Color-Kinematics Duality at Five Loops

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Plan

- Motivation
- Five-loop construction
- Summary and outlook

Motivations

QCD meets Gravity

Infrared structure in gauge theories

UV (non)renormalizability of gravity theories

we need to go to (non-planar) higher loops

IR divergences

Infrared structure of amplitudes: $\mathcal{A}_m = \mathcal{H}_m \cdot Z$

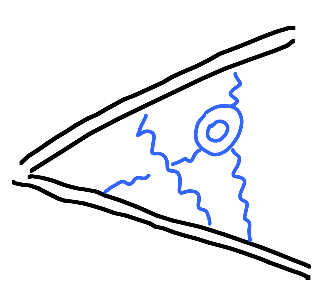
[see e.g. Gardi, Magnea; Becher, Neubert 2009]

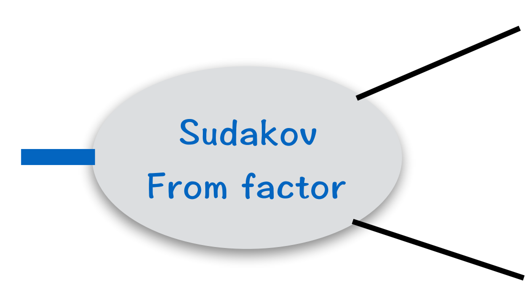
$$Z = \exp\left\{ \int_0^{\mu^2} \frac{d\lambda^2}{\lambda^2} \left[\frac{1}{8} \hat{\gamma}_K \left(\alpha_S(\lambda^2, \epsilon)\right) \sum_{(i,j)} \ln\left(\frac{2 p_i \cdot p_j \, e^{i\pi\lambda_{ij}}}{\lambda^2} \right) \mathbf{T}_i \cdot \mathbf{T}_j - \frac{1}{2} \sum_{i=1}^{m} \gamma_{J_i} \left(\alpha_S(\lambda^2, \epsilon)\right) \right] \right\}$$

Leading IR singularity -> Cusp anomalous dimension

We still know very little about non-planar cusp anomalous dimension:

Leading order -> Four loops





UV divergences

A general belief: gravity theories are non-renormalizable and must be UV divergent at some loop order:

based on power counting

Is $\mathcal{N} = 8$ Supergravity Ultraviolet Finite?

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hep-th/0611086

Conventional wisdom holds that no four-dimensional gravity field theory can be ultraviolet finite. This understanding is based mainly on power counting. Recent studies confirm that one-loop $\mathcal{N} = 8$ supergravity amplitudes satisfy the so-called "no-triangle hypothesis", which states that triangle and bubble integrals cancel from these amplitudes. A consequence of this hypothesis is that for any number of external legs, at one loop $\mathcal{N} = 8$ supergravity and $\mathcal{N} = 4$ super-Yang-Mills have identical superficial degrees of ultraviolet behavior in D dimensions. We describe how the unitarity method allows us to promote these one-loop cancellations to higher loops, suggesting that previous power counts were too conservative. We discuss higher-loop evidence suggesting that $\mathcal{N} = 8$ supergravity has the same degree of divergence as $\mathcal{N} = 4$ super-Yang-Mills theory and is ultraviolet finite in four dimensions. We comment on calculations needed to reinforce this proposal, which are feasible using the unitarity method.

UV bounds for N=8 SUGRA

Finiteness bound via symmetry/duality arguments:

[Bossard, Howe, Stelle; Green, Russo, Vanhove; Green, Bjornsson; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger; Bossard, Hillmann, Nicolai; Ramond, Kallosh; …]

- expected divergence at 5-loop in D=24/5 for N=8
- expected divergence at 7-loop in D=4 for N=8
- expected divergence at 3-loop in D=4 for N=4

ongoing bets

(pictures from Zvi Bern’s talk at MHV30)

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- expected divergence at 5-loop in D=24/5 for N=8
- expected divergence at 7-loop in D=4 for N=8
- expected divergence at 3-loop in D=4 for N=4

Explicit construction via CK duality showed it does not!

