Windings of twisted strings

QCD meets Gravity, Bhaumik Institute, UCLA

12/12/2017

Piotr Tourkine
CERN, Geneva

- Ongoing work, based on

[arXiv:1606.05636] JHEP **1611** (2016) 036
On the null origin of the ambitwistor string
E. Casali, P. Tourkine

[arXiv:1710.01241]
Winding modes of tensionful ambitwistor strings
E. Casali, P. Tourkine

- In collaboration with Eduardo Casali, Mathematical Institute, Oxford

Outline

- Review & conundrums
- Twisted strings
- Results : winding sector
  - Spectrum
  - Amplitudes

The scattering equations

$$\mathrm{Res} P^2\big|_{z_i} = \sum_{i,j} \frac{k_i \cdot k_j}{z_i - z_j} = 0$$

- Solve for z_i in terms of $k_i \,.\, k_j$
- SL(2,C) invariance
- $(n-3)!$ solutions

Cachazo-He-Yuan formulae (2013)

$$\text{tree-level amplitude} = \int \prod_{i=1}^{n} dz_i \, \bar{\delta}(\text{sc. eqns.}) \, F(k_i, \epsilon_j, z_i)$$

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scattering data:
kinematics, polarisations,
colour structure, …

Cachazo-He-Yuan formulae (2013)

$$\text{tree-level amplitude} = \int \prod_{i=1}^{n} dz_i \, \bar{\delta}(\text{sc. eqns.}) \, F(k_i, \epsilon_j, z_i)$$

$$\sum_{\text{solutions}} \left(\frac{F}{\text{Jac}} \right) = \text{n-point field theory amplitude}$$

massless theories mostly

any dimension

example of how formula leads to understand something new about field theory:

⇨ there is a string that produces them; the ambitwistor string

Ambitwistor string

Mason, Skinner 2013

- A chiral string in ambitwistor space

$$S_{\mathbb{A}} = \int_{\Sigma} P \cdot \bar{\partial} X - \frac{e}{2} P^2$$

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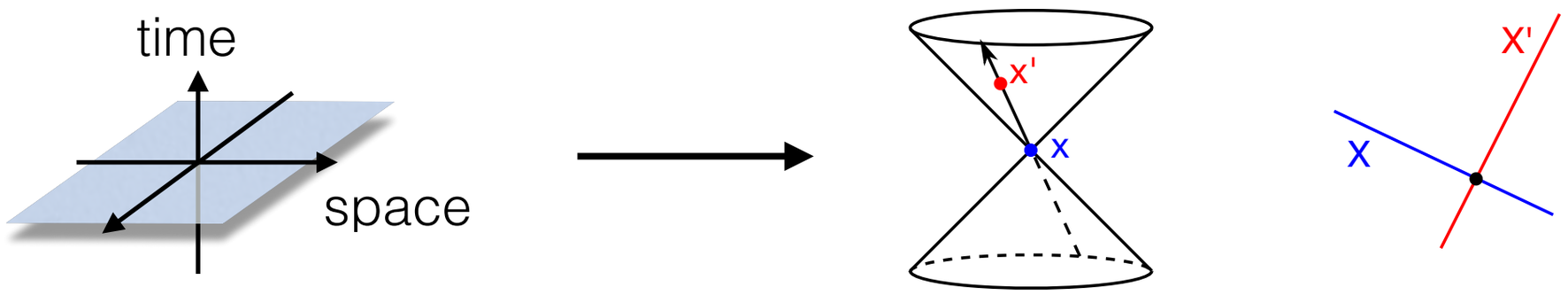

ambitwistor space = space of null geodesics of complexified Minkowski spacetime

Ambitwistor string

Mason, Skinner 2013

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$$S_{\mathbb{A}} = \int_{\Sigma} P \cdot \bar{\partial} X - \frac{e}{2} P^2$$

- Spectrum: type II supergravity

NO STRINGY MODES

$\alpha' = 0$ of string theory.

Ambitwistor string

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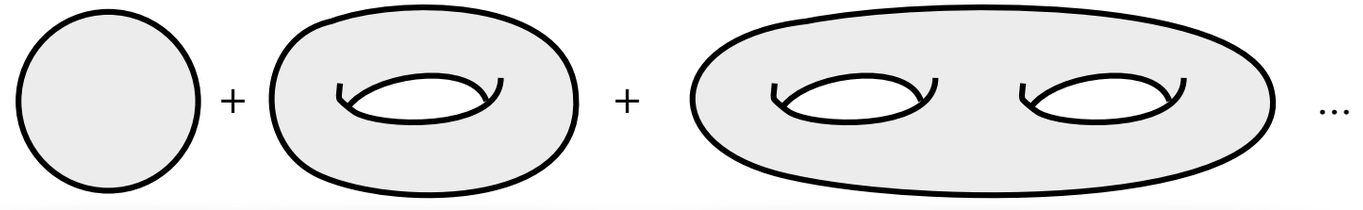
NO STRINGY MODES

$\alpha' = 0$ of string theory.

- Extension to loops possible



Adamo, Casali, Skinner '14, Ohmori '15

Paradox : high vs low energy ?

