarks and leptons.
- Apparently, all the fundamental properties of the muon and the electron are exactly the same.

Difference between electron and muon g-2

Vacuum polarization is the source of the difference between the *g*-factors of the electron and muon. In QED these particles differ only by their masses. Vacuum polarization comes in at 4th order.

The calculation is given also in PSF, vol. 2 (1973), although of course it was known much earlier [Suura and Wichmann PR **105,** 1930 (1957); Petermann, PR **105,** 1931 (1957)]:

$$(g_\mu - g_e)/2 = \alpha^2/(3\pi^2)[\ln(\mu/m) - 25/12 + 3\pi^2/4(m/\mu)]$$

$$= 0.590 \times 10^{-5}$$

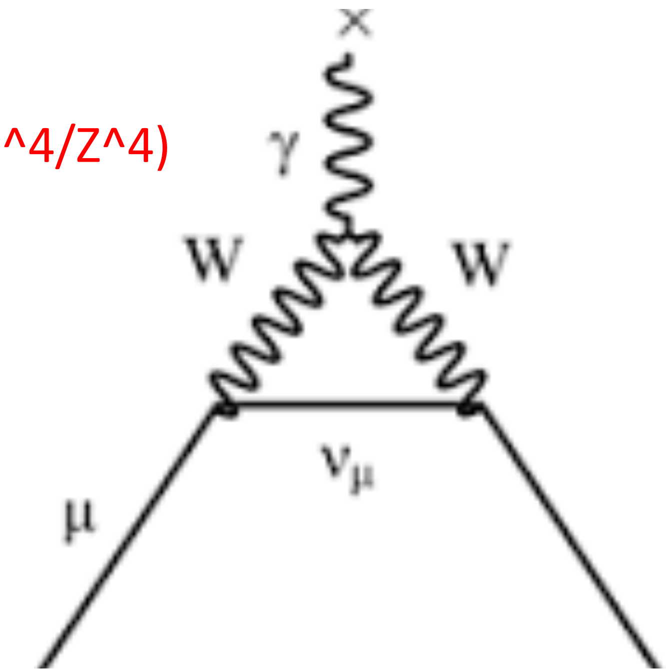
where m and μ denote the masses of the electron and muon.

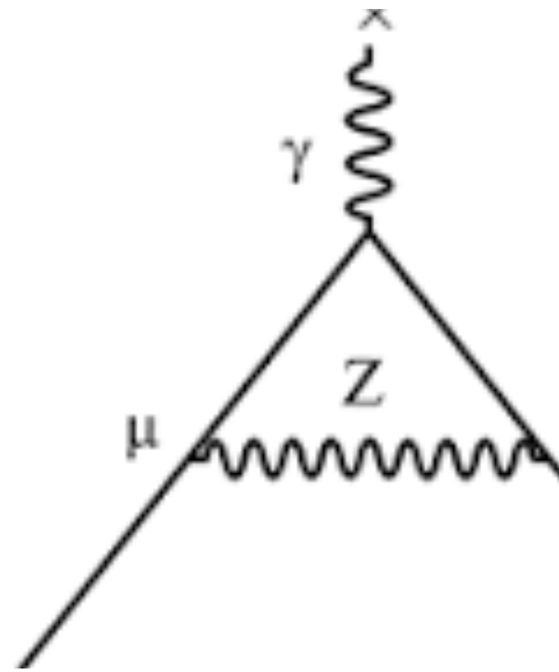
This accounts for 90% of the difference….

Weak corrections to muon magnetic moment

-1)=G μ^2/(8 π^2√2)
10/3 +(4/3)(1-6 W^2/Z^2+4 W^4/Z^4)