[Bern, Davies, Dennen, Huang 2012]

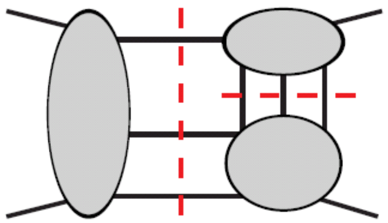
More hidden structure than captured by known symmetries

Loop constructions

We need five-loop amplitudes construction.

We provide a (related) five-loop construction by Sudakov form factor in N=4 SYM.

Tools: color-kinematics duality + unitarity

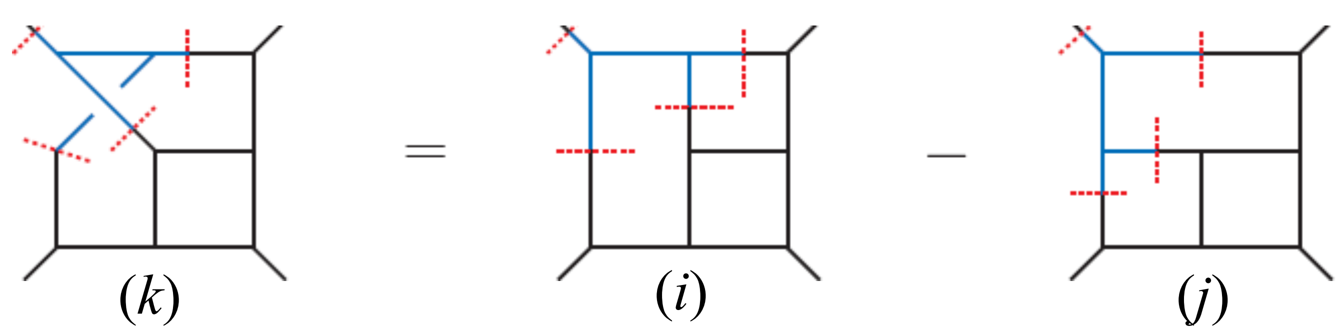


Color-Kinematics duality

Claim (conjecture):

[Bern, Carrasco, Johansson 2008]

We can find a cubic graph representation such that color and kinematics satisfy the same algebraic (Jacobi) equations.



Loop level

$$\mathcal{A}_n^{L\text{-loop}} = \sum_{\Gamma_i} \int \prod_j^L d^D \ell_j \frac{1}{S_i} \frac{c_i\, n_i}{\prod_a D_a}$$

$$c_k = c_i - c_j \quad \longrightarrow \quad n_k = n_i - n_j$$

Jacobi identity

dual Jacobi relation

Power of the CK duality

Obtain non-planar from planar for free!

Obtain gravity from YM for free!

colour factor $\longrightarrow$ kinematic numerator

$\boldsymbol{c_k}$ $\qquad\qquad$ $\boldsymbol{n_k}$

$$\mathcal{A}_4^{\text{tree}}(1,2,3,4) = \frac{c_s\, n_s}{s} + \frac{c_t\, n_t}{t} + \frac{c_u\, n_u}{u} \qquad \mathcal{M}_4^{\text{tree}}(1,2,3,4) = \frac{\tilde{n}_s\, n_s}{s} + \frac{\tilde{n}_t\, n_t}{t} + \frac{\tilde{n}_u\, n_u}{u}$$

Loop constructions

The colour-kinematics duality is still a conjecture at loop level, and relies on explicit construction.

Difficulty: without a underlying principle, a prior it is not guaranteed to work $\longrightarrow$ effort + luck

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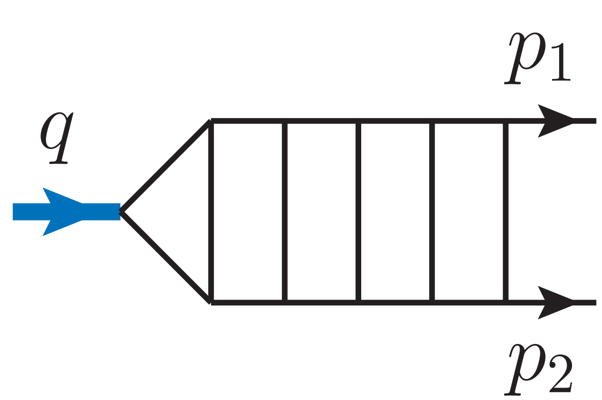
Sudakov form factor in N=4 SYM