- Scattering equations are also the saddle point of strings scattering at high energy

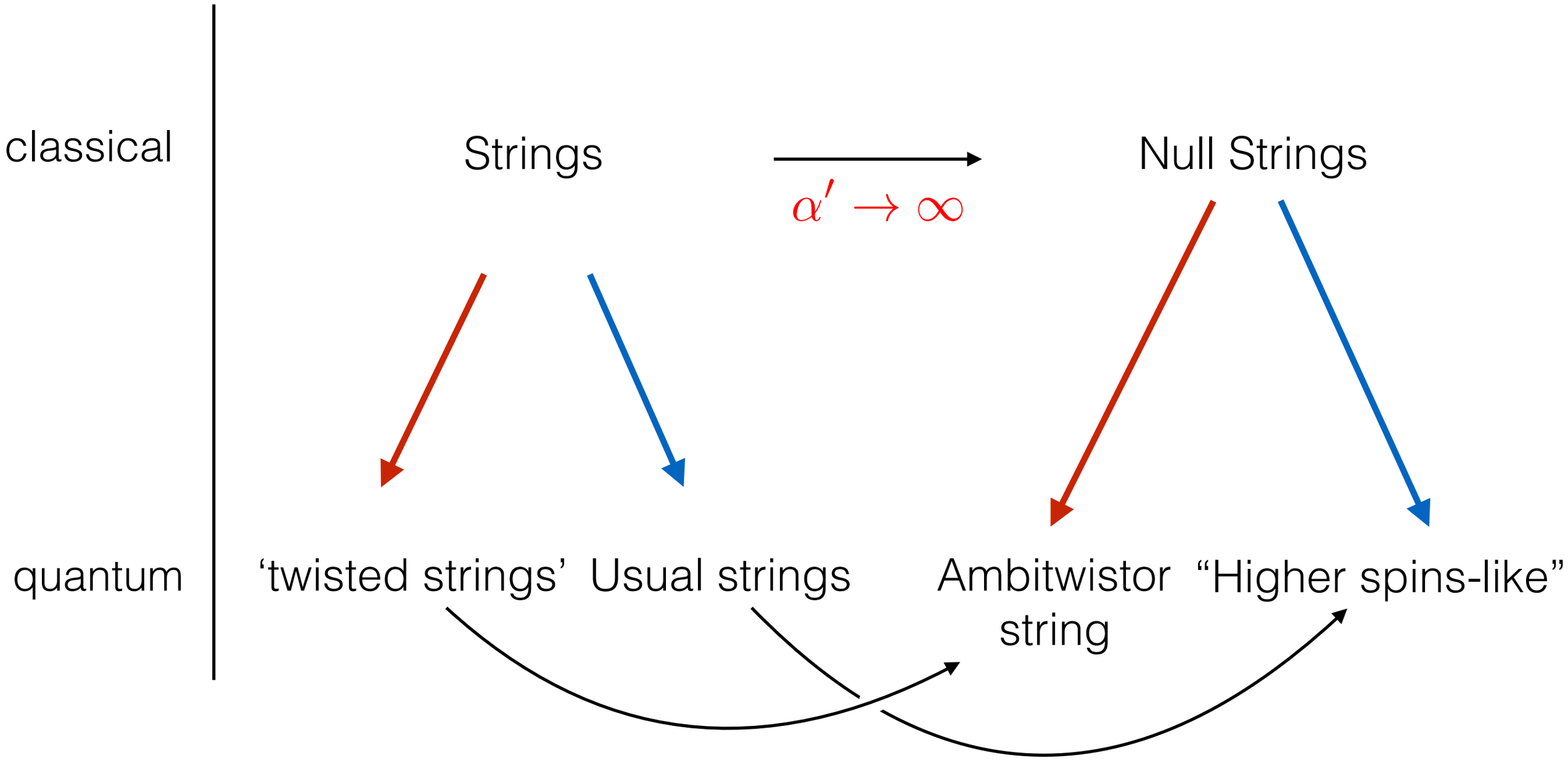
- But CHY formulae = field theory = low energy of string theory

- Solution is a subtle point, linked to a *quantization ambiguity*

Gamboa Ramirez Ruiz-Altaba 1989

Siegel Huang Leite Li Yuan 2015,2016,2017;
Casali, Herfray, Tourkine ; 2016, 2017;
Lee, Rey, Rosabal 2017

global picture



quantum

Strings

$\alpha' \to \infty$

Null Strings

'twisted strings'

Usual strings

Ambitwistor string

"Higher spins-like"

spectrum

m^2

$4/\alpha'$

0

$-4/\alpha'$

quantum

Strings

$\alpha' \to \infty$

Null Strings

'twisted strings'

Usual strings

Ambitwistor string

"Higher spins-like"

spectrum

m^2

0

same amplitudes

first order actions

$$S_H = \int P \cdot \partial_t X - L$$

string theory:

$$S_H = \int P \cdot \partial_t X - \frac{e}{2}(P^2 + \frac{1}{\alpha'^2}(\partial_t X)^2) - \lambda P \cdot \partial_\sigma X$$

ambitwistor string theory:

$$S_{\mathbb{A}} = \int P \cdot \bar{\partial} X - \frac{e}{2} P^2 - \lambda P \cdot \partial X$$