/, Z = masses of corresponding
icles



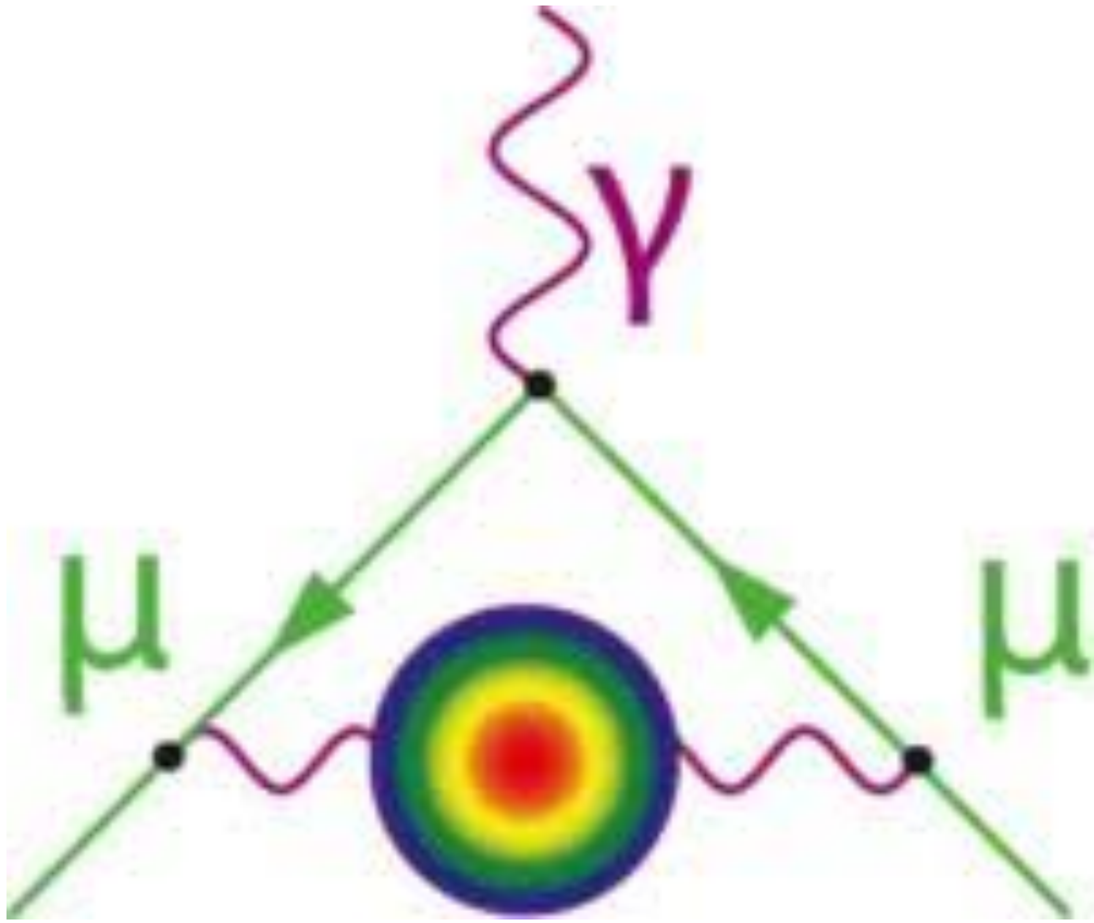


nese were computed as soon as the electroweak synthesis was established: (Schwinger)-Glashow-Weinberg-S

1972: Jackiw & Weinberg; Bars & Yoshimura; Bardeen, Gasrmans &Lautrup; Fujikawa, Lee & Sanda

1974: Tsai, DeRaad, Milton recalculated using Schwinger's source theory mass operator approach

Because of its higher mass, VP brings in important hadronic corrections for the muon *g-2*



Electron and muon anomalous magnetic moments

According to PDG:

$$g_e/2-1 = 1.15965218091\ (26) \times 10^{-3}$$

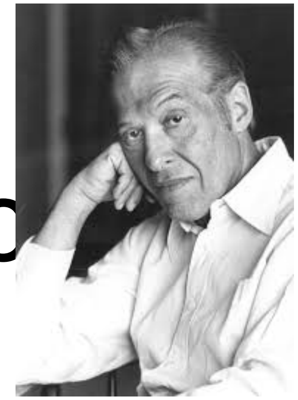
$$g_\mu/2-1 = 1.1659209\ (6) \times 10^{-3}$$

The former constitutes the most striking confirmation of QED. For some years after its precision measurement by Gabrielse et al., it could not even be tested against theory, since α was not known precisely enough. Now g_e is the most accurate prediction in history, since α can be measured using matter-wave interferometry (Cs-133) [Parker et al., Science, 2018]:

$$1/\alpha = 137.035999046\ (27)$$

The anomalous magnetic moment of the muon is in some tension with theory, however, which is the subject of this conference.

Julian Schwinger's legac



Schwinger was first to calculate the correction to Dirac's value of the magnetic moment of the electron.

Together with all the higher corrections which have been heroically calculated through the years, and equally brilliant experimental work we now have in QED the most magnificent theory the world has ever known.

Neither Feynman nor Schwinger would have thought QED (now the cornerstone of the standard model) would have reigned uncorrected two decades into the 21st century.

But maybe the muon is hinting at something beyond?

Further Historical Reading

Silvan S. Schweber, *QED and the Men Who Made It: Dyson, Feynman, Schwinger, and Tomonaga*, Princeton University Press, 1994.

Jagdesh Mehra and Kimball A. Milton, *Climbing the Mountain: The Scientific Biography of Julian Schwinger,* Oxford University Press, 2000.