Operator in the stress tensor supermultiplet

$\longrightarrow$ UV finite at four dimensions

e.g. $\langle \phi(p_1)\phi(p_2)|\mathrm{Tr}(\phi^2)|0\rangle$

Five-loop graphs: 15 propagators



$$p_1^2 = p_2^2 = 0, \quad q^2 = (p_1 + p_2)^2 \neq 0$$

Strategy

See e.g.
[Bern, Carrasco, Dixon, Johansson, Roiban 2012;
Boels, Kniehl, Tarasov, GY 2012;
Carrasco 2015]

$\boxed{\text{Color-kinematic duality}}$

provides an ansatz of the integrand

$\boxed{\text{Unitarity}}$

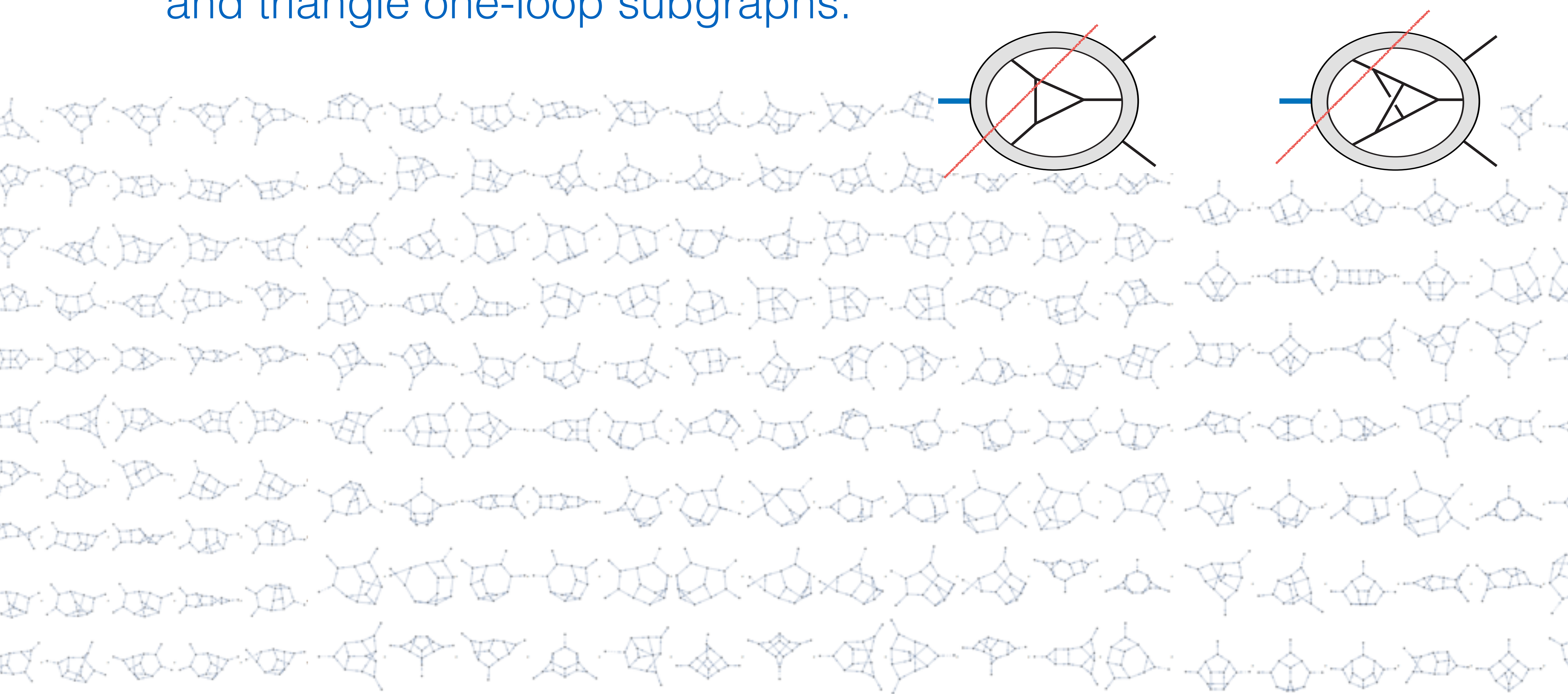
provides physical constraints as well as checks