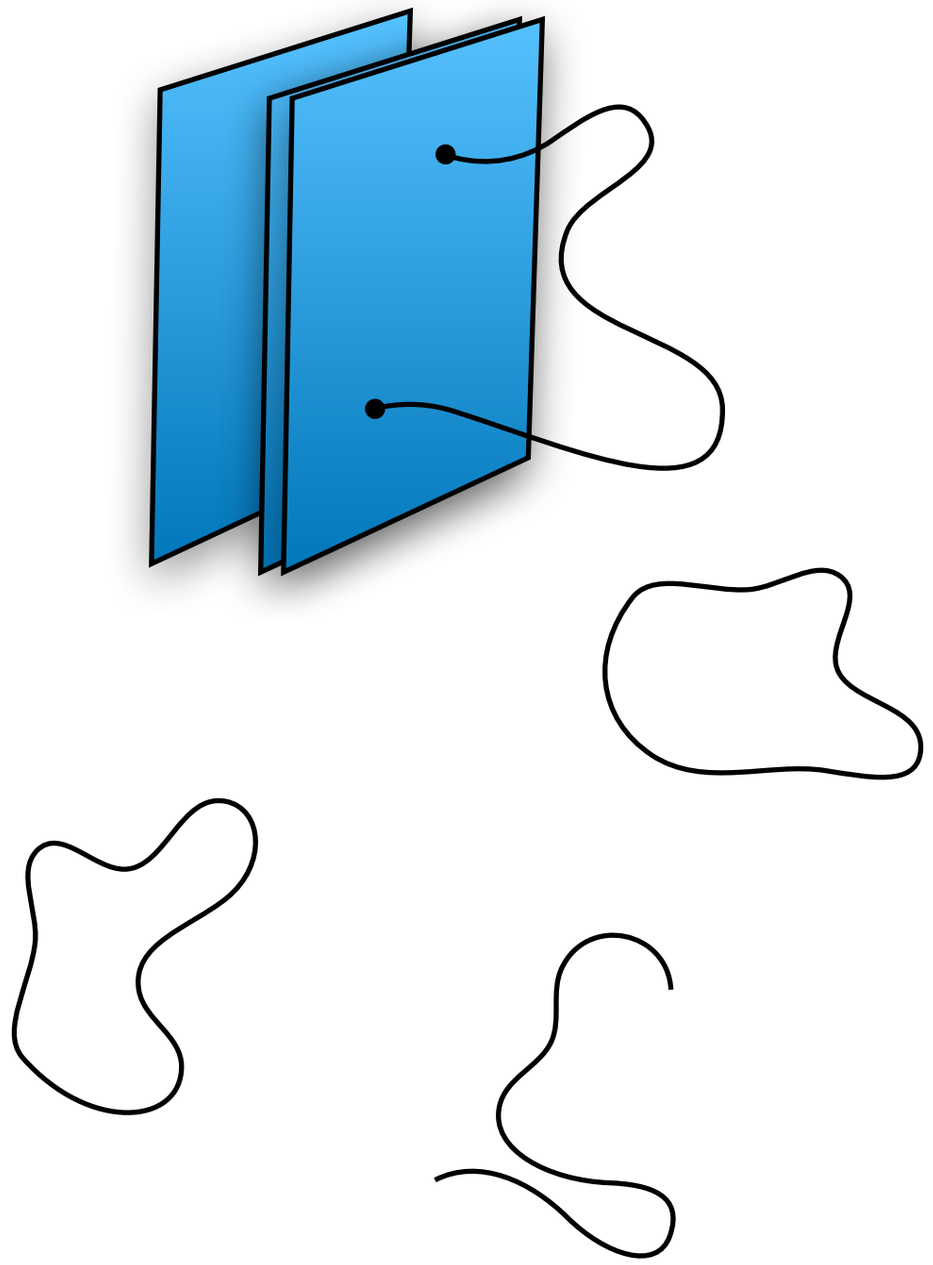
conundrum #2: modular invariance

- discrete symmetry that guarantees UV finiteness of string theory
- conjectured to exist for ambitwistor string
- but supergravity not finite for d>4

Branes ? Open strings ?

- Traditionally open strings carry gauge degrees of freedom. Give a way to implement fully consistent gauge interactions.

- Also, D-branes ? Non-pertubative solutions in supergravity, how can they be described in this context ?



twisted strings

twisted strings

Goal:

first explain how these string theories can be truncated to massless states **with no limit to be taken**,

then we'll see that the winding sector allows for an oscillator spectrum

Literature

[hep-th/9804003] J.Math.Phys. **40** (1999) 4639-4657
A general BRST approach to string theories with ζ function regularizations
S. Hwang, R. Marnelius, P. Saltsidis

[arXiv:1512.02569]
Amplitudes for left-handed strings
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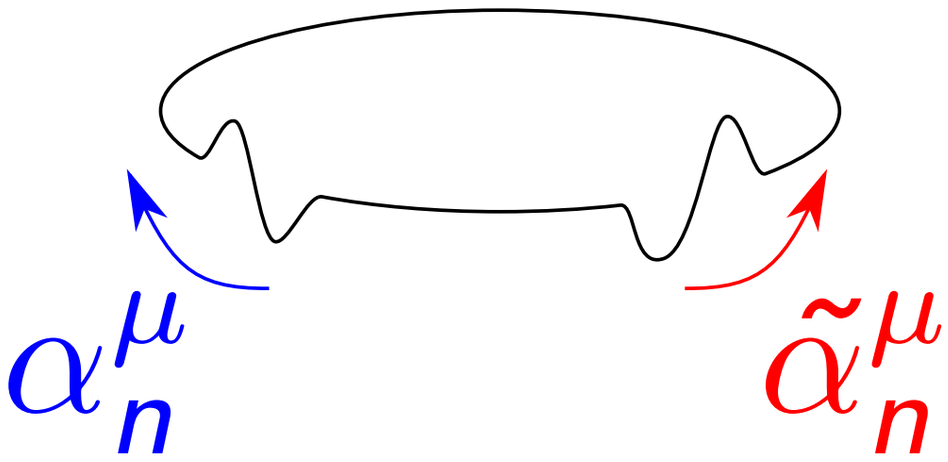
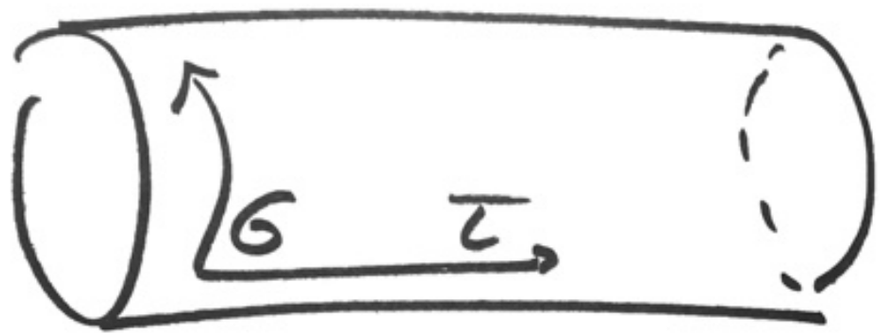
Eqs of motion : left- and right-movers





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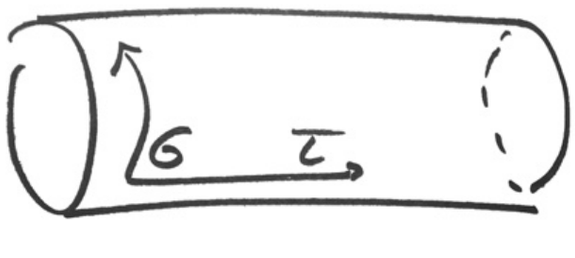

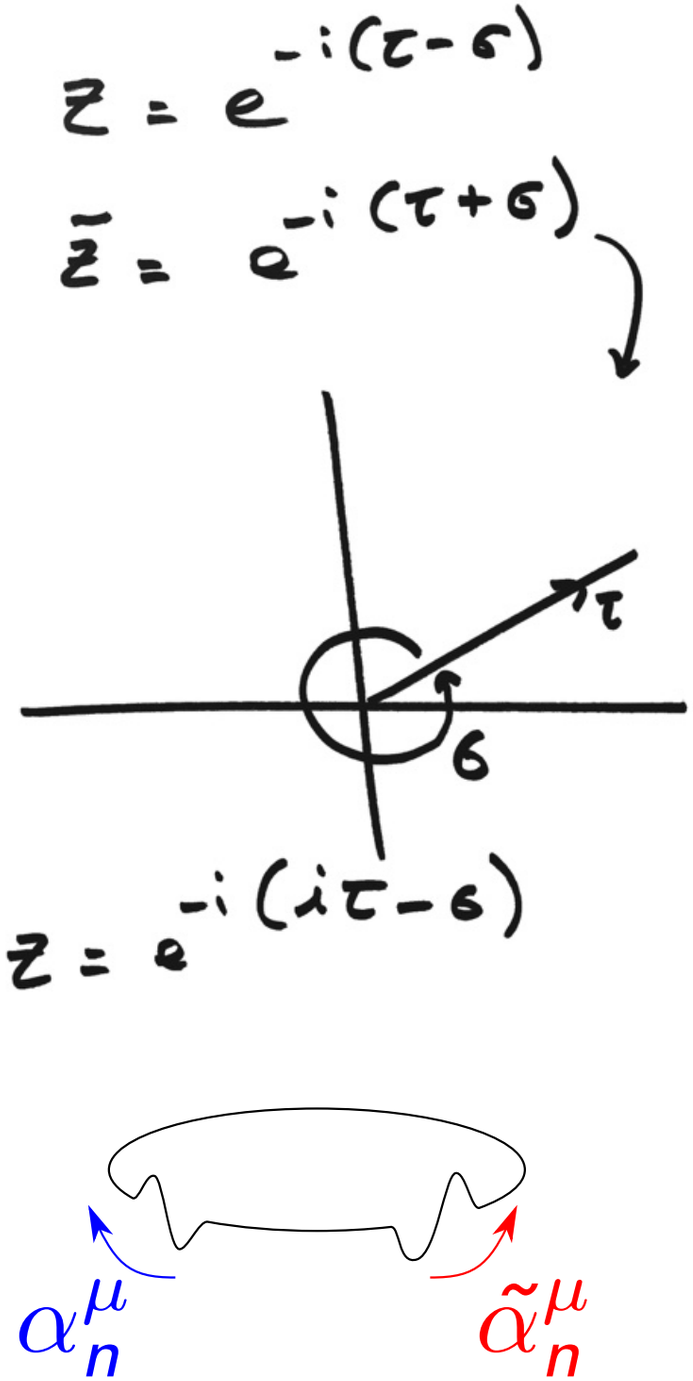

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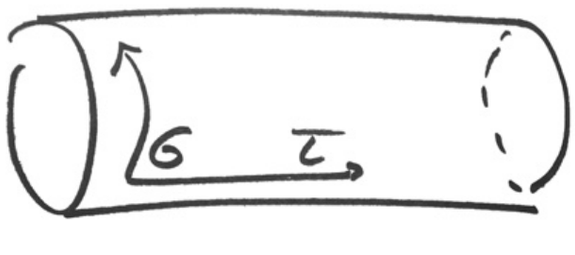