- A linear algebra problem
- Integrand results in a compact form

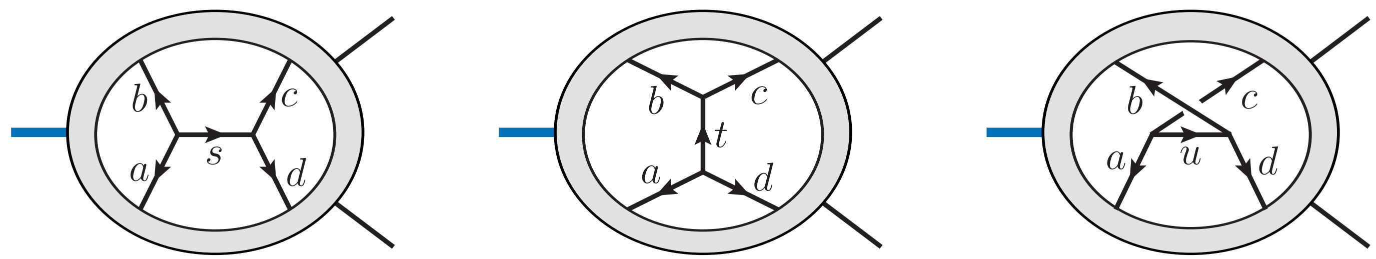
Cubic graphs

There are 306 trivalent topologies to consider.

For N=4 SYM: exclude those containing tadpole, bubble and triangle one-loop subgraphs.