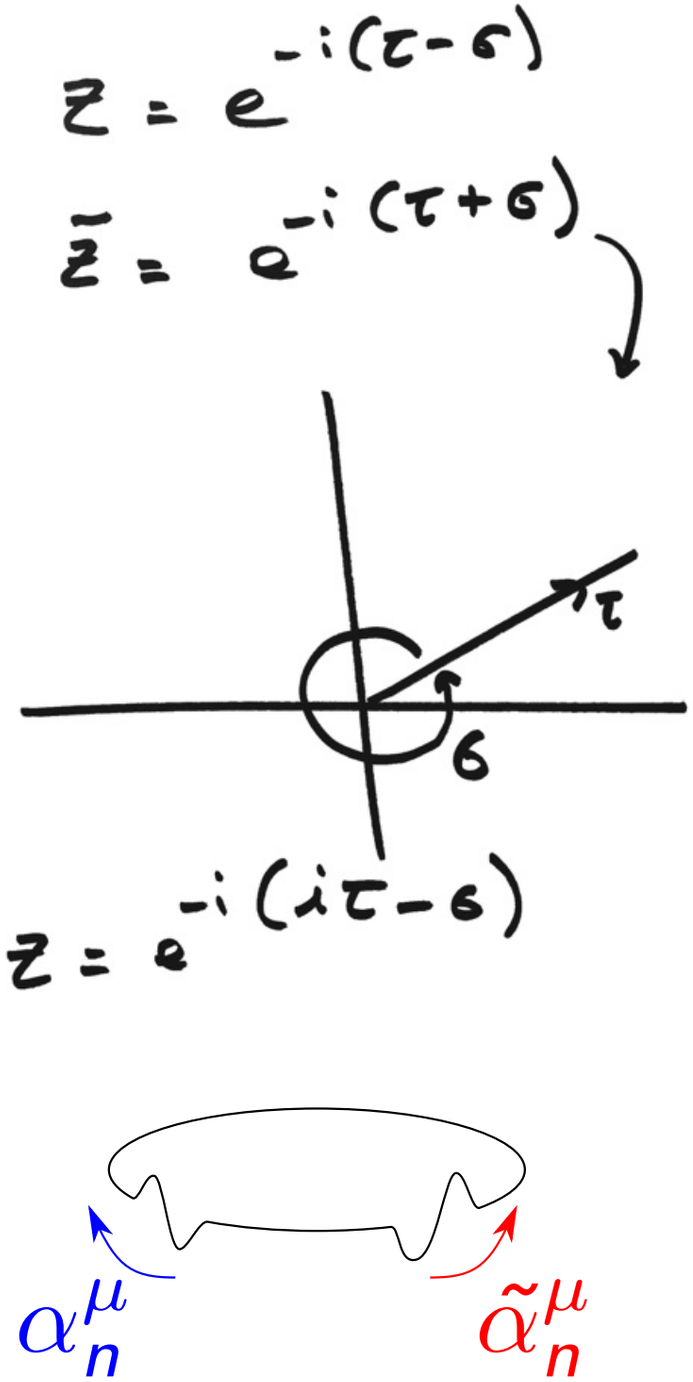

$$X \sim \sum_n \left(\frac{\alpha_n}{z^n} + \frac{\tilde{\alpha}_n}{\bar{z}^n} \right)$$

Quantization, quantizations ?

traditional strings:

$$\alpha_n|0\rangle = 0, \quad \tilde{\alpha}_n|0\rangle = 0, \quad n > 0$$

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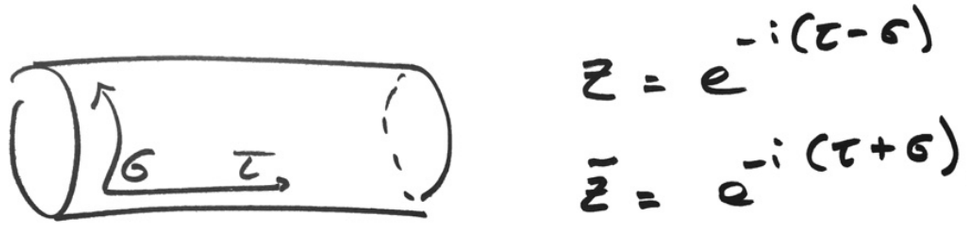

other option:

$$\alpha_n|0\rangle_{\mathbb{A}} = 0, \quad \tilde{\alpha}_{-n}|0\rangle_{\mathbb{A}} = 0, \quad n > 0$$

Exchange between space and time on worldsheet ?

Siegel 2015

Quantization, quantizations ?

$$\alpha_n|0\rangle = 0, \quad \tilde{\alpha}_n|0\rangle = 0, \quad n > 0 \longrightarrow \alpha_n|0\rangle_{\mathbb{A}} = 0, \quad \tilde{\alpha}_{-n}|0\rangle_{\mathbb{A}} = 0, \quad n > 0$$

$$: \alpha_n^\mu \alpha_{-m}^\nu := \alpha_{-m}^\nu \alpha_n^\mu \longrightarrow \text{unchanged}$$

$$: \bar{\alpha}_n^\mu \bar{\alpha}_{-m}^\nu := \bar{\alpha}_{-m}^\nu \bar{\alpha}_n^\mu \longrightarrow : \bar{\alpha}_n^\mu \bar{\alpha}_{-m}^\nu := \bar{\alpha}_n^\mu \bar{\alpha}_{-m}^\nu$$

but we keep the usual commutation relations

$$[\alpha_n^\mu, \alpha_m^\nu] = [\tilde{\alpha}_n^\mu, \tilde{\alpha}_m^\nu] = n\delta_{n+m}\eta^{\mu\nu}$$

Spectrum

$$\langle phys' | T_{\alpha\beta} | phys \rangle = 0$$

Spectrum $\langle \text{phys}|\, T_{\alpha\beta} |\text{phys}'\rangle = 0$

"Old canonical quantization", works as well for light-cone and BRST

$$T = T(z) + \bar{T}(\bar{z}) = \partial X \partial X + \bar{\partial} X \bar{\partial} X \qquad L_n = \frac{1}{2i\pi} \oint T(z) z^{n+1} dz$$

$$L_0 = \frac{1}{2} \sum_{m \in \mathbb{Z}} : \alpha_{-m} \alpha_m :$$

same for barred version

$$L_n |\text{phys}\rangle = 0 \qquad \bar{L}_n |\text{phys}\rangle = 0$$

$$(L_0 - 1)|\text{phys}\rangle = 0 \qquad (\bar{L}_0 - 1)|\text{phys}\rangle = 0$$

Spectrum

$$L_0 = \frac{1}{2}\alpha_0^2 + \sum_{m>0} \alpha_{-m}\alpha_m \qquad \bar{L}_0 = \frac{1}{2}\bar{\alpha}_0^2 + \sum_{m>0} \bar{\alpha}_{-m}\bar{\alpha}_m$$

Number operator

Constraints:

$$L_0 - \bar{L}_0 = 0 \qquad L_0 + \bar{L}_0 - 2 = 0$$

Spectrum

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Number operator

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Use:

$$\alpha_0^\mu = \sqrt{\frac{\alpha'}{2}}\, p_L^\mu$$

$$\bar{\alpha}_0^\mu = \sqrt{\frac{\alpha'}{2}}\, p_R^\mu$$

$$p_L = p_R = p \qquad p^2 = -m^2$$

Spectrum

$$L_0 = \frac{1}{2}\alpha_0^2 + \sum_{m>0} \alpha_{-m}\alpha_m \qquad \bar{L}_0 = \frac{1}{2}\bar{\alpha}_0^2 + \sum_{m>0} \bar{\alpha}_{-m}\bar{\alpha}_m$$

Number operator

Constraints: $N - \bar{N} = 0, \qquad N + \bar{N} - 2 - \frac{\alpha' m^2}{2} = 0$

$$\alpha_0^\mu = \sqrt{\frac{\alpha'}{2}}\, p_L^\mu$$

$$\bar{\alpha}_0^\mu = \sqrt{\frac{\alpha'}{2}}\, p_R^\mu$$

$$p_L = p_R = p \qquad p^2 = -m^2$$

Spectrum

$$\bar{L}_0 = \frac{1}{2}\bar{\alpha}_0^2 + \sum_{m>0} \bar{\alpha}_{-m}\bar{\alpha}_m$$

$$\bar{L}_0^{twist} = \frac{1}{2}\bar{\alpha}_0^2 + \sum_{m>0} (\bar{\alpha}_m\bar{\alpha}_{-m} + [\bar{\alpha}_{-m}, \bar{\alpha}_m])$$

Spectrum

$$\bar{L}_0 = \frac{1}{2}\bar{\alpha}_0^2 + \sum_{m>0} \bar{\alpha}_{-m}\bar{\alpha}_m$$

$$\bar{L}_0^{twist} = \frac{1}{2}\bar{\alpha}_0^2 + \sum_{m>0} (\bar{\alpha}_m\bar{\alpha}_{-m} + [\bar{\alpha}_{-m}, \bar{\alpha}_m])$$

=-m.(d-2)

Minus number operator

$$\sum_{m=1}^{\infty} m = \zeta(-1) = -1/12 \quad d=26$$

$$\bar{L}_0^{twist} = \frac{1}{2}\bar{\alpha}_0^2 - \bar{N}^{twist} + 2$$

Twisted Virasoro conditions

Huang Siegel Yuan 2016

$$L_0 - \bar{L}_0 = 0 \qquad L_0 + \bar{L}_0 - 2 = 0$$

twist ↓

$$N + \bar{N}^{twist} - 2 = 0, \qquad N - \bar{N}^{twist} - \frac{\alpha' m^2}{2} = 0$$

N,N^{twist}	1,1	2,0	0,2
state	gravity	massive spin 2's	
mass	0	+4/α'	-4/α'

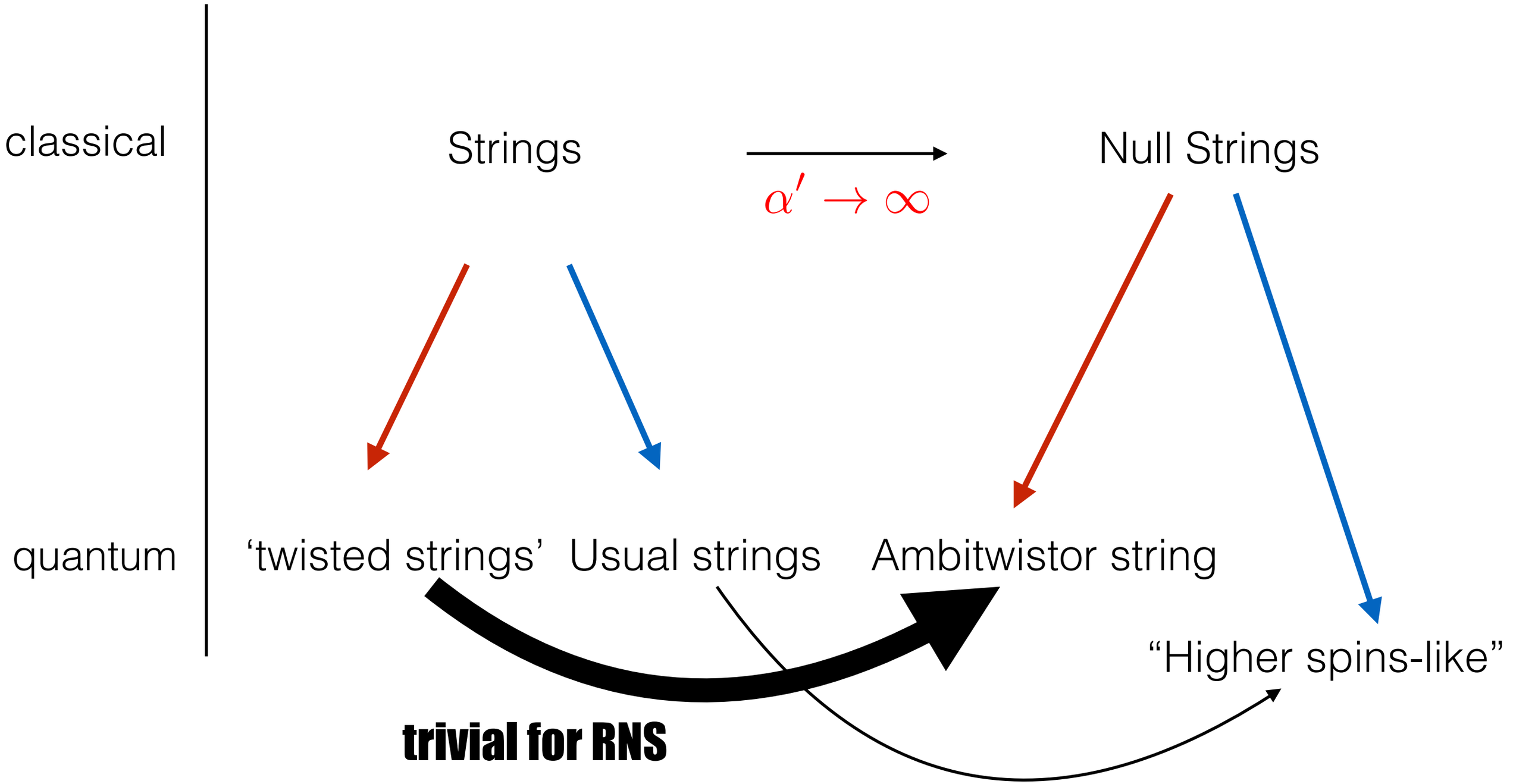
Twisted Virasoro conditions

Huang Siegel Yuan 2016

For RNS model (worldsheet supersymmetric)