Dual Jacobi relations



$$c_s = c_t + c_u \quad \Rightarrow \quad n_s = n_t + n_u$$

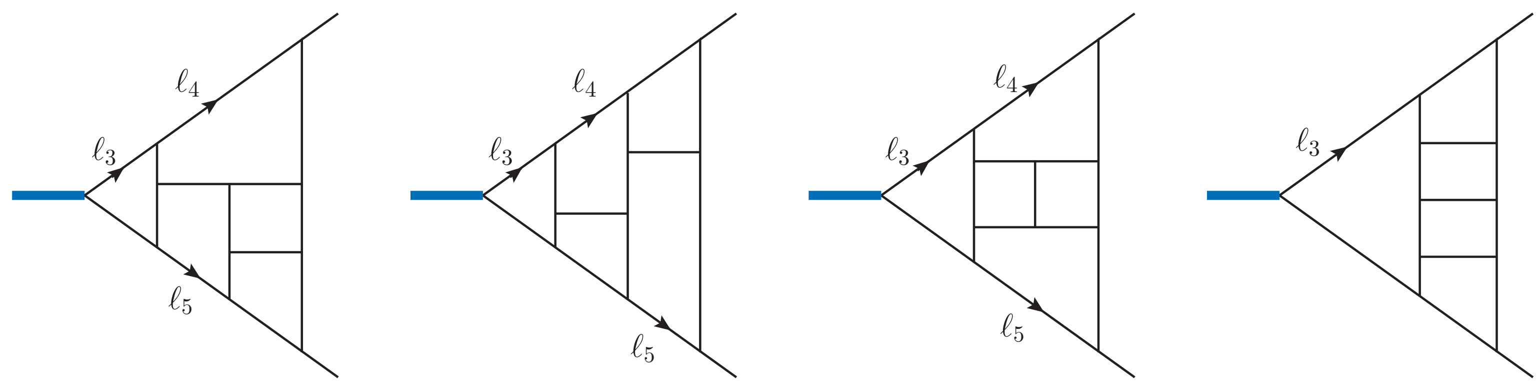
Jacobi identity

dual Jacobi relation

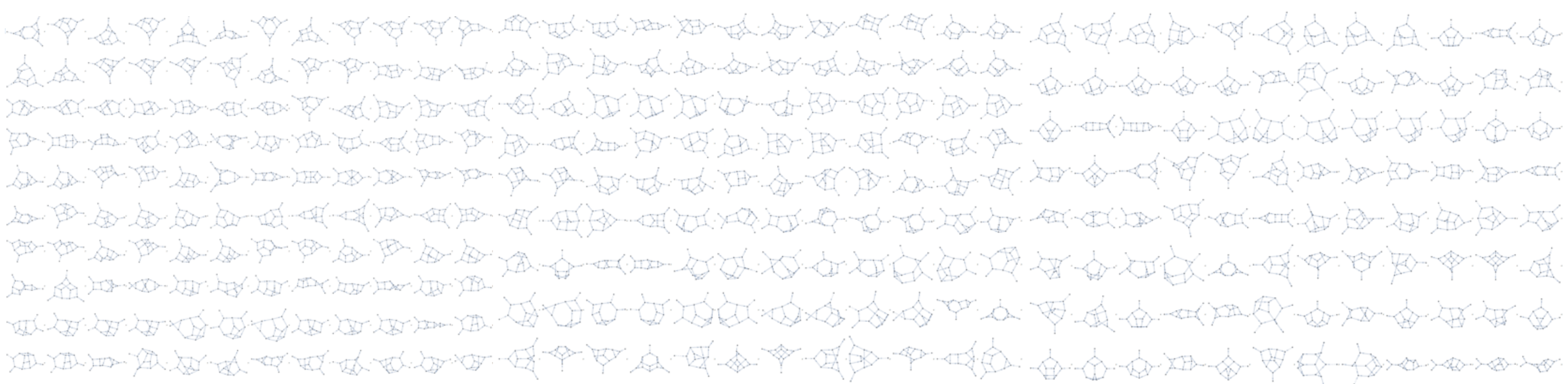
A highly constrained set of equations on the numerators.

Master graphs

Four master graphs obtained via dual Jacobi relation:



All other graphs can be generated from the master graphs by using dual Jacobi relations.



Statistics

TABLE I. Number of cubic and master graphs up to five loops.

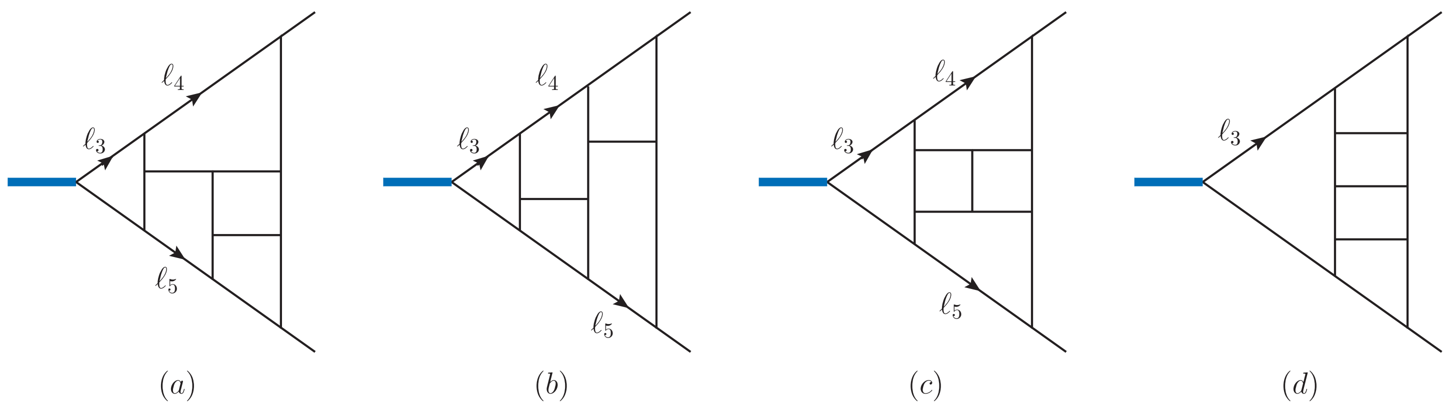
L loops	$L{=}1$	$L{=}2$	$L{=}3$	$L{=}4$	$L{=}5$
# of topologies	1	2	6	34	306
# of planar masters	1	1	1	2	4

Compare to four-point amplitude in N=4 SYM:

L loops	$L{=}1$	$L{=}2$	$L{=}3$	$L{=}4$
# of topologies	1	2	12	85
# of masters	1	1	1	2

Ansatz of master graphs

Power counting property for N=4 SYM:



$$M^{(\mathrm{a})} = \{\tau_{i3}\tau_{j4}\tau_{k5},\ s_{12}\tau_{i3}\tau_{45},\ s_{12}\tau_{i4}\tau_{35},\ s_{12}\tau_{i5}\tau_{34},\ s_{12}\tau_{i3}\tau_{j4},\ s_{12}\tau_{i3}\tau_{j5},\ s_{12}\tau_{i4}\tau_{j5},\ s_{12}^2\tau_{34},\ s_{12}^2\tau_{35},\ s_{12}^2\tau_{45},\ s_{12}^2\tau_{i3},\ s_{12}^2\tau_{i4},\ s_{12}^2\tau_{i5}, s_{12}^3\}$$

$$N^{(a)} = \sum_{j=1}^{36} a_j M_j^{(a)}$$

$\tau_{ij} = 2k_i \cdot \ell_j$ or $2\ell_i \cdot \ell_j$

parameters to fix
(four masters in total 162 parameters)

Using the dual Jacobi equations: $N_s = N_t + N_u$
we get numerators of all other graphs

Full ansatz

$$\mathcal{F}_2^{\text{5-loop}} = s_{12}^2\, F_2^{\text{tree}} \sum_{\sigma_2} \sum_{i=1}^{306} \int \prod_j^L d^D\ell_j \, \frac{1}{S_i} \, \frac{C_i\, N_i}{\prod_{\alpha_i} P_{\alpha_i}^2}$$

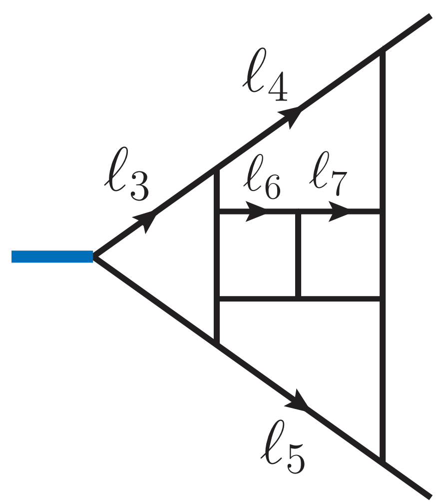
The numerators depend linearly on 162 parameters

We need to fix these parameters:

- Automorphism symmetry
- Unitarity constraints

Automorphism symmetry

Example:



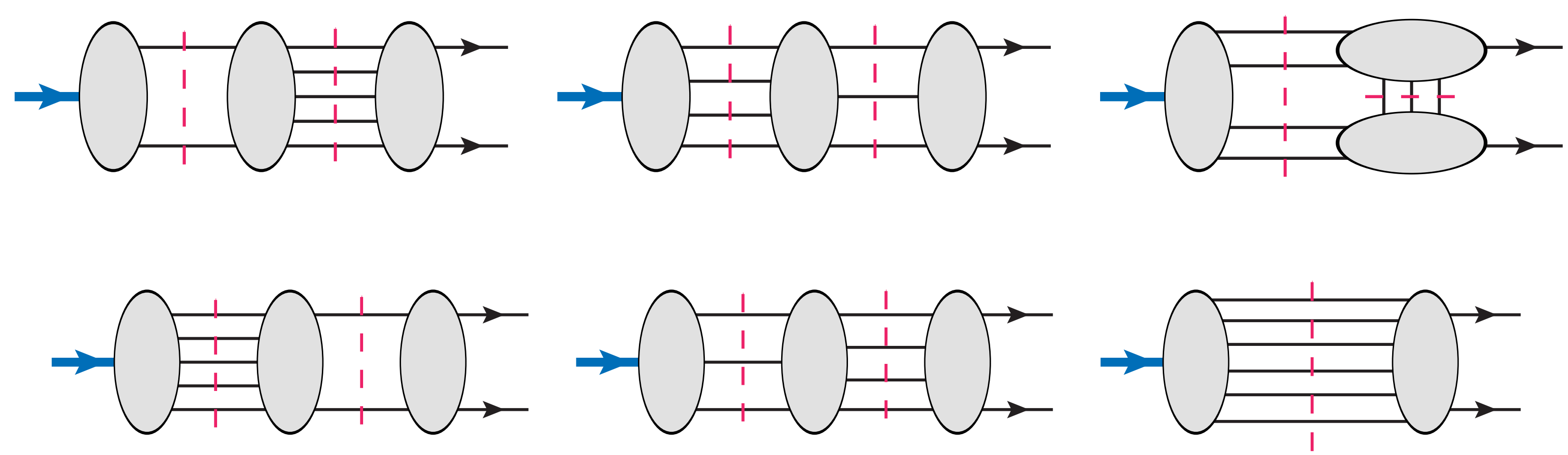
$$\{\ell_3 \to p_1 + p_2 - \ell_3, \quad \ell_4 \leftrightarrow \ell_5,$$
$$\ell_6 \to p_1 + p_2 - \ell_4 - \ell_5 - \ell_6,$$
$$\ell_7 \to p_1 + p_2 - \ell_4 - \ell_5 - \ell_7\}.$$

Fix 115 parameters!

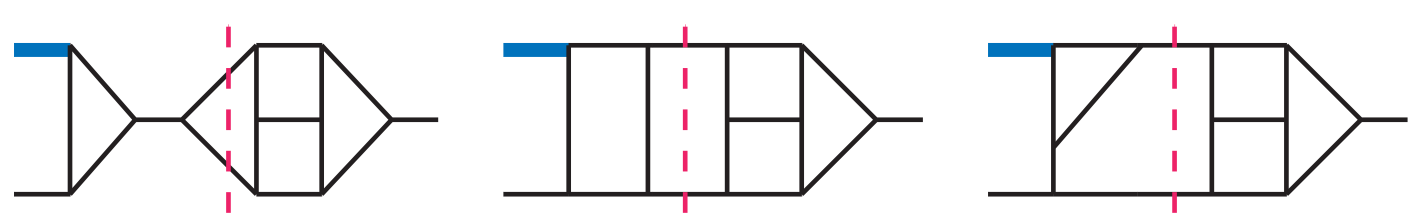
Unitarity checks

$$\text{cut}(\sum \text{cubic graphs}) = \sum_{\text{states}} F^{\text{tree}} \prod_I A_I^{\text{tree}}$$

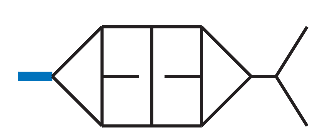
from the ansatz



Subtle checks:



Fixing parameters

Starting point:	162
Automorphism symmetry:	↓ 47
Simple maximal cuts:	↓ 20
All dual Jacobi relations:	↓ 10
Non-trivial cuts:	↓ 3
if no 1/s poles:	↓
	1

Final result

$$\mathcal{F}_2^{5\text{-loop}} = s_{12}^2\, F_2^{\text{tree}} \sum_{\sigma_2} \sum_{i=1}^{306} \int \prod_j^L d^D\ell_j \frac{1}{S_i} \frac{C_i\, N_i}{\prod_{\alpha_i} P_{\alpha_i}^2}$$

Numerators saturate the finiteness bound for N=4 SYM amplitudes:

$$D < 4 + \frac{6}{L}\,, \qquad L > 1$$

Double copy indicate possible UV divergences at D=22/5.

$\boxed{\text{possible enhanced cancellation?}}$

see [Bern, Davies, Dennen 2014]

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Summary

- Realization of the colour-kinematics duality at five loops
- A compact five-loop integrand of Sudakov form factor in N=4 SYM
- Indication to UV property of N=8 SUGRA via double copy

Outlook

- Enhanced cancellation?
- Interpretation for double-copy for form factor?
- Six-loop?
- Generalization to other operators/theories(QCD)?
- What is the underlying principle of the duality?

Thank you for your attention!