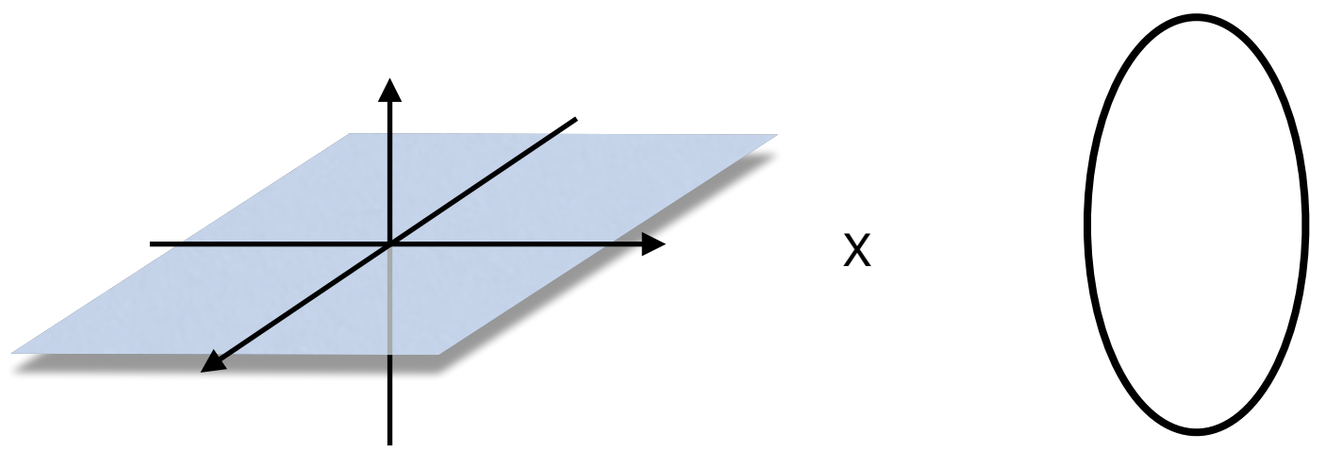
N,N^{twist}	1,1
state	supergravity
mass	0

again


classical
quantum
Strings
$\alpha' \to \infty$
Null Strings
'twisted strings'
Usual strings
Ambitwistor string
"Higher spins-like"
trivial for RNS

results : winding sector has non-trivial excitations

Casali, Tourkine 2017



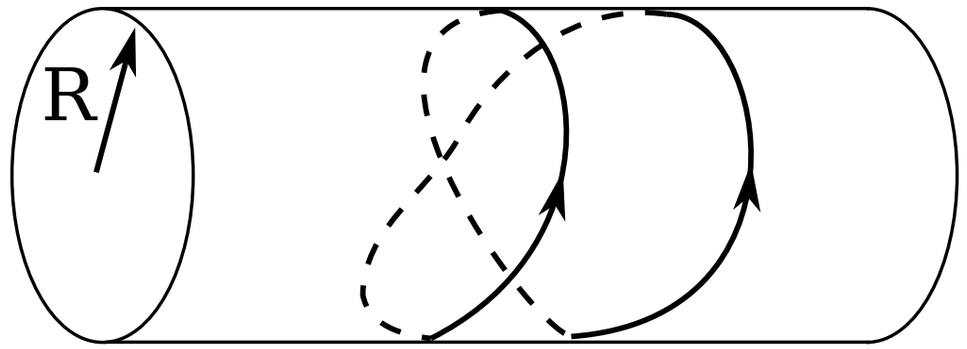
results : winding sector

Casali, Tourkine 2017

- Spectrum

Non trivial space-time topology:

ex.:



w=2

- Amplitudes

$$X^{25}(\sigma + 2\pi) = X^{25}(\sigma) + 2w\pi R$$

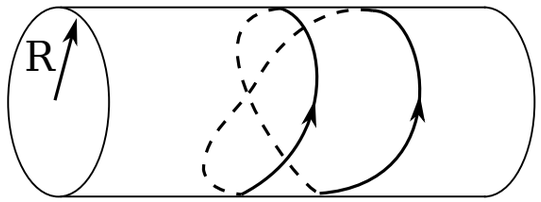
that is something a pointlike object can't do

Compact charges

Kaluza-Klein momentum

Windings

$$X^{25}(\sigma + 2\pi) = X^{25}(\sigma) + 2w\pi R$$



$$p_L^{25} = \left(\frac{n}{R} + \frac{Rw}{\alpha'} \right)$$

$$p_R^{25} = \left(\frac{n}{R} - \frac{Rw}{\alpha'} \right)$$

Spectrum : bosonic model

$$m^2 = \frac{n^2}{R^2} + \frac{R^2 w^2}{\alpha'^2} + \frac{2}{\alpha'}(N - \bar{N})$$

$$N + \bar{N} + nw = 2.$$

Spectrum : bosonic model

$$m^2 = \frac{n^2}{R^2} + \frac{R^2 w^2}{\alpha'^2} + \frac{2}{\alpha'}(N - \bar{N})$$
$$N + \bar{N} + nw = 2.$$

- Many many new solutions as soon for *nw<0*

$$(\tilde{\alpha}_{\bar{N}})^{\bar{N}} (\alpha_{-N})^{N} |n, w, k\rangle$$

- Interacting massive higher-spin theory
- Has ghosts (*cf.* later)
- How many states ? Hagedorn transition ?

T-duality

$$m^2 = \frac{n^2}{R^2} + \frac{R^2 w^2}{\alpha'^2} + \frac{2}{\alpha'}(N - \bar{N})$$
$$N + \bar{N} + nw = 2.$$

Spectrum is invariant under T-duality

$$R \leftrightarrow \alpha'/R$$
$$n \leftrightarrow w$$

T-duality

$$m^2 = \frac{n^2}{R^2} + \frac{R^2 w^2}{\alpha'^2} + \frac{2}{\alpha'}(N - \bar{N})$$
$$N + \bar{N} + nw = 2.$$

Spectrum is invariant under T-duality

$$R \leftrightarrow \alpha'/R$$
$$n \leftrightarrow w$$

At self-dual radius, $R = \sqrt{\alpha'}$
infinitely many massless states arise :

$$0 = (n + w)^2 + 2(N - 2)$$

Families of solutions for N=1, N=2
(i.e. spin 1's, and 'charged' gravitons)

Persists for RNS model

Negative-norm states

see Huang Siegel Yuan 2016; Lee, Rey, Rosabal 2017

Graviton $\alpha^{\mu}_{-1}\tilde{\alpha}^{\nu}_{1}|0,k\rangle$

Massive spin 2 $\tilde{\alpha}^{\mu}_{1}\tilde{\alpha}^{\nu}_{1}|0,k\rangle$

$$||\alpha^{\mu}_{-1}\tilde{\alpha}^{\nu}_{1}|0,k\rangle||^2 = \langle 0,k|\alpha_{1\,\mu}\tilde{\alpha}_{-1\,\nu}\alpha^{\mu}_{-1}\tilde{\alpha}^{\nu}_{1}|0,k\rangle$$
$$= -\langle 0|0\rangle$$

Negative-norm states

see Huang Siegel Yuan 2016; Lee, Rey, Rosabal 2017

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Solution : set $\langle 0|0\rangle < 0$

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$$= -\langle 0|0\rangle$$

Solution : set $\langle 0|0\rangle < 0$

But then

$$||\tilde{\alpha}^{\mu}_{1}\tilde{\alpha}^{\nu}_{1}|0,k\rangle||^2 = \langle 0|0\rangle$$

has negative norm…

Negative-norm states

see Huang Siegel Yuan 2016; Lee, Rey, Rosabal 2017

With windings, you have many more options to create these negative norm states

Schematically: $(\tilde{\alpha}_{\bar{N}})^N (\alpha_{-N})^N |n, w, k\rangle$

Open question : could there be backgrounds on which these states do not arise ?

RNS twisted model

- flat space : d=10 type II supergravity
- toroidal compactification : higher-spins, positive mass only, T-dual background
- negative norm states remain in toroidal compactification

Amplitudes / normal strings

Virasoro — Shapiro formula:

$$\int_{\mathbb{C}} d^2z|z|^{2a}|1-z|^{2b} = \frac{\Gamma(a)\Gamma(b)\Gamma(c)}{\Gamma(a+b)\Gamma(a+c)\Gamma(b+c)}$$

Infinitely many poles in all channels, ***s-m², t-m², u-m²*** with

$$m^2 = 4(N-1)/\alpha'$$

Amplitudes / twisted strings

Flat space

Huang Siegel Yuan 2016, related : Mende 1994

Twisted string amplitudes formula:

$$\int_{\mathbb{C}} d^2z \left(\frac{z}{\bar{z}}\right)^a \left(\frac{1-z}{1-\bar{z}}\right)^b = \frac{(-a)(-b)}{(-1-a-b)(-a-b)(1-a-b)}$$

Amplitudes / twisted strings

Casali, Tourkine 2017

Amplitudes of twisted strings with windings:

$$\int \prod_{i=1}^{n-3} d^2 z_i \prod_{i<j} \left(\frac{z_i - z_j}{\bar{z}_i - \bar{z}_j} \right)^{\frac{1}{2}\alpha' q_i \cdot q_j} |z_i - z_j|^{n_i w_j + n_j w_i}$$

Finitely many poles !

Amplitudes / twisted strings

Casali, Tourkine 2017

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Four-point generic formula:

$$\int d^2 z \left(\frac{z}{\bar{z}} \right)^a |z|^{2n} \left(\frac{1-z}{1-\bar{z}} \right)^b |1-\bar{z}|^{2m} = \frac{(-a-n)^{2n+1}(-b-m)^{2m+1}}{(-1-a-b-n-m)^{2n+2m+3}}$$

where : $(a)^n = \frac{\Gamma(a+n)}{\Gamma(a)}$ and if n is integer $(a)^n = a(a+1)\dots(a+n-1)$

Pochhammer symbol

Amplitudes / twisted strings

Casali, Tourkine 2017

Amplitudes of twisted strings with windings:

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Most importantly, amplitude is a rational function,
so the high energy scattering is power-law suppressed :
typical from field theoretic behaviour

Amplitudes / twisted strings

Casali, Tourkine 2017

Amplitudes of twisted strings with windings:

$$\int \prod_{i=1}^{n-3} d^2 z_i \prod_{i<j} \left(\frac{z_i - z_j}{\bar{z}_i - \bar{z}_j} \right)^{\frac{1}{2}\alpha' q_i \cdot q_j} |z_i - z_j|^{n_i w_j + n_j w_i}$$

Non-convergent and multivalued

Amplitudes / twisted strings

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⚠ Non-convergent and multivalued ⚠

Need a cycle : KLT (Huang Siegel Yuan 2016)
Picard–Lefschetz cycles (Mizera 2017)

One loop partition functions

- Easy to obtain following standard rules
- Unclear what it means, does not converge, may hint to complexification of worldsheet/spacetime

$$Z(\tau,\tilde{\tau}) = \int d^{26}k\, e^{i\pi(\tau-\tilde{\tau})\frac{\alpha' k^2}{2}} \sum_{n,m\in\mathbb{Z}^2} \frac{q^{p_L^2/2}\tilde{q}^{p_R^2/2}}{\eta(q)^{24}\eta^{24}(1/\tilde{q})}$$

Towards CHY for higher spins ?

- What theories ? No-go's about massless higher spins in flat (transverse) space

- We just know the formulae we get make sense in the scattering equation way

$$\int F(z_i) \prod_{i<j} (z_i - z_j)^{n_i w_j + n_j w_i} \bar{\delta}(E_i).$$

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Conclusion

- Described exotic field theories, halfway between strings and conventional field theories
- They exhibit T-duality
- Power-law suppressed high energy behaviour
- *Negative-norm